

Strats

In a model-driven business, the models are the business

Models Will Run the World

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Chain-Ladder
First Link:
The Method

- We observe some objects that have changed over time (the circles)
- We observe two new objects (the squares)
- What is an estimate of their changed values?

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Let's play a game

* biased coin idea thanks to <https://zbicki.me/blog/how-to-create-an-unfair-coin-and-prove-it-with-math.html>

Predictions are not certain, prediction bands.

With Excel

Original model:
 $y = bx + \sqrt{x}e$

Equivalent model:
 $y' = bx' + e$

Three stats from equivalent model's data are applied to original model's data

Original data	Equivalent 12-24 month data	Model parameters
1	1	$\mu = 0.792$
2	1	$\sigma^2 = 0.000$
3	1	$\sigma = 0.000$
4	2	$\mu = 0.40795$
5	1	$\sigma^2 = 0.000$
6	1	$\sigma = 0.000$
7	5	$\mu = 0.000$
8	2	$\sigma^2 = 0.000$
9	2	$\sigma = 0.000$
10	2	$\mu = 0.000$

* m, v notation c/o Wikipedia (lognormal)

Chain-Ladder
First Link:
The Model

$y = bx + \sqrt{x}e$ ← error term makes it a model

equivalent model

$y' = bx' + e$
where
 $y' = \frac{y}{\sqrt{x}}$
and
 $x' = \frac{x}{\sqrt{x}}$

Value of b that minimizes

$\sum (y' - bx')^2$ *Calculus*

is

$b = \frac{\sum x' y'}{\sum x'^2} = \frac{\sum \frac{x}{\sqrt{x}} \frac{y}{\sqrt{x}}}{\sum \left(\frac{x}{\sqrt{x}}\right)^2} = \frac{\sum y}{\sum x}$

Chain-Ladder
First Link:
An Example

- Apply the model
 $y = bx + \sqrt{xe}$
to this skinny triangle

	x	y
1	129.28	218.24
2	135.47	255.51
3	94.53	232.66
4	77.33	165.16
5	130.29	296.19
6	9.10	35.77
7	131.50	233.45
8	85.19	114.70
9	85.79	112.39
10	54.03	161.14
11	94.19	169.68
12	190.87	416.01
13	118.53	263.72
14	126.01	244.73
15	62.47	150.62
16	140.85	385.98
17	77.33	
18	131.50	

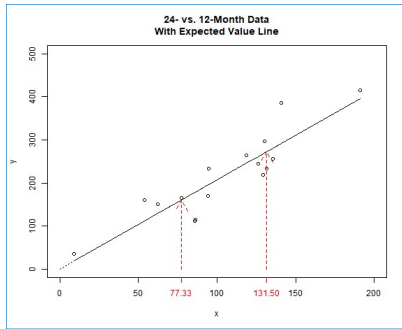
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12-24 Month
Development
Experience

$$b = \frac{\sum y}{\sum x} = 2.074$$

- 2.074 = slope of the line through origin
- prediction of new initial observations:
77.33 -> 160.4
131.5 -> 272.7

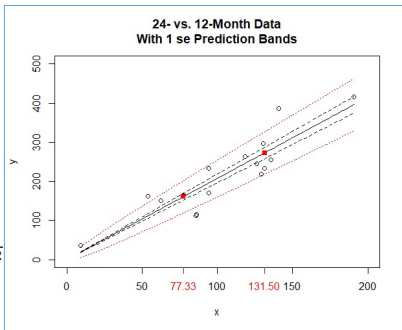


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Predictions are not
certain: prediction
bands

- --- Parameter risk Δ
Variability of estimated mean
- - - - Process risk Γ
Variability around theoretical mean
- Total risk $= \sqrt{\Delta^2 + \Gamma^2}$
Variability of a predicted outcome




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* notation by Ali Majidi

Predictions are not certain: prediction bands

With 

```

ChainLadder::makeChainLadder(ChL, est.sigma = "Mack",
ChainLadder::makeChainLadder(triangle = tri, est.sigma = "Mack",
Latest Dev. To.Date Ultimate ISMR Mack s.e cv(CSNR)
1 218.2 1.000 218.2 0 0.0 NaN
2 255.5 1.000 255.5 0 0.0 NaN
3 232.7 1.000 232.7 0 0.0 NaN
4 185.2 1.000 185.2 0 0.0 NaN
5 296.2 1.000 296.2 0 0.0 NaN
6 35.6 1.000 35.6 0 0.0 NaN
7 233.4 1.000 233.4 0 0.0 NaN
8 114.7 1.000 114.7 0 0.0 NaN
9 112.4 1.000 112.4 0 0.0 NaN
10 161.1 1.000 161.1 0 0.0 NaN
11 169.7 1.000 169.7 0 0.0 NaN
12 416.0 1.000 416.0 0 0.0 NaN
13 261.7 1.000 261.7 0 0.0 NaN
14 244.7 1.000 244.7 0 0.0 NaN
15 150.6 1.000 150.6 0 0.0 NaN
16 386.0 1.000 386.0 0 0.0 NaN
17 77.3 0.482 160.4 83 41.1 0.495
18 131.5 0.482 272.7 141 54.4 0.385
Totals
Latest: 3,664.78
Dev: 0.94
Ultimate: 3,889.04
ISMR: 224.26
Mack.s.e 70.00
cv(CSNR): 0.31

```

* ChainLadder package by Markus Gesmann et al.

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Why does the prediction envelope fan out only at the high end?

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Chain-Ladder link

$$y = bx + \sqrt{xe}$$

hint

- Assumption is, The higher the initial value, the greater the variability of the subsequent value
- When might you have less variability the larger the beginning value?

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How do prediction bands look under different models?

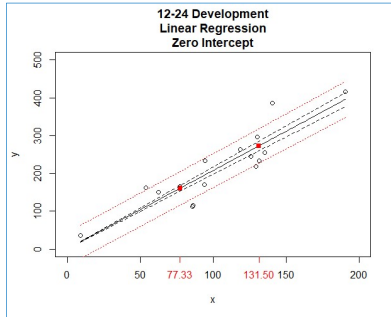
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Prediction bands without square-root-heteroskedasticity

$$y = bx + e$$

- --- parameter risk
- total risk



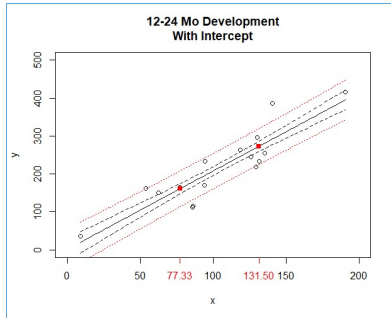
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Prediction bands when there's an intercept

$$y = a + bx + e$$

- --- parameter risk
- total risk



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Homework

1. What would the graph of the model look like if the simple average is the optimal link ratio?
 - Does the answer change if "optimal" is a matter of actuarial judgment?
2. What could be drivers of a non-zero intercept?
3. How to model the BF method within the Chain-Ladder paradigm?
4. How to model the first column within the Chain-Ladder paradigm?
5. Prove that our game satisfies the assumptions of the model

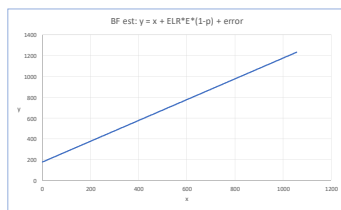
$$y = bx + \sqrt{x}e$$

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Bornhuetter-Ferguson

- What is the slope of the line?
- What is the intercept?



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**Chain-Ladder
Second Link:
Add Another
Column**

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	359.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.53
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.79	112.39	159.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	
14	126.01	244.73	
15	62.47	150.62	
16	140.84	385.98	
17	77.33		
18	131.50		

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**Chain-Ladder
Second Link:
Add Another
Column**

- $b_y = 1.181$
- $\sigma_{b_y} = 0.083$
- $\sigma_{a_y} = 4.1$

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Chain-Ladder predicts the future recursively

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	359.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.53
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.79	112.39	158.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	311.45
14	126.01	244.73	289.03
15	62.47	150.62	177.88
16	140.84	385.98	455.84
17	77.33	160.38	189.41
18	131.50	272.73	322.10
b	2.074	1.181	

- Orange projections are products of a scalar and an estimated parameter **Which is which?**
 - Formulas for Parameter Risk and Process Risk can be found in slides above
- Red projections are products of an estimate and an estimated parameter
 - Formulas for Parameter Risk and Process Risk are derived from the **Law of Total Variance**

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Law of Total Variance

sidebar

- Wikipedia:

$$Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$$
- "In actuarial science, specifically credibility theory, the first component is called the expected value of the process variance (**EVPV**) and the second is called the variance of the hypothetical means (**VHM**)."
 • Retrieved June 25, 2015
- See Majidi and Bardis formula derivations, "A Family of Chain-Ladder Models," *Variance*, Vol 6, Issue 2, pp. 157-158

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Recursive projections with statistics – complete many squares

- Expected Value
- Parameter Risk – Δ

$$\Delta_y^2 = x^2 \cdot \hat{\sigma}_b^2$$

$$\Delta_z^2 = \hat{y}^2 \cdot \hat{\sigma}_b^2 + \hat{b}^2 \cdot \Delta_y^2 + \hat{\sigma}_b^2 \cdot \Delta_y^2$$

- Process Risk – Γ

$$\Gamma_y^2 = x \cdot \hat{\sigma}_x^2$$

$$\Gamma_z^2 = \hat{y} \cdot \hat{\sigma}_y^2 + \hat{b}^2 \cdot \Gamma_y^2$$

Data		
x	y	z
1	xxx	yyy 222
2	xxx	yyy
3	xxx	yyy

Parameter estimates		
b_1	b_1	
b_2	b_2	
σ_b^2	σ_b^2	

Expected Values		
x	y	z
1		
2		$z^A = yyy * b_1$
3		$y^A = xxx * b_1, z^A = y^A * b_1$

Parameter Risk		
x	y	z
1		
2		Δ_z
3		Δ_z

Process Risk		
x	y	z
1		
2		Γ_z
3		Γ_z

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Tricks for Risk Estimates for the Total/sum Row

Expected Values		
x	y	z
1		
2	yyy	$z^A = yyy * b_1$
3	$y^A = xxx * b_1$	$z^A = y^A * b_1$
future sum	Y^A	$Z^A = (yyy + Y^A) * b_1$

Parameter Risk		
x	y	z
1		
2		Δ_z
3	Δ_y	Δ_z
future sum	Δ_y	same formula

Process Risk		
x	y	z
1		
2		Γ_z
3	Γ_y	Γ_z
future sum	Γ_y	$\Gamma_z^2 = \Gamma_y^2 + \Gamma_z^2$

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Why not directly estimate the 12-36 month link ratio?

- What if you learned $b_{xz} = \text{sum}(z) / \text{sum}(x) = 2.337$
- Why not say the expected 36-month value of $x = 77.33$ is

$$77.33 * 2.337 = 180.7 \text{ (se 44.7)}$$

rather than

$$77.33 * 2.074 * 1.181 = 189.4 \text{ (se 72.1 see above)}$$

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MackChainLadder(WCIRB Indemnity + Medical Combined Incurred, tail = 1.025)

Year	Dev/To Date	Ultimate	IBNR	Mack Ldr	CYIBNR
1986	2,527	0,970	2,623	84	10
1987	2,855	0,974	2,953	79	38
1988	3,208	0,974	3,282	84	40
1989	3,633	0,974	3,759	96	43
1990	4,027	0,974	4,147	112	46
1991	4,728	0,973	4,872	133	50
1992	5,820	0,971	5,920	150	43
1993	6,978	0,971	7,170	92	40
1994	8,209	0,970	8,370	99	42
1995	9,360	0,924	9,724	358	1,024
1996	9,797	0,901	10,057	410	1,096
1997	4,483	0,921	4,973	480	1,204
1998	6,655	0,920	6,983	628	1,376
1999	6,305	0,920	6,904	788	1,487
2000	7,240	0,898	8,070	824	1,598
2001	10,614	0,890	11,846	1,182	2,025
2002	10,617	0,895	11,889	1,202	2,038
2003	10,614	0,882	11,224	1,120	1,954
2004	7,823	0,880	8,572	940	1,037
2005	6,470	0,880	7,364	834	1,006
2006	6,591	0,880	7,480	898	1,029
2007	7,020	0,873	8,064	1,027	1,090
2008	7,189	0,864	8,323	1,134	1,030
2009	6,804	0,860	8,064	1,206	1,090
2010	6,925	0,835	8,293	1,266	1,037
2011	6,543	0,815	8,020	1,480	1,097
2012	6,454	0,790	8,175	1,721	1,017
2013	6,209	0,770	8,253	2,044	1,041
2014	6,382	0,730	8,854	2,572	1,709
2015	6,113	0,680	9,476	3,262	2,782
2016	5,382	0,540	8,481	4,010	3,842
2017	6,500	0,370	8,494	4,444	4,750
Year	162,450	0,370	218,762	46,713	60,023

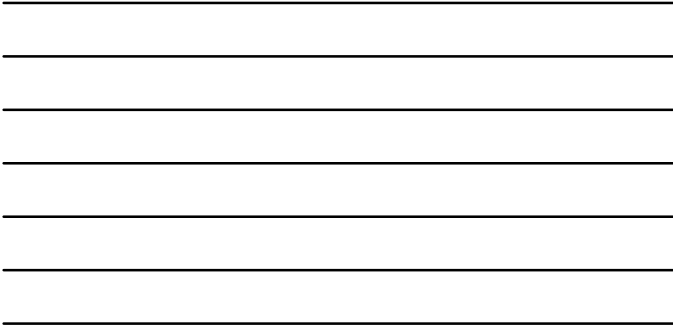
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Reported industry IBNR @ 3/31/2018 = \$36,196 ~58%-ile

Very close to MackChainLadder central estimate = \$36,713

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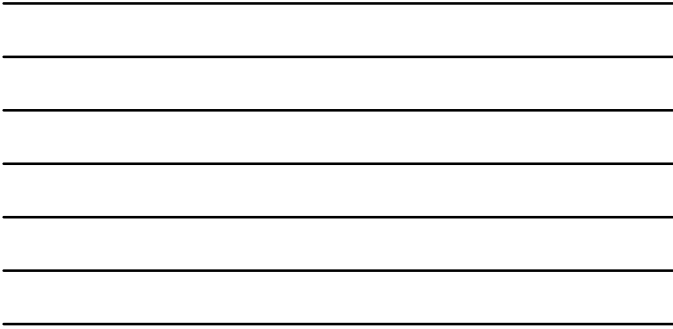
ChainLadder sample GL triangle 'GenIns' (in thousands)

GenIns is a triangle first published in Taylor & Ashe paper (1983) and repeatedly studied in the literature

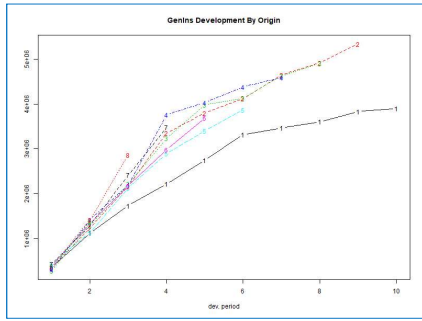
origin	1	2	3	4	5	6	7	8	9	10
1	358	1,125	1,735	2,218	2,746	3,320	3,466	3,606	3,834	3,901
2	352	1,236	2,170	3,253	3,799	4,120	4,648	4,914	5,339	
3	291	1,292	2,219	3,235	3,986	4,133	4,629	4,909		
4	311	1,419	2,195	3,757	4,030	4,382	4,588			
5	443	1,136	2,128	2,898	3,403	3,873				
6	396	1,333	2,181	2,986	3,692					
7	441	1,288	2,420	3,483						
8	359	1,421	2,864							
9	377	1,363								
10	344									

origin	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
1	3,143	1,543	1,278	1,238	1,209	1,044	1,04	1,063	1,018
2	3,511	1,755	1,545	1,133	1,084	1,128	1,057	1,086	
3	4,448	1,717	1,458	1,232	1,037	1,112	1,061		
4	4,568	1,547	1,712	1,073	1,087	1,047			
5	2,564	1,873	1,362	1,174	1,138				
6	3,366	1,636	1,369	1,236					
7	2,923	1,878	1,439						
8	3,953	2,016							
9	3,619								

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ChainLadder::
plot(GenIns)



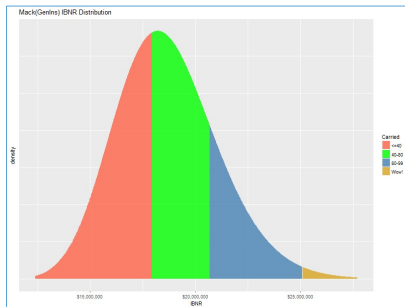
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MackChainLadder(GenIns)

origin	Latest	Dev.To.Date	Ultimate	IBNR	Mack.S.E	CV(IBNR)
1	3,901	100.0%	3,901	0	0	
2	5,339	98.3%	5,434	95	72	75.9%
3	4,909	91.3%	5,379	470	119	25.4%
4	4,588	86.6%	5,298	710	132	18.5%
5	3,873	79.7%	4,858	985	261	26.5%
6	3,692	72.2%	5,111	1,419	410	28.9%
7	3,483	61.5%	5,661	2,178	558	25.6%
8	2,864	42.2%	6,785	3,920	875	22.3%
9	1,363	24.2%	5,642	4,279	971	22.7%
10	344	6.9%	4,970	4,626	1,363	29.5%
sum	34,358		53,039	18,681	2,441	13.1%

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Safety Levels of
GenIns Carried
IBNR



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- Wrap-up: What are possible uses of an IBNR distribution?
- Rather than a distribution, can Mack/Murphy be used in predictive analytics?

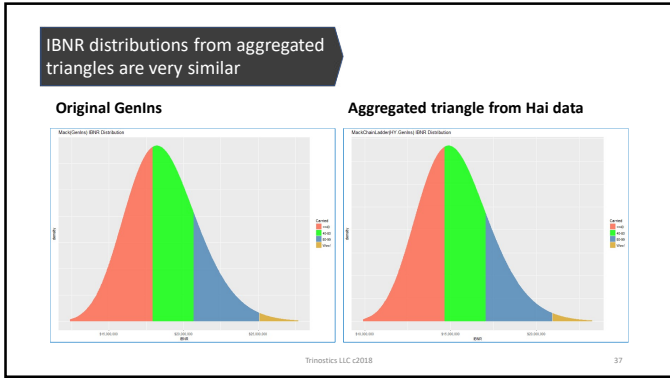
What happens when Mack/Murphy is run on detail data?

- Suppose x and y are actually observations from 4 companies in 4 accident years
- Will link ratios from aggregated data and detail data always be the same?
- What about the risk statistics?

	x	y	x	y
1	129.28	218.24	436.61	871.57
2	135.47	255.51		
3	94.53	222.66		
4	77.33	165.16		
5	130.29	296.19	357.09	680.11
6	9.10	35.77		
7	131.50	223.45		
8	86.19	114.70		
9	85.79	112.39	424.88	859.22
10	54.03	161.14		
11	94.19	169.68		
12	190.87	416.01		
13	118.53	263.72	447.86	1045.05
14	126.01	244.73		
15	62.47	150.62		
16	140.85	385.98		
17	77.28			
18	117.98			
	1666.44	3455.95	1666.44	3455.95
	Mean =	2.074	Mean =	2.074

GenIns at the Claim Level

- Hai You generated simulations of over 6000 synthetic claims whose accident year aggregation is "close in shape" to GenIns
 - We pegged the 13% cv as the primary measurement of similarity
- Hai's claim-characteristic choices included:
 - Frequency distribution
 - Severity distribution
 - Distribution for the number of payments per claim
 - Report lag and payment lag
- The purpose of this exercise was to compare the Mack results on the claim detail with the statistics from the aggregate triangle



GenIns at the Claim Level:
Claim detail sample in triangle format

	Latest	Ultimate	IBNR	Mack.S.E	CV(IBNR)
GenIns	34,358	53,039	18,681	2,441	13.1%
HY.GenIns	32,556	47,866	15,310	2,127	13.9%
HY.detail	32,556	40,909	8,353	695	8.3%

3304	
3305	
4330	9,102
5538	3,144
5911	1,007
6289	273
6300	1,425

Why the IBNR drop?
Why the CV drop?

* simulated claims by Hai You Trinostics LLC ©2018 38

Is the weighted average development factor appropriate?

Dev Factor	12-24
GenIns	3.491
HY.GenIns	3.413
HY.detail	1.288

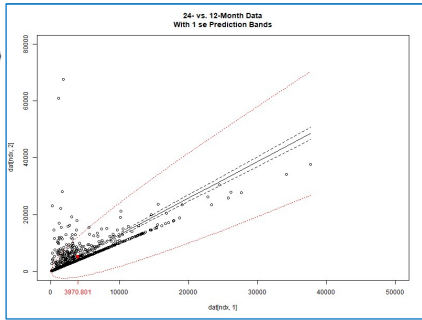
What happened to the 12-24 factor from the claim detail?!?

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The 12-24 month relationship from the claim detail

- Are Chain-Ladder assumptions violated by the detailed data?

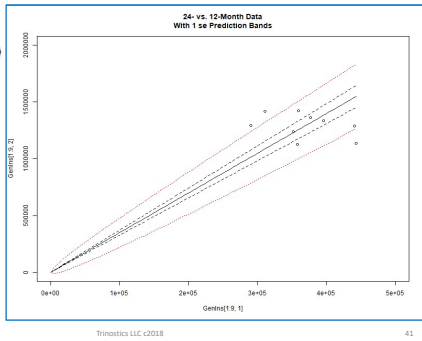
linear regression in R:
 $\text{lm}(y \sim x)$
 Coefficients:
 (Intercept) x
 1330.6 0.96



The 12-24 month relationship from the GenIns triangle

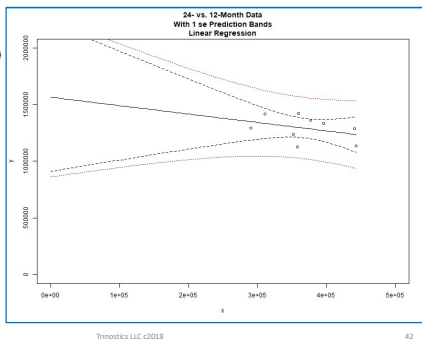
- Are Chain-Ladder assumptions violated by the aggregate data?

Is that the line you would draw through that data?



The 12-24 month relationship from the GenIns triangle

- Rhetorical question:
 Why should this model not be considered for projecting the 12-month value?



What's next?

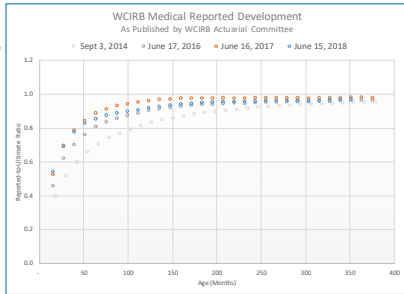
- How to model serial correlation?
 - ARMA
 - Michael Wacek, "The Path of the Ultimate Loss Ratio Estimate", *eForum*
- Growth curves
 - Sherman; Clark; Guszczka
- Bayes
- Wüthrich
 - Individual Claim Development with Machine Learning (2017)
 - Neural Networks Applied to Chain-Ladder Reserving (2018)



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Can InsureTech jump the curve?



* graphics by Kirsten Singer

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Summary

- Despite all its problems, the Chain-Ladder Mack/Murphy *model* is *useful*
 - The regression tale of development is easy to understand
 - Distributions help our principals make decisions
- Exciting actuarial analysis in the future
 - Combining methods mid-stream
 - AI modeling of the path to ultimate
- Stories/models with clarity sell best
 - Everybody likes pictures



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Q&A

Thank you for coming!
Dan Murphy
dmurphy@trinoagnostics.com

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