Introduction

Instructional Approach to Mack and ODP Bootstrap Models

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Overview

- Introductions
- Notation
- Ranges vs. Distributions
- Mack Model
- ODP Bootstrap Model

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Logistics Overview

- Facilities
- Time / Breaks
- Language, Terminology & Currency
- Your view blocked? / Can't See?
- Feedback forms
- Don't Skip Ahead
- Have Fun / Ask Questions
- Examples NOT Commercial Software

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Notation

w – denotes an Accident (or Policy) period ("when").

d – denotes a development age ("delay").

k – denotes a diagonal or constant *w* + *d*.

c(*w*,*d*) – denotes cumulative losses for accident (or policy) period *w*, and development age *d.*

c(*w*,*n*) – denotes ultimate losses for accident (or policy) period *w* [also denoted as *U*(*w*)].

R(*w*,*d*) – denotes future development for accident (or policy) period *w*, and development age *d*.

Note: *R*(*w*,*d*) = *U*(*w*) – *c*(*w*,*d*)

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Notation

q(*w*,*d*) – denotes incremental losses for accident (or policy) period *w*, and development age *d.*

f(*d*) – denotes the factor applied to *c*(*w*,*d*) to estimate *q*(*w*,*d*+1).

F(*d*) – denotes the factor applied to *c*(*w*,*d*) to estimate *c*(*w*,*d*+1). For example, the *d* to *d*+1 age-to-age factor.

[Note: WP Report uses *c(w,n)*]

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Notation

G(*w*) – denotes a factor relating to accident (or policy) year *w*.

h(*w*+*d*) – denotes a factor relating to diagonal *k* (or calendar year) in which *w*+*d* is constant.

e(*w*,*d*) – denotes the mean of zero random fluctuation which occurs in cell *w*, *d*.

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Notation

- $E(x)$ – denotes the expectation of the random variable x. (Also μ_x)
- $Var(x)$ – denotes the variance of the random variable x. (Also σ_x^2 or $Sigma_x^2$)
- W – Weight
- N – Total number of Accident (Policy) periods
- n – Total number of Development periods
- $\hat{x}$ – Estimate of x.

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Notation Example

Cumulative Losses

AY	0	1	2
1981	5,012	8,269	10,907
1982	106	4,285	
1983	3,410		

Notation

AY	0	1	2
1981	c(1981,0)	c(1981,1)	c(1981,2)
1982	c(1982,0)	c(1982,1)	
1983	c(1983,0)		

w — Can be Year (i.e., 1981) or Index (i.e., 1)

d

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Notation Example

Incremental Losses

AY	0	1	2
1981	5,012	3,257	2,638
1982	106	4,179	
1983	3,410		

Calendar Effect denoted *h*(1982+1) or *h*(1983) or *h*(3)

Notation

AY	0	1	2
1981	q(1981,0)	q(1981,1)	q(1981,2)
1982	q(1982,0)	q(1982,1)	
1983	q(1983,0)		

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Notation Example

Age-to-Age Factors

AY	1	2
1981	1.650	1.319
1982	40.425	
Mean	21.037	1.319

Notation

AY	1	2
1981	*F*(1981,1)	*F*(1981,2)
1982	*F*(1982,1)	
Mean	*F*(1)	*F*(2)

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Ranges vs. Distributions

- A **Range** is not the same as a **Distribution**
- *A Range of Reasonable Estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable.
- A *Distribution* is a statistical function that attempts to quantify probabilities of all possible outcomes.

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Ranges vs. Distributions

- A **Range** is generally considered to be either a subset of "*central estimates*" or a subset of the "*possible outcomes*".
- For a "*central estimate*" the incremental values will essentially have the random movements "*averaged*" or "*smoothed*" out.
- A "*possible outcome*" will generally include random movements in the incremental values (e.g., calendar period payments within each accident period).

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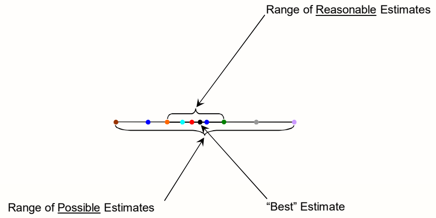
Ranges vs. Distributions



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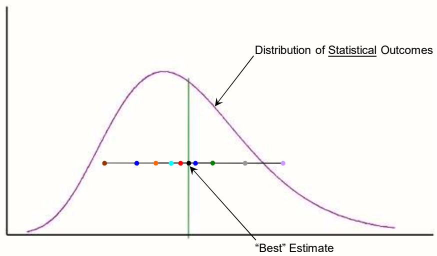
Ranges vs. Distributions



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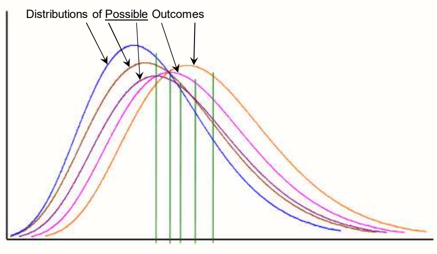
Ranges vs. Distributions



With multiple models:
You can evaluate the relative strengths of each model!

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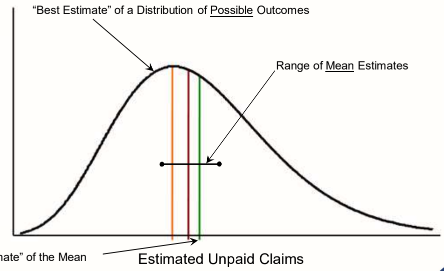
Ranges vs. Distributions



With multiple models:
You can use credibility weights to get your "best estimate"!

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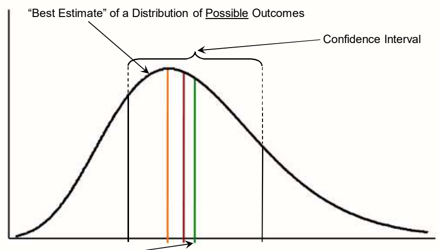
Ranges vs. Distributions



With multiple models:
You can calculate confidence intervals.

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Basic Models

- And Now for Something Completely Different…
  - Mack Model
  - ODP Bootstrap Model

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Final Questions?

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