AR-3: Reserve Ranges – Outcomes vs. Estimates

Casualty Loss Reserve Seminar

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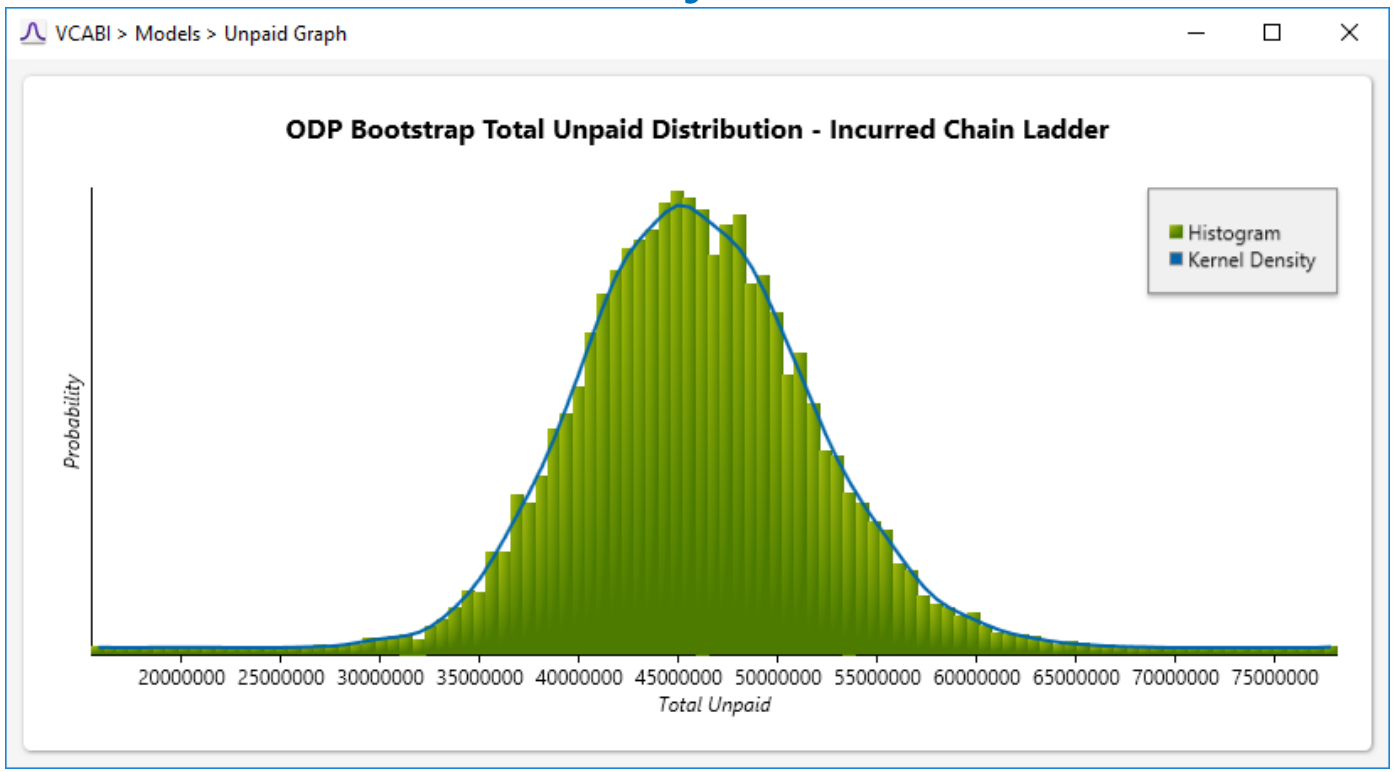
The question

- How much useful information about ranges of reasonable estimates can we expect from stochastic models?
- Some challenges:
  - Basis of presentation: ranges vs distributions
  - Process risk
  - Limitations inherent to triangle-based models

Basis of presentation

- Typical output from a stochastic model
- What does it tell you?



Do the numbers help?

Company Line of Business Evaluated as of June 30, 2018 ODP Bootstrap Unpaid – Incurred Chain Ladder									
Accident Year	Mean	Standard Error	Coefficient of Variation	50.0 %	75.0 %	80.0 %	85.0 %	90.0 %	95.0 %
2006	7	22	300.4 %	0	5	8	14	24	45
2007	19	37	192.9 %	6	25	33	44	62	93
2008	40	53	132.8 %	23	59	71	87	107	146
2009	169	284	168.2 %	66	200	254	338	467	737
2010	455	446	98.1 %	321	636	737	859	1,051	1,365
2011	1,247	787	63.1 %	1,100	1,670	1,832	2,038	2,309	2,761
2012	1,489	812	54.6 %	1,361	1,955	2,112	2,304	2,587	3,020
2013	2,898	1,085	37.4 %	2,784	3,553	3,760	4,019	4,343	4,857
2014	3,531	1,255	35.6 %	3,417	4,300	4,549	4,826	5,162	5,754
2015	7,474	1,588	21.2 %	7,379	8,495	8,772	9,091	9,542	10,226
2016	8,747	1,935	22.1 %	8,613	9,963	10,303	10,742	11,286	12,129
2017	14,716	3,294	22.4 %	14,507	16,663	17,233	17,946	18,994	20,484
2018	5,071	2,782	54.9 %	4,834	6,612	7,117	7,736	8,588	10,017
Total	45,865	5,704	12.4 %	45,662	49,560	50,559	51,703	53,250	55,540

Process risk

- How well do you understand the workings of your model?
- How does it treat process risk?
- Can you control the treatment?

Limitations of triangle data

Evaluated as of June 30, 2018
Incurred Loss - Incremental

Accident Year	6	18	30	42	54	66	78	90	102	114	126	138	150	162	174	186
2003	881	3,401	1,405	561	(12)	130	(3)	1	0	0	(19)	0	0	0	0	0
2004	865	3,694	1,237	500	973	(146)	(40)	180	(61)	(3)	0	0	0	0	0	
2005	1,233	2,769	1,396	429	546	(201)	236	0	4	(35)	1	0	49	0		
2006	1,537	3,815	2,149	705	355	(406)	(52)	333	285	0	2	0	0			
2007	1,326	3,683	3,022	1,432	1,033	973	2,548	120	14	(55)	(244)	0				
2008	2,162	5,364	2,855	3,218	1,790	1,161	25	235	352	(1)	(4)					
2009	1,575	6,118	2,460	1,384	1,426	(89)	187	(736)	740	(316)						
2010	1,528	8,294	3,638	1,710	693	332	82	279	571							
2011	1,308	8,957	7,346	4,139	1,409	1,083	(23)	446								
2012	1,885	8,495	4,590	2,346	935	648	(73)									
2013	2,857	6,507	5,324	2,839	2,451	1,730										
2014	1,664	5,152	3,758	2,952	406											
2015	2,619	7,708	3,392	2,247												
2016	1,709	4,921	3,019													
2017	1,818	6,625														
2018	1,175															

Should we expect to see an event like this from 66 to 78 months once every 10 years?

Do we think there is chance of development from 126 to 133 months?

May or may not be material depending on what model you are using, how careful you are, and the intended use of your estimates.

More tricky triangles

Evaluated as of December 31, 2017
Incurred Loss - Incremental

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168
12-1998	1,475	870	20	(1)	0	0	0	0	0	0	0	0	0	0
12-1999	1,500	2	(0)	236	(68)	(0)	0	0	0	0	0	(0)	0	0
12-2000	0	1,500	(0)	0	0	0	0	(0)	0	0	0	0	0	0
12-2001	1,450	250	650	3,144	1,582	(220)	129	170	18	(0)	0	0	(0)	0
12-2002	1,450	2,304	809	300	(14)	767	(204)	96	(0)	400	500	500	0	0
12-2003	250	350	600	551	2,399	2,151	(800)	(0)	(13)	0	0	0	200	(200)
12-2004	200	(0)	610	1,386	2,749	(100)	500	219	(350)	0	0	0	0	0
12-2005	1,150	925	1,329	164	1,432	1,000	359	(413)	200	714	21	500	0	
12-2006	350	4,414	(860)	(369)	696	(20)	0	0	0	0	0	0		
12-2007	340	424	572	1,460	2,039	1,556	(500)	252	0	0	0			
12-2008	236	982	1,145	630	501	1,051	0	200	1,300	0				
12-2009	8	100	8	2,496	1,996	0	0	100	145					
12-2010	4	3,140	992	4	0	1,096	(9)	0						
12-2011	870	1,472	4,655	2,162	2,096	430	489							
12-2012	216	(200)	1,492	496	396	893								
12-2013	136	2,986	1,551	683	0									
12-2014	120	2,028	1,051	1,658										
12-2015	62	1,265	4,833											
12-2016	12	0												
12-2017	504													

The bottom line

- A range of reasonable estimates is a statement about the potential impact of actuarial judgment
- Many stochastic models are designed to produce statistical indications that are blind to actuarial judgment
- Bayesian approaches are the obvious way to bridge this gap

Model design considerations

- Clear link between prior distributions and actuarial judgment
- Prior distributions should account for as many sources of risk as possible
  - Process risk as observed in the data
  - Reliance on industry benchmarks
  - Systemic risk factors not easily quantifiable based on the data or the benchmarks

Illustrative example

- Two prior distributions:
  - A priori expected loss ratios
  - Paid loss development patterns
- Commercial auto liability
- Rigorous use of publicly available industry data
- Potentially tempered by judgment or to fit the intended purpose

A priori loss ratios: data

- Schedule P
  - Net earned premium
  - Net ultimate loss & ALAE
  - Accident years 1989-1998
  - Annual Statement years 1998-2007
  - Why so stale?
- 167 companies
  - At least $100K NEP in each year

A priori loss ratios: assumptions

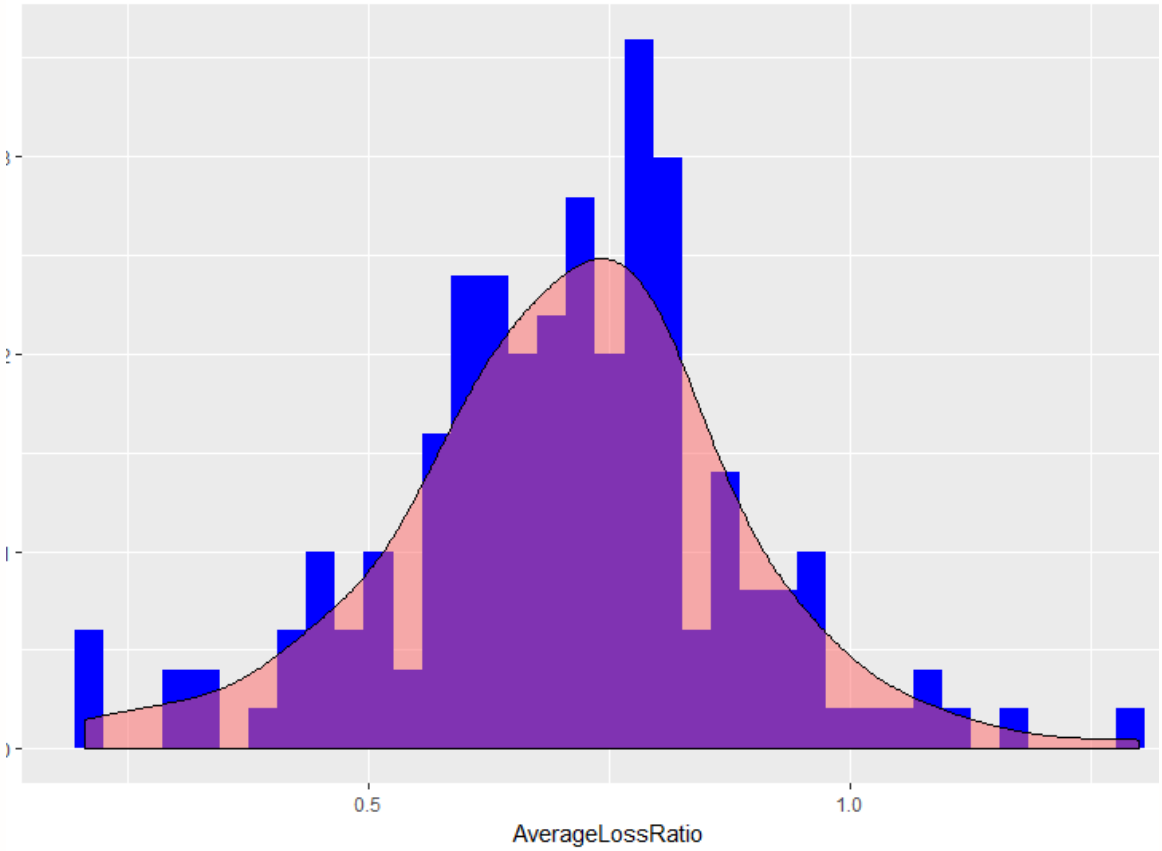
- Each company has a constant expected loss ratio, θ:

$$\mathrm{E}[\text{ Ultimate Loss }] = \theta \times \text{Premium}$$

- Each company has a fixed process risk parameter, φ

$$\mathrm{Var}[\text{ Ultimate Loss }] = \varphi \times \text{Premium}$$

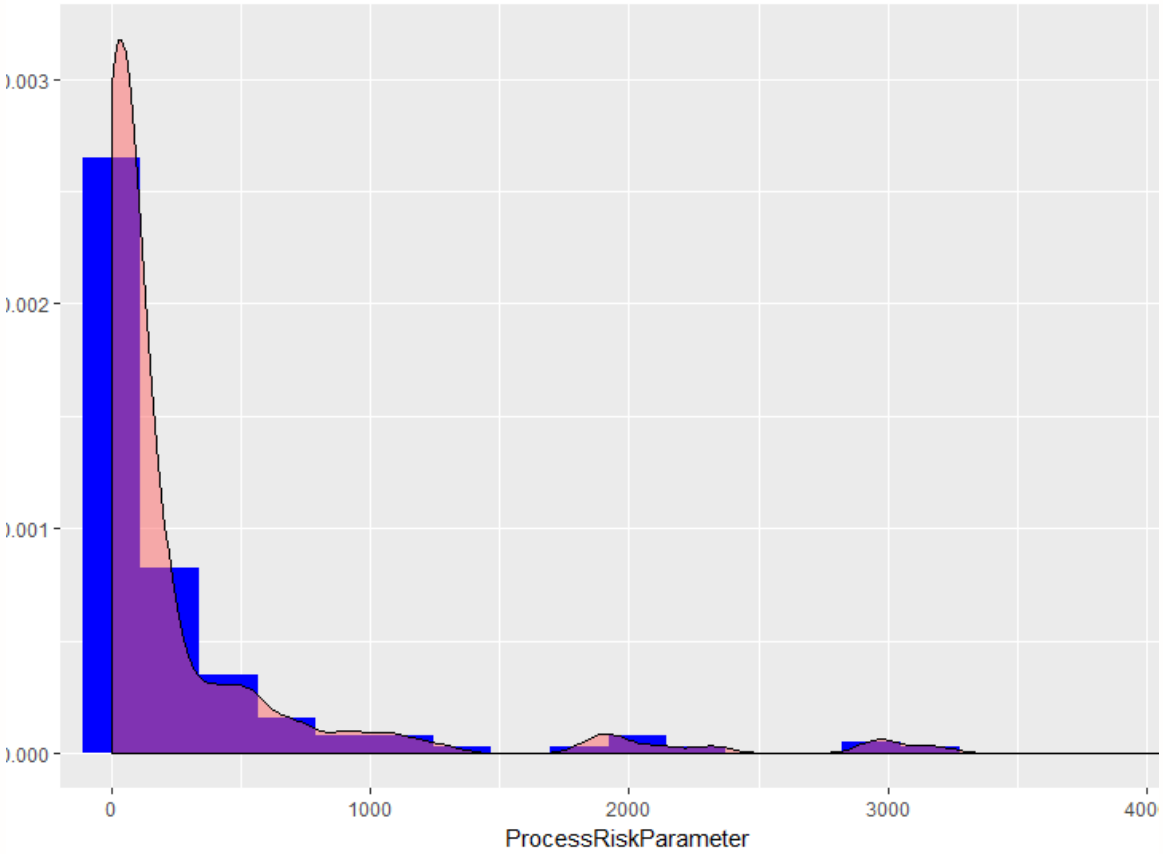
- Parameters differ between companies
- Reasonable?



Average loss ratios

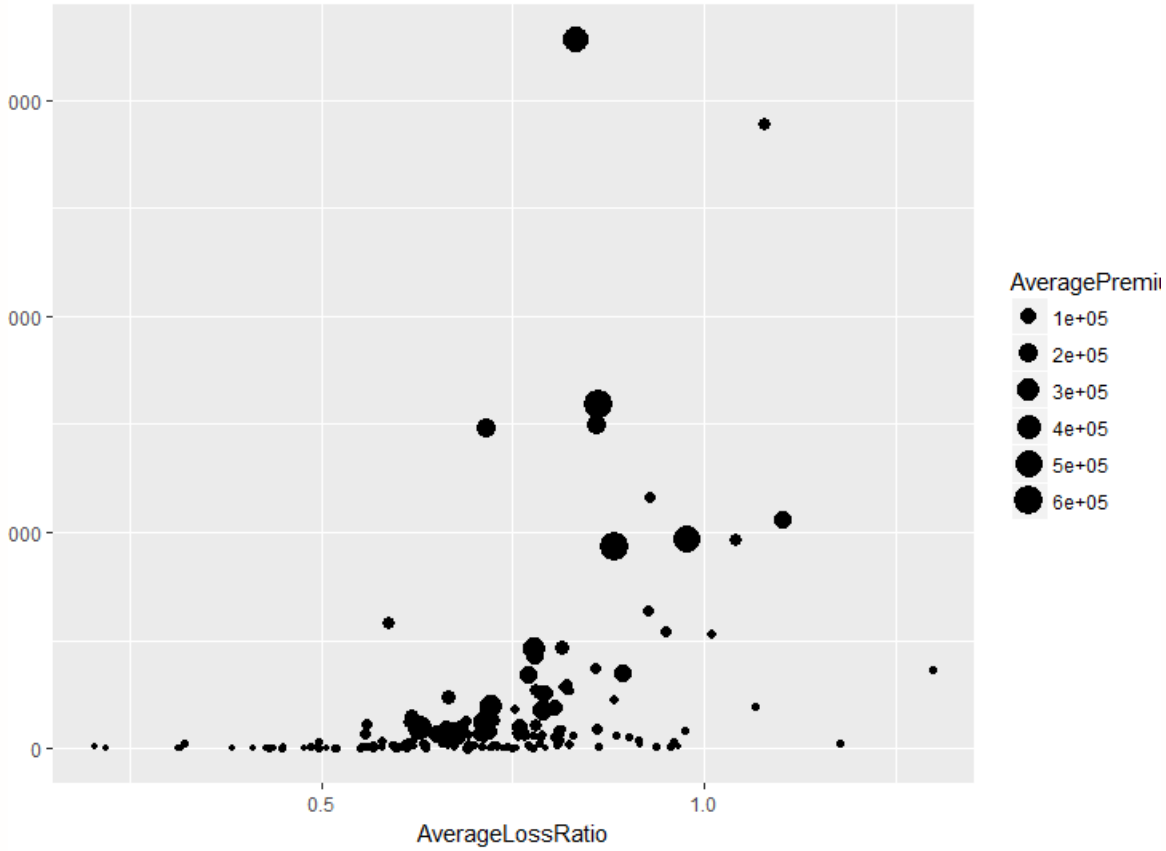
Let's use method of moments to select a lognormal a priori

Maybe even a range of reasonable estimates already?



Process risk parameter

Pareto perhaps?



Process risk vs loss ratio

Possible dependence structure?

Joint distribution or copula?

Is it material?

Payment pattern: data

- Schedule P Part 3
  - Same companies
  - Same years
- But how do we parse it?
  - Development factors?
  - Incremental percentage of ultimate?
  - Incremental percentage of reserves?

A priori payment pattern: assumptions

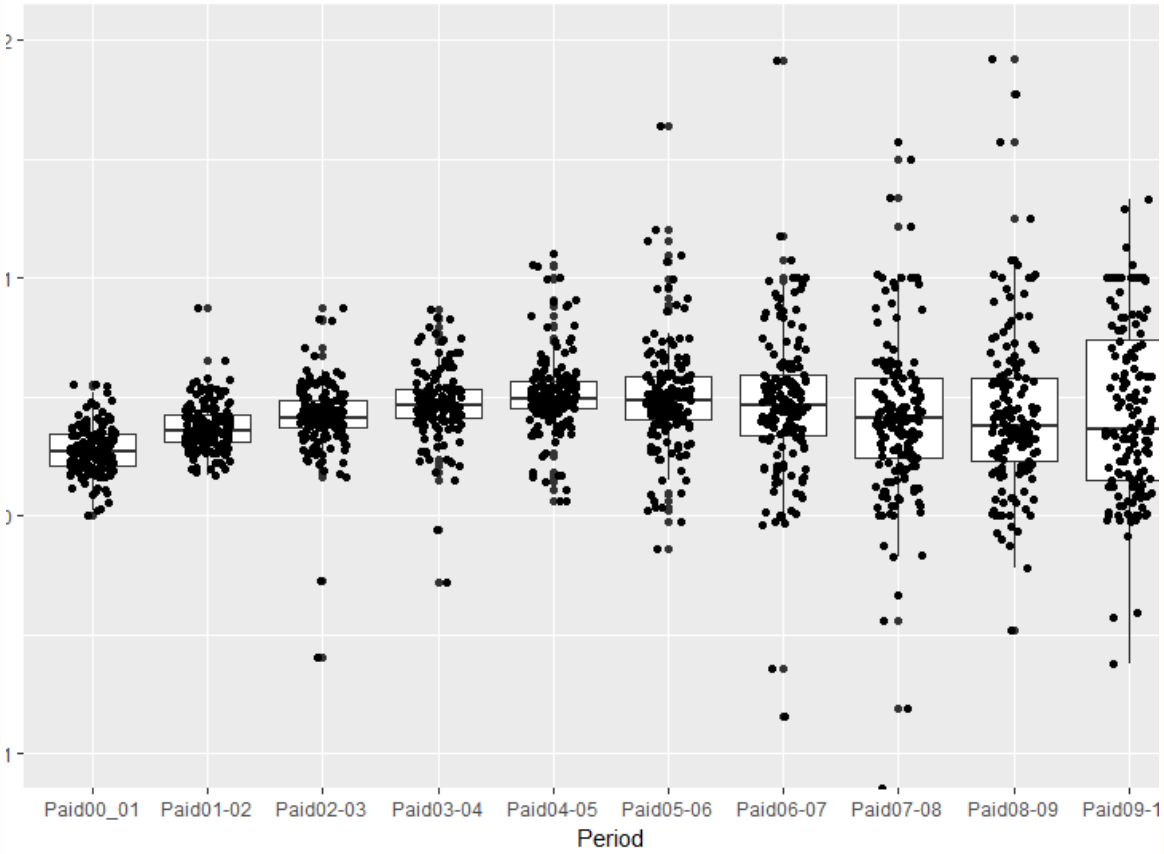
- At development period j, each company has an parameter η_j with:

  $$\mathrm{E}[\text{ Incremental Paid}] = \eta_j \times \text{Starting Reserve}$$

- With an associated fixed process risk parameter, ξ_j

  $$\mathrm{Var}[\text{ Incremental Paid}] = \xi_j \times \text{Starting Reserve}$$

- Parameters differ between companies
- Reasonable? Central Limit Theorem?



Expected average paid

Is this what you expected?

What about a tail factor?

Questions and Discussion