Convolutions In Reserving with a Focus on the BF Method

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Outline

1. Convolution
2. Distributions
3. CY Trend/Reserve Cycle
4. Factoring a Triangle
5. New Directions

What is a Convolution?

Convolution is a fancy name for the structure that is common to almost all of our traditional reserving methods.

Traditional Methods

Period	Data Type	Factor	Estimate
2010	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2011	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2012	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2013	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2014	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2015	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2016	Value(AY)	Factor(age)	Value(AY)×Factor(age)
2017	Value(AY)	Factor(age)	Value(AY)×Factor(age)
		Total	Σ Estimate(Reserve Date)

$h(x+y) = \Sigma\, f(x)\, g(y)$

Premise of Convolution

- In the real world $h = f \times g$ occurs in continuous time.
- We are unable to measure instantaneous quantities, we can only measure over intervals.

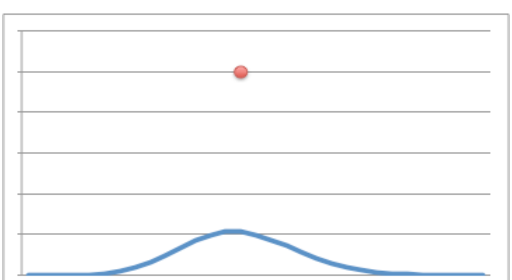
Consequence

- What we see as a sum of interval observations is actually the result of an integral process.
- All of the properties of integrals apply to convolution sums.
- All integration theorems and known solutions of the integrals of products can be used without proving the special case.

What We Have Learned So Far

- Traditional reserving methods have a sound theoretical basis.

Distributions



Process vs. Method

- A Process describes our theory of the way that a system works.
- A Method is a formula or algorithm for generating an estimate of the output of the system.

Example: Bornhuetter-Ferguson

- Method:
  IBNR(AY) = EL(AY) × (1-1/LDF(age))
- Process:
  Inc.(AY,age) = EL(AY) × ΔEmg(age)

The Process describes what happens in each cell of the triangle.

Prospective vs. Retrospective

- Methods are strictly prospective, while processes are also retrospective.
- If we model our triangle as being generated by a particular process then:
  - We can find residuals;
  - We can construct an error distribution.

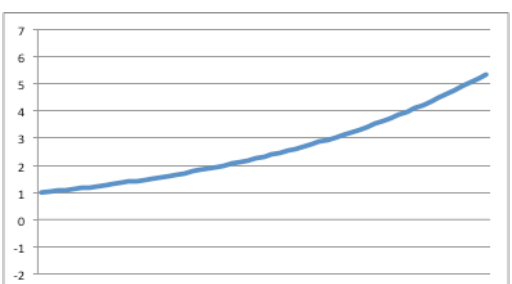
Convolution Process

- A convolution equation is an expression of the belief that the underlying process is multiplicative at each instant (in each cell).
- Every traditional method reflects an assumption about the convolutional generating process.
- Traditional (Convolutional) Methods are distributional.

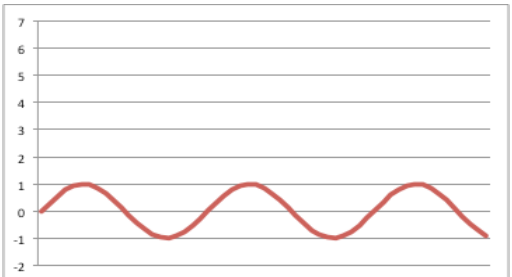
What We Have Learned So Far

- Traditional reserving methods have a sound theoretical basis.
- Traditional Methods are not deterministic, they are distributional.

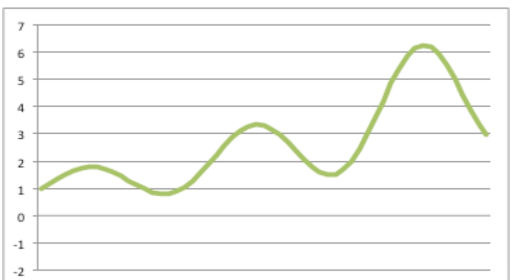
CY Trend/Reserve Cycle



CY Trend/Reserve Cycle



CY Trend/Reserve Cycle



Three Factor Model

GLMs made a significant advance when actuaries began using models with three factors, for AY, age, and CY.

Prior to that, it had been shown that two-factor regression models could be matched by selecting an "average of all" development patterns.

Three Factor Model

Incremental:

ΔPaid(AY,age) = X(AY) Y(CY) Z(age)

GLM:

ln(ΔPaid) = ln(X) + ln(Y) + ln(Z)

Three Factor Model

ΔPaid(AY, age) = X(AY) Y(CY) Z(age)

- X(AY) is an independent variable.
- Z(age) is a pattern that depends on the line of business and data type.
- Y(CY) the CY Trend/Reserve Cycle is a source of variability in the model.

Three Factor Model

Incremental:

ΔPaid(AY, age) = X(AY) Y(CY) Z(age)

Reserve:

Unpaid $= \Sigma\ \Sigma\ \Delta$Paid

Convolution:

Unpaid $= \Sigma\ \Sigma\ \Delta$Paid $= X * Y * Z$

GLMs are Convolutions

The three factor model is still just multiplication in the individual cells, which we sum to get reserves. It is a convolution.

Can we use the mathematics of convolutions to better understand GLM models?

Convolutions Enable the Modeling of CY Trend/Reserve Cycles

- A pure exponential trend (constant rate of change) does not induce additional variability in losses.
- Only the Reserve Cycle component adds to the variability of losses.
- We can use convolutions to easily find the Reserve Cycle component.

Causes of CY Reserve Cycles

- Changes in the rate of inflation
- Major court decisions or reform laws
- Financial shocks

What We Have Learned So Far

- Traditional reserving methods have a sound theoretical basis.
- Traditional Methods are not deterministic, they are distributional.
- GLMs increase accuracy only if they model the CY Reserve Cycle (Trend alone is not enough).

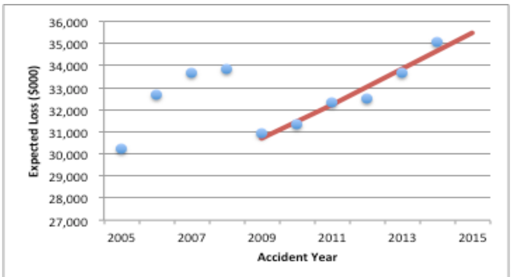
How to Find the Cycle Using Triangles

WC Net Paid - US Totals

(Randomized)

Accident Year	1	2	3	4	5	6	7	8	9	10
1995										17,751
1996									17,917	19,394
1997								19,225	20,670	20,899
1998							20,510	21,899	22,296	22,519
1999						19,654	21,135	21,694	21,974	22,463
2000					20,918	22,333	23,150	23,596	24,083	24,942
2001				20,511	22,194	23,251	23,836	24,621	25,467	25,509
2002			17,391	20,415	22,092	23,054	24,050	25,051	25,165	26,470
2003		13,079	17,630	20,110	21,808	23,079	24,064	24,356	25,833	26,314
2004	6,516	13,612	17,161	19,533	21,181	22,475	22,799	24,513	25,147	25,697
2005	7,064	13,638	17,373	19,735	21,553	22,111	24,024	24,725	25,381	25,784
2006	7,116	14,100	18,293	21,244	22,366	24,635	25,559	26,470	27,000	27,447
2007	7,112	14,623	19,105	21,271	24,140	25,547	26,664	27,441	28,048	
2008	7,127	14,764	18,710	22,410	24,380	25,796	26,833	27,539		
2009	6,832	13,466	18,092	20,686	22,522	23,839	24,630			
2010	6,610	14,028	18,220	20,811	22,660	23,712				
2011	7,083	14,375	18,649	21,408	23,079					
2012	6,920	14,180	18,274	20,731						
2013	6,823	14,111	18,115							
2014	6,979	14,243								
2015	6,874									

Expected Losses
(Prior Year Ultimates)



Convolution Process

h(x+y) = Σ f(x) g(y)

1. Multiplication
2. Sum
3. Rotation

Inverse Process

Perform three operations on a triangle:

1. Division
2. Difference
3. Rotation

Cumulative Payment Pattern

(Divide Out Expected Loss)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										0.890
1996									0.797	0.863
1997								0.792	0.852	0.861
1998							0.782	0.835	0.850	0.858
1999						0.754	0.811	0.832	0.843	0.862
2000					0.710	0.758	0.786	0.801	0.818	0.847
2001				0.654	0.708	0.741	0.760	0.785	0.812	0.813
2002			0.535	0.628	0.680	0.709	0.740	0.771	0.774	0.814
2003		0.408	0.549	0.627	0.680	0.719	0.750	0.759	0.805	0.820
2004	0.210	0.439	0.554	0.631	0.684	0.726	0.736	0.791	0.812	0.829
2005	0.229	0.442	0.563	0.640	0.699	0.717	0.779	0.802	0.823	0.836
2006	0.215	0.426	0.553	0.642	0.676	0.745	0.773	0.800	0.816	0.830
2007	0.209	0.430	0.562	0.626	0.710	0.751	0.784	0.807	0.825	
2008	0.209	0.433	0.548	0.657	0.714	0.756	0.786	0.807		
2009	0.217	0.428	0.576	0.658	0.717	0.758	0.784			
2010	0.211	0.447	0.581	0.664	0.723	0.756				
2011	0.217	0.440	0.571	0.655	0.707					
2012	0.210	0.431	0.555	0.630						
2013	0.202	0.418	0.536							
2014	0.197	0.403								
2015	0.190									

Incremental Payment Pattern

(Difference Cumulative Pattern)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										
1996										0.066
1997									0.060	0.009
1998								0.053	0.015	0.008
1999							0.057	0.021	0.011	0.019
2000						0.048	0.028	0.015	0.017	0.029
2001					0.054	0.034	0.019	0.025	0.027	0.001
2002				0.093	0.052	0.030	0.031	0.031	0.004	0.040
2003			0.142	0.077	0.053	0.040	0.031	0.009	0.046	0.015
2004		0.229	0.115	0.077	0.053	0.042	0.010	0.055	0.020	0.018
2005	0.229	0.213	0.121	0.077	0.059	0.018	0.062	0.023	0.021	0.013
2006	0.215	0.211	0.127	0.089	0.034	0.069	0.028	0.028	0.016	0.014
2007	0.209	0.221	0.132	0.064	0.084	0.041	0.033	0.023	0.018	
2008	0.209	0.224	0.116	0.108	0.058	0.041	0.030	0.021		
2009	0.217	0.211	0.147	0.083	0.058	0.042	0.025			
2010	0.211	0.237	0.134	0.083	0.059	0.034				
2011	0.217	0.223	0.131	0.084	0.051					
2012	0.210	0.221	0.124	0.075						
2013	0.202	0.216	0.119							
2014	0.197	0.205								
2015	0.190									

CY Incremental Payment Pattern

(Rotate Incremental Pattern)

Calendar Year	12	24	36	48	60	72	84	96	108	120
2005	0.229	0.229	0.142	0.093	0.054	0.048	0.057	0.053	0.060	0.066
2006	0.215	0.213	0.115	0.077	0.052	0.034	0.028	0.021	0.015	0.009
2007	0.209	0.211	0.121	0.077	0.053	0.030	0.019	0.015	0.011	0.008
2008	0.209	0.221	0.127	0.077	0.053	0.040	0.031	0.025	0.017	0.019
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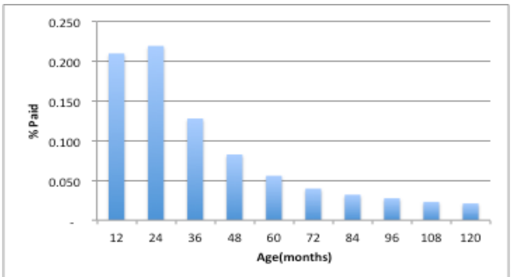
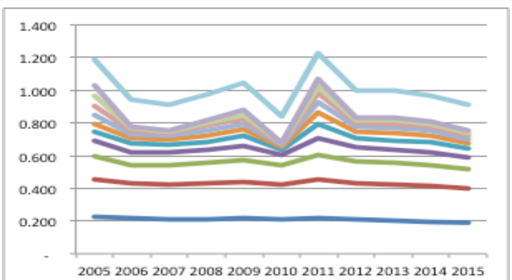
2010	0.211	0.211	0.116	0.064	0.034	0.018	0.010	0.009	0.004	0.001
2011	0.217	0.237	0.147	0.108	0.084	0.069	0.062	0.055	0.046	0.040
2012	0.210	0.223	0.134	0.083	0.058	0.041	0.028	0.023	0.020	0.015
2013	0.202	0.221	0.131	0.083	0.058	0.041	0.033	0.028	0.021	0.018
2014	0.197	0.216	0.124	0.084	0.059	0.042	0.030	0.023	0.016	0.013
2015	0.190	0.205	0.119	0.075	0.051	0.034	0.025	0.021	0.018	0.014

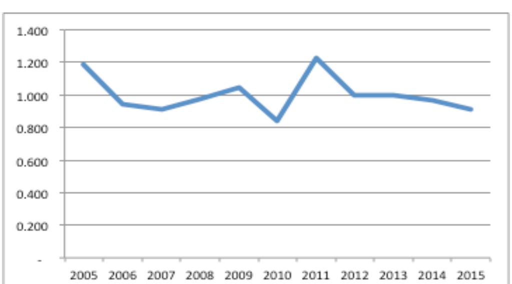
Incremental Payment Pattern



CY Cycle



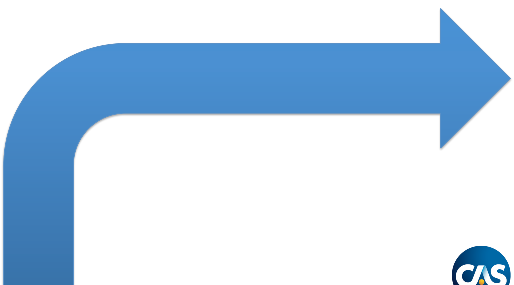
CY Cycle



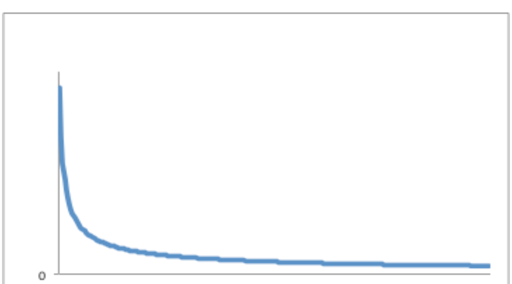
What We Have Learned So Far

- Traditional reserving methods have a sound theoretical basis.
- Traditional Methods are not deterministic, they are distributional.
- GLMs increase accuracy only if they model the CY Cycle (not Trend).
- We can find the CY Cycle through a series of simple manipulations of a Triangle.

New Directions



Alternate Severity



All Claims Severity Distribution

We normally think of severity distributions in terms of the reported claims. But when building individual claim development models we need a separate model for IBNR.

Instead, if we consider the severity distribution of all claims (reported or not) then the unreported claims are represented by a point mass at zero.

The mathematics of convolution equations allows for the handling of point masses.

Partial Severities

(Divide by Ult. Claim Counts)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										2,519
1996									2,659	2,878
1997								2,966	3,188	3,224
1998							3,019	3,224	3,282	3,315
1999						2,985	3,210	3,295	3,338	3,412
2000					3,059	3,266	3,385	3,451	3,522	3,648
2001				3,215	3,479	3,645	3,737	3,860	3,992	3,999
2002			2,966	3,482	3,768	3,932	4,102	4,273	4,292	4,515
2003		2,504	3,375	3,850	4,175	4,418	4,606	4,662	4,945	5,037
2004	1,280	2,675	3,372	3,839	4,162	4,417	4,480	4,817	4,942	5,050
2005	1,433	2,767	3,524	4,003	4,372	4,485	4,873	5,015	5,149	5,230
2006	1,472	2,917	3,784	4,395	4,627	5,096	5,287	5,476	5,586	5,678
2007	1,557	3,202	4,183	4,657	5,285	5,593	5,838	6,008	6,141	
2008	1,684	3,489	4,422	5,297	5,762	6,097	6,342	6,509		
2009	1,824	3,595	4,831	5,523	6,014	6,365	6,576			
2010	1,756	3,726	4,839	5,527	6,018	6,298				
2011	1,828	3,709	4,812	5,524	5,955					
2012	1,783	3,654	4,709	5,342						
2013	1,780	3,681	4,725							
2014	1,752	3,576								
2015	1,685									

Incremental Partial Severities

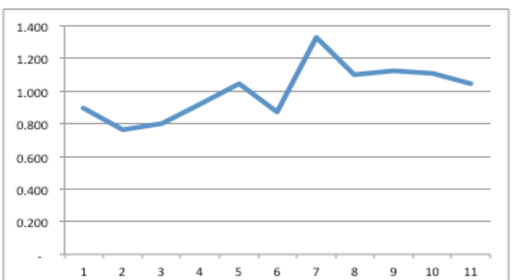
(Difference)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										
1996										219
1997									223	35
1998								204	58	33
1999							225	85	43	74
2000						207	119	65	71	126
2001					264	166	92	123	133	7
2002				516	286	164	170	171	19	223
2003			871	475	325	243	189	56	283	92
2004		1,395	697	466	324	254	64	337	125	108
2005	1,433	1,334	758	479	369	113	388	142	133	82
2006	1,472	1,445	867	611	232	469	191	188	110	92
2007	1,557	1,644	981	474	628	308	245	170	133	
2008	1,684	1,805	933	875	466	335	245	167		
2009	1,824	1,771	1,235	693	490	352	211			
2010	1,756	1,970	1,114	688	491	279				
2011	1,828	1,882	1,103	712	431					
2012	1,783	1,871	1,055	633						
2013	1,780	1,901	1,044							
2014	1,752	1,824								
2015	1,685									

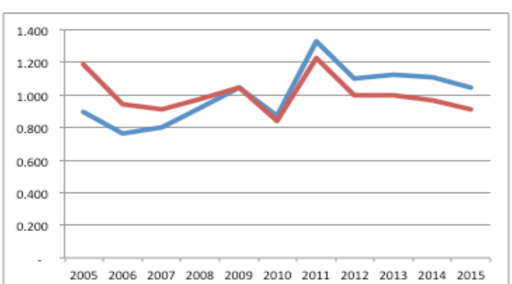
CY Incremental Partial Severities
(Rotate)

Calendar Year	12	24	36	48	60	72	84	96	108	120
2005	1,433	1,395	871	516	264	207	225	204	223	219
2006	1,472	1,334	697	475	286	166	119	85	58	35
2007	1,557	1,445	758	466	325	164	92	65	43	33
2008	1,684	1,644	867	479	324	243	170	123	71	74
2009	1,824	1,805	981	611	369	254	189	171	133	126
2010	1,756	1,771	933	474	232	113	64	56	19	7
2011	1,828	1,970	1,235	875	628	469	388	337	283	223
2012	1,783	1,882	1,114	693	466	308	191	142	125	92
2013	1,780	1,871	1,103	688	490	335	245	188	133	108
2014	1,752	1,901	1,055	712	491	352	245	170	110	82
2015	1,685	1,824	1,044	633	431	279	211	167	133	92

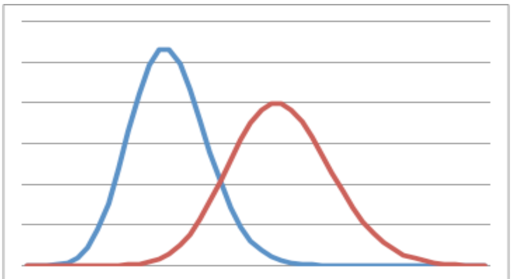
Trend-Cycle



Cycle vs. Trend-Cycle



Additive Risk



Statistical Meaning of Convolution

- In statistics, convolution is the process by which independent distributions are added together.
- This allows us to look at our convolutional reserving methods in a completely different way, as the sums of distributions.

Decomposition of Reserve Variability

- BF:
  Reserve Risk = ELR Risk * Emergence Risk
- Freq-Sev:
  Reserve Risk = Freq Risk * Severity Risk
- 3 Factor Model:
  Reserve Risk = Insurance Risk * Inflation Risk