Inflation in Traditional Methods

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Purpose

1. Explore the structure of loss triangles

2. Present a technique for making traditional methods inflation sensitive

Session Outline

- Inflation as a CY phenomenon
  - AY vs. CY data
  - Factoring losses
- Changing Basis by Linear Transformation
  - Linear transformation from CY to AY
  - Familiar example
  - Build a method to apply it

Incremental Loss Model

Incremental Paid(AY,age) = X(AY) Y(CY) Z(age)

- 3 variables in a 2 dimensional array
- age = CY – AY
- Choose 2 variables as the basis and the third will fall on a diagonal

Example 1
X(AY) = 1, Y(CY)=1, Z(age)=1

Incremental AY Triangle

	Age			
	0	1	2	3
AY				
1				1
2			1	1
3		1	1	1
4	1	1	1	1
5	1	1	1	
6	1	1		
7	1			

Example 1
X(AY) = 1, Y(CY)=1, Z(age)=1

Incremental AY Triangle

	Age			
AY	0	1	2	3
1				1
2			1	1
3		1	1	1
4	1	1	1	1
5	1	1	1	
6	1	1		
7	1			

Incremental CY Array

	Age			
CY	0	1	2	3
1				
2				
3				
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1

Example 2 – Inflation in CY 7
Y(7)=2

Incremental AY Triangle

	Age			
AY	0	1	2	3
1				1
2			1	1
3		1	1	1
4	1	1	1	2
5	1	1	2	
6	1	2		
7	2			

Incremental CY Array

	Age			
CY	0	1	2	3
1				
2				
3				
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	2	2	2	2

Example 3 – Exposure Growth
X(5), X(6), X(7) = 2

Incremental AY Triangle

	Age			
AY	0	1	2	3
1				1
2			1	1
3		1	1	1
4	1	1	1	1
5	2	2	2	
6	2	2		
7	2			

Incremental CY Array

	Age			
CY	0	1	2	3
1				
2				
3				
4	1	1	1	1
5	2	1	1	1
6	2	2	1	1
7	2	2	2	1

Factorization of Losses

- Claim Count / Severity Method
  - Ultimate Claim Count(AY)
  - Partial Severity(age)
  - Inflation Index(CY)

AY Incremental Partial Severities

Age

AY	12	24	36	48	60	72	84	96	108
1998									61
1999								121	47
2000							136	86	81
2001						231	109	87	148
2002					302	173	78	195	144
2003				541	269	159	168	206	128
2004			791	473	243	202	253	192	12
2005		1,351	808	437	284	300	238	35	87
2006	1,625	1,484	869	487	295	397	90	150	
2007	1,758	1,580	863	607	543	213	225		
2008	1,995	1,596	1,052	831	375	322			
2009	1,936	1,895	1,233	585	495				
2010	2,025	1,957	1,059	753					
2011	2,128	1,928	1,173						
2012	2,140	1,910							
2013	2,071								

CY Incremental Partial Severities

Age

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Averages in Two Directions

CY	Age 12	24	36	48	60	72	84	96	Average	Inflation Index	Y-O-Y Change
2006	1,625	1,351	791	541	302	231	136	121	637	0.83	
2007	1,758	1,484	808	473	269	173	109	86	645	0.84	1%
2008	1,995	1,580	869	437	243	159	78	87	681	0.89	6%
2009	1,936	1,596	863	487	284	202	168	195	716	0.93	5%
2010	2,025	1,895	1,052	607	295	300	253	206	829	1.08	16%
2011	2,128	1,957	1,233	831	543	397	238	192	940	1.22	13%
2012	2,140	1,928	1,059	585	375	213	90	35	803	1.05	-15%
2013	2,071	1,910	1,173	753	495	322	225	150	887	1.16	10%
									767		
Average	1,960	1,713	981	589	351	250	162	134	6,139		
CY Pattern	0.32	0.28	0.16	0.10	0.06	0.04	0.03	0.02			

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Incremental Loss Model

Incremental Paid(AY,age) = X(AY) Y(CY) Z(age)

Cumulative AY Losses

Cum. Paid(AY,t) = $\int_{age=o}^{t} X(AY)Y(CY)Z(age)\,dage$

Cumulative AY Losses

Cum. Paid(AY,t) = $\int_{age=o}^{t} X(AY)Y(CY)Z(age)\,dage$

Cum. Paid(AY,t) = $\int_{CY=AY}^{AY+t} X(AY)Y(CY)Z(CY-AY)\,dCY$

Pull X(AY) Outside the Integral

Cum. Paid(AY,age) = $X(AY)\int_{CY=AY}^{AY+t} Y(CY)Z(CY-AY)\,dCY$

Question

What is the significance of convolution equations?

- Statistical Meaning
  - The sum of two independent random variables
- Algebraic Meaning
  - A linear transformation to or from diagonals

Joint Probability of Two Dice

	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Sum of Two Dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Convolution of Two Dice

Prob(7) = $\sum$ Prob(A) Prob(7-A)

	1	2	3	4	5	6
1						1/36
2					1/36	
3				1/36		
4			1/36			
5		1/36				
6	1/36					

Takeaway:

Convolution maps basis vectors to diagonals

Convolution allows us to change basis from CY to AY

Convolution from CY to AY

		CY Pattern					
		12	24	36	48	60	72
Inflation Rate	Cost Index	0.28	0.25	0.14	0.09	0.05	0.04
6%	1.338						0.054
6%	1.263					0.063	
6%	1.191				0.107		
6%	1.124			0.157			
6%	1.060		0.265				
	1.000	0.280					

Averages in Two Directions

Age

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Convolution from CY to AY

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	1.000	0.280					

Other Factorizations

- Pure Premium Approach
  - Exposure(AY)
  - Partial Pure Premium(age)
  - Pure Premium Index(CY)

- GLM Approach
  - Loss at first report(AY)
  - Incremental ratio to first report(age)
  - Loss cost trend(CY)

Conclusions

- Convolutions map to or from diagonals
- The constant dollar emergence pattern can be obtained as averages from a CY data array.
- Convolution of the incremental constant dollar emergence pattern with the corresponding severity index yields the cumulative AY emergence pattern.

Extensions

- The convolution approach works with incurred losses as well as paid. No log transforms were used.
- GLMs can be viewed as convolution models in which the AY factor is emerged losses at 12 mo.
- "Reserving cycles" reflect the image under convolution of variations in the inflation rate.