Michael R Larsen
Director & Actuary
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Linking Reserve Variability to Variability in Future Accident Year Loss Ratio Forecasts

Agenda

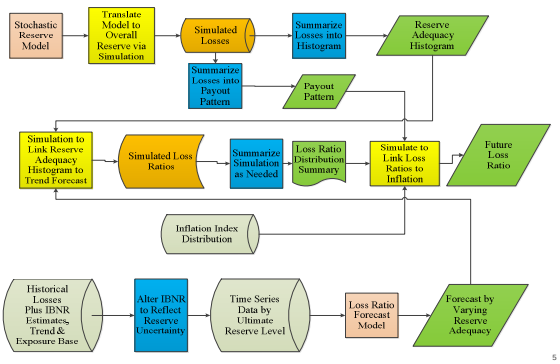
- Goal of session: Provide a framework to attack the problem
- Problem Description
- Solution Outline
- Example Implementing Solution
- Considerations in Practice
- Conclusion

Problem Description

- Client requires a distribution around projected loss ratio
  - Underwriters want reasonable range of results
  - DFA model requires loss ratio distribution
- Sources of uncertainty
  - Trend estimates
  - Reserve Estimates
  - Written activity
  - Inherent volatility in loss ratio distribution
- Forecast loss ratio estimates linked to reserve estimates
  - Recent history not at ultimate
  - Variation in ultimate affects forecast

Process Outline

Solution Flow Chart



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Solution Assumptions

Models available

- Stochastic Reserve Model
  - Isolates Effect of Trend
  - Produces simulated losses
  - Payment pattern model
- Stochastic Loss Ratio Model
  - Captures random variation in losses
- Can program to build simulation linking loss distributions

Data Available

- Historical inflation index with forecast
- Suitable exposure base
- Enough history to build models

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Simulation to Link Reserve Variability to Loss Ratio Variability

- Algorithm similar to Compound Poisson on Exam S or Aggregate Loss Distributions on Exam 4/C
- For each time series fit to a given reserve adequacy level
  - Link the relative reserve level histogram expected counts to the parameters from the time series fit to data developed using those relative reserve level
  - Set the sample size for each set of time series parameters describing a loss ratio forecast using the expected counts from the reserve histogram
  - Set up a Do Loop using that sample size
    - Invert the loss ratio forecast distribution using the time series loss ratio estimate parameters
    - Write the result to a file
    - Repeat until reach sample size limit

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Simulation to Loss Ratio Variability to Trend Variability

- For each pre-inflated simulated loss ratio
  - Simulate a balanced payment pattern to ultimate
  - Multiply loss ratio by the percent split by development year
  - Multiply split losses by cumulative loss cost trend
  - Total split losses for that simulation to obtain trended, simulated loss ratio

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Example

Models Underlying Example

- Stochastic Reserve Model
  - Linear Mixed Model as covered on Modern Actuarial Statistics II
  - See repeated measures examples in West textbook
  - Variation of reserve model in “Best Estimate for Reserves” by Zehnwirth
- Stochastic Loss Ratio Model
  - ARIMA as covered on Modern Actuarial Statistics I or Exam S
  - First degree moving average with differencing
- Data
  - Model on log scale
  - Real data but transformed
    - Rescaled
    - Slight randomization

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Data Transformation Prior to Joining Reserving & Loss Ratio Forecast

Reserving

- Forecast with no inflation for simulation
- Bootstrapped modeled results
  - With Bayesian MCMC simulation already available
- Summarize total reserve distribution to link to time series
- Summarize total reserve distribution to arrive at payment pattern distribution by development year

Loss Ratio Forecast

- De-trend data.
- Adjust IBNR estimate with set number of adjustments
- Label new set of time series to link back to reserve histogram

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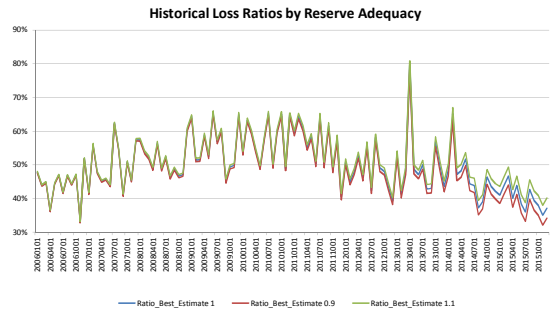
Effect of Reserve Adequacy on Time Series Data Input



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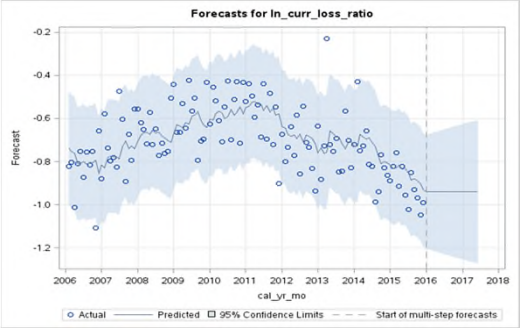
Times Series Model Actual vs. History for Reserve Adequacy 1.0 on Log Scale by Calendar Month



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Time Series Model Results by Relative Reserve Adequacy Group

Forecast Accident Year Loss Ratio on Natural Log Scale		
Reserve Ratio	**Mu**	**Sigma**
0.6	-1.32	0.15
0.7	-1.21	0.13
0.8	-1.11	0.12
0.85	-1.06	0.12
0.9	-1.02	0.11
0.95	-0.98	0.11
1	-0.94	0.10
1.05	-0.90	0.10
1.1	-0.87	0.09
1.15	-0.83	0.09
1.2	-0.80	0.09
1.3	-0.74	0.08
1.4	-0.68	0.08
1.5	-0.63	0.08
1.6	-0.58	0.08
1.7	-0.53	0.08
1.8	-0.49	0.09

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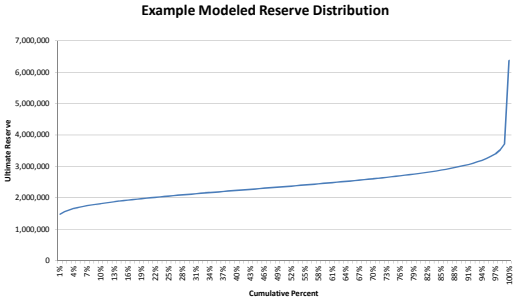
Summarized Results of Example Reserve Model Bootstrapping



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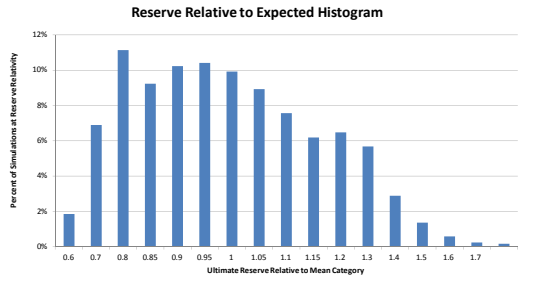
Translating Smooth Curve to Histogram to Link to Time Series Results



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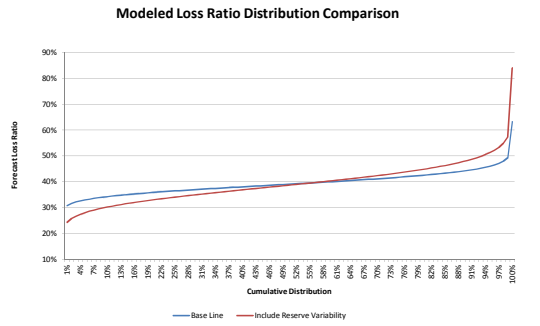
Effect of Including Reserve Variability



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Considerations in Practice

- Need to delay including experience until some aging occurs
  - Similar to classic ratemaking concern
- Capping simulation results
  - Limitation on modeling accuracy
  - Results from inverting parameter estimates at tail may be unstable
- Limit to usable length of time series
  - Underwriting practices could change
  - Need similar variance level across time for ARIMA

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CLOSING SLIDE

Conclusions

- We have tools to build forecast loss distributions
  - Material now covered on exams
  - Software becoming friendlier
- Move from point estimate to distributions
  - Realistic picture of inherent variability
  - Starting point to evaluate actual to expected results
- Lots of moving parts
  - Only covered logical flow to link components
  - Constructing underlying models topic for separate sessions

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