# The shape of uncertainty
# &
# Fat Tails in Risk Models

Jamie Mackay and David Ingram
CLRS
2018

# AR8: The Shape of Uncertainty

- Session Overview

| | |
|---|---|
| 1. The Shape of Uncertainty | Jamie Mackay |
| 2. Fat Tails in Risk Models | Dave Ingram |

# The shape of uncertainty

- "What is your unpaid loss reserve?"

**"How certain are you about that number?"**

***"What do you mean by that?"***

***"At what point in the future?"***

***"How do you measure uncertainty?"***

***"Range in the* estimate *or variability around the* eventual outcome*?"***

***"How certain do you want me to be?"***

***"Why do you ask?"***

# Shaped by your requirements

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**RISK PROFILE**

**TIME HORIZON**

**RISK TOLERANCE**

**RISK MEASURE**

# Shaped by your requirements

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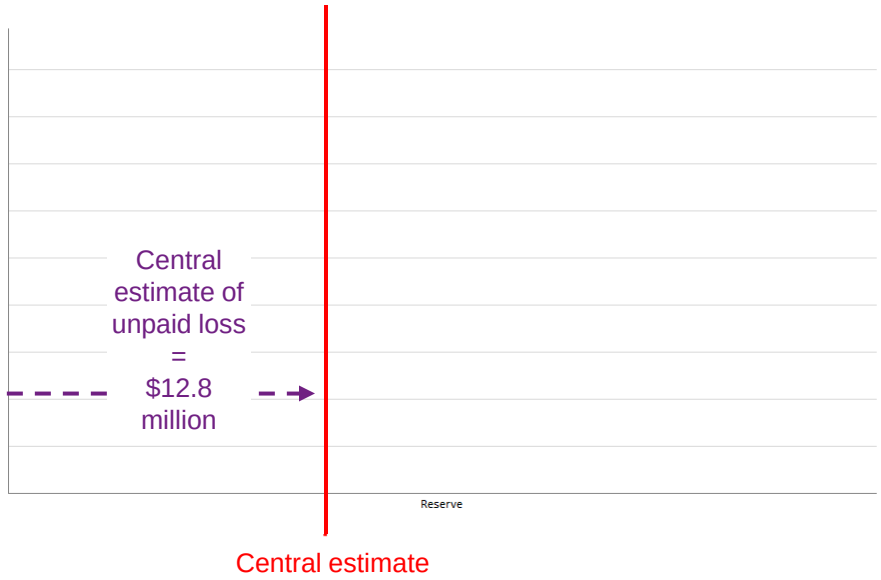
RISK TOLERANCE



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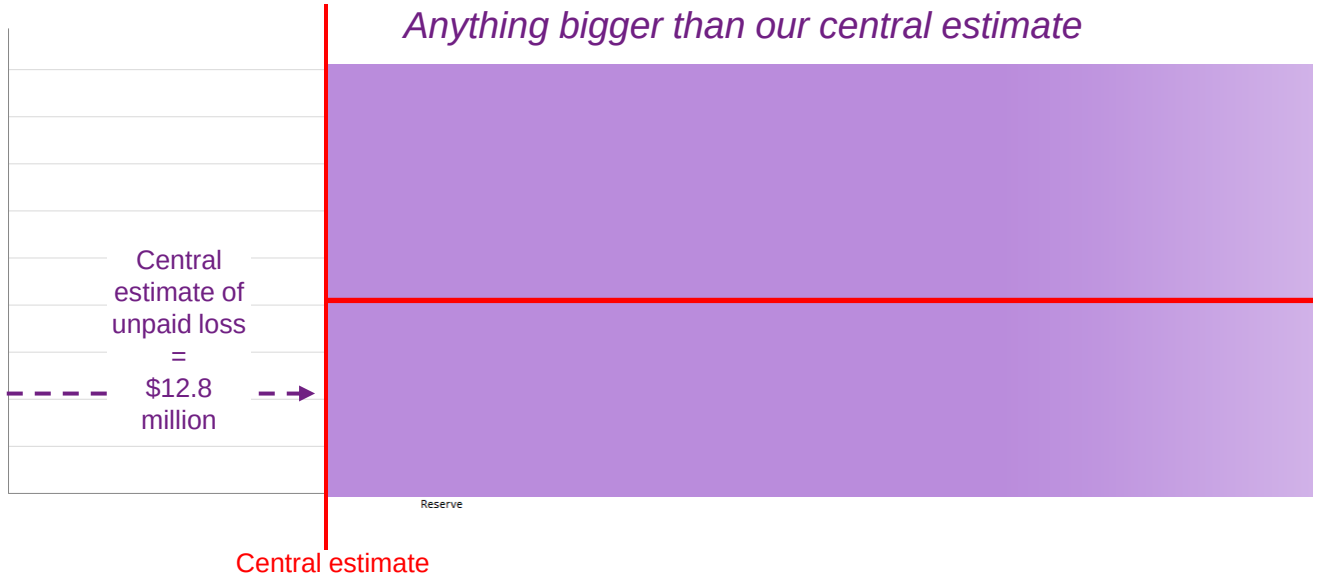

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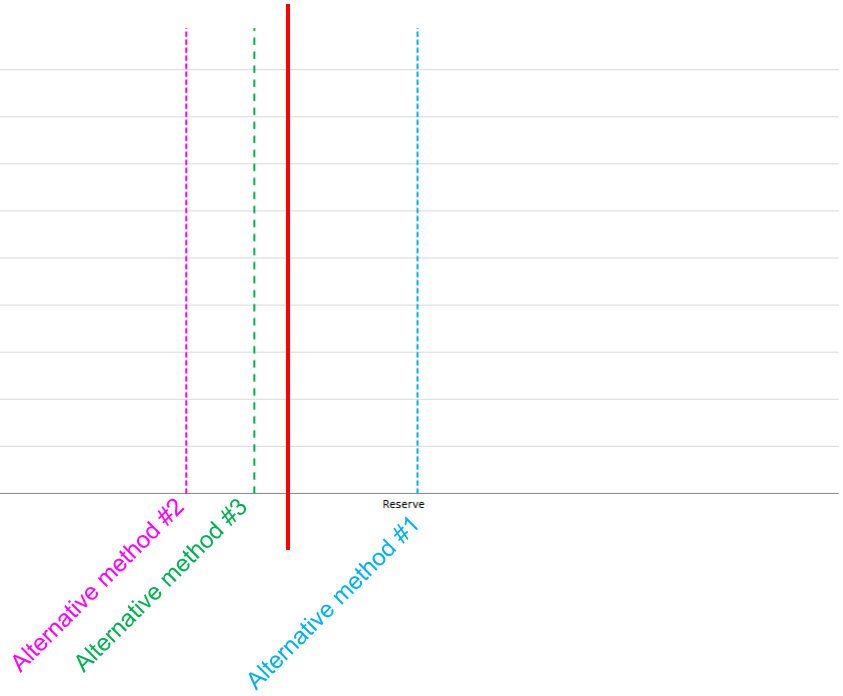

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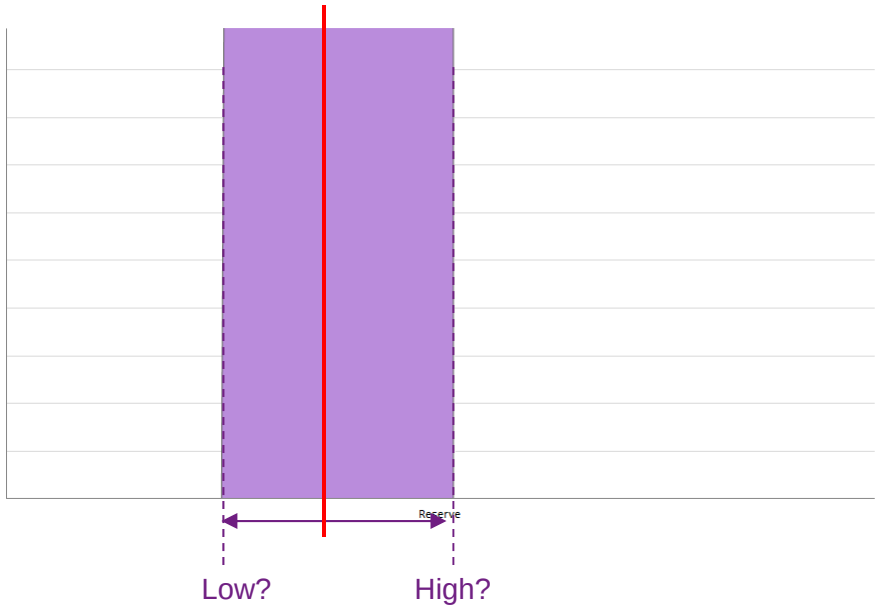

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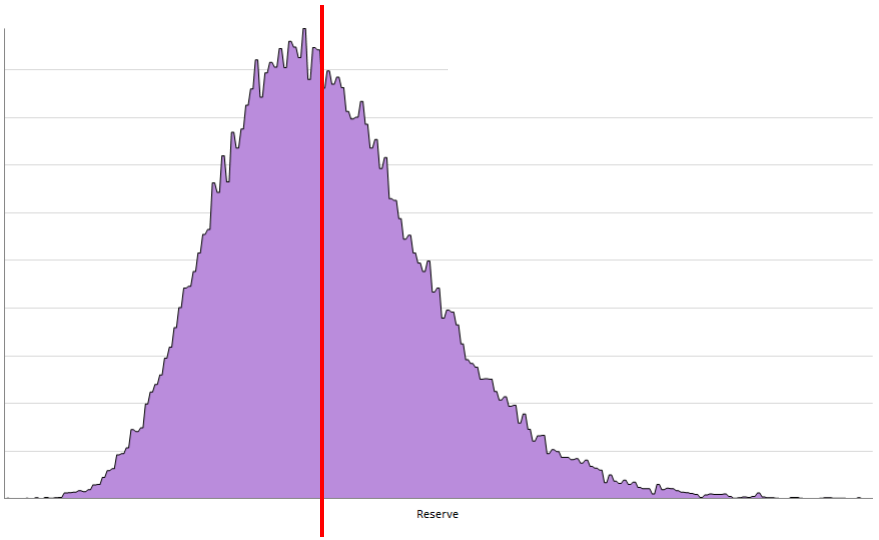
A predictive distribution of ~~estimates?~~ outcomes?



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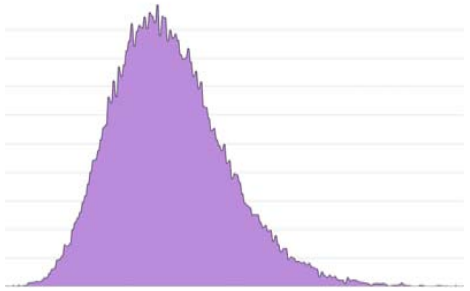
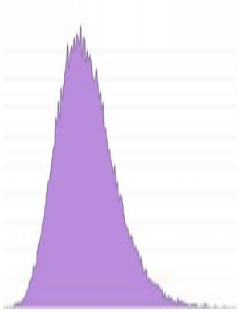
The **ultimate** view

The **one-year** view

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## The **ultimate** view

**Purpose:** How much could the eventual outcome differ from our current estimate?

**Methods:** Mack, Bootstrapping, MCMC, practical stochastic, etc

**Notes:** Provides diagnostic insight into the uncertainty of a reserve estimate; provides useful support for assessing reasonableness of a booked reserve provision; cash-flows (adjusted or otherwise) are key input into capital models

## The **one-year** view

**Purpose**: How much could our estimate change over the next year?

**Methods:** Re-reserving / “actuary-in-a-box”, Merz & Wuthrich, etc

**Notes:** We would not expect our ‘central estimate’ to be any different in one year than it is now. However, this approach helps quantify how different it *could be*, based on expected volatility in cashflows
Typically used for for projecting balance sheets as part of a capital modeling framework within the context of ORSA, Solvency II, etc

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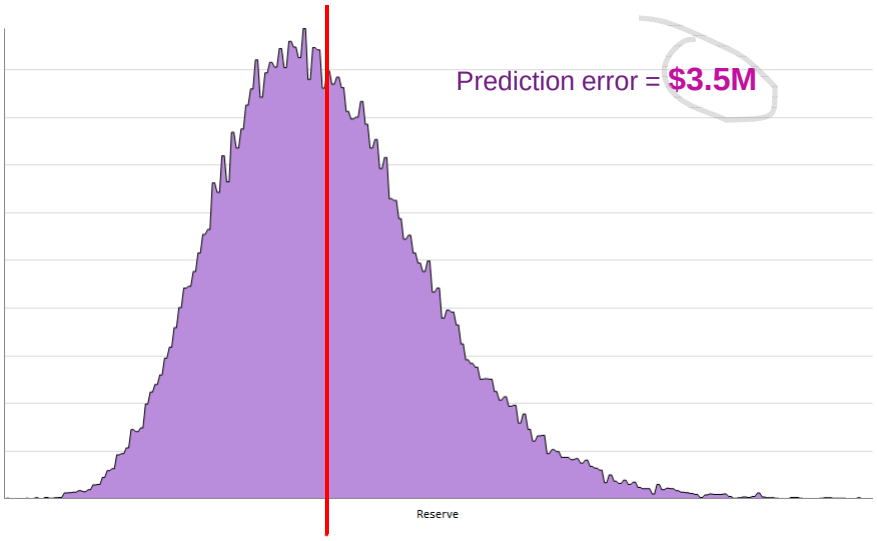


The **prediction error** describes the characteristics of the distribution and provides a dollar value of the inherent volatility (similar to standard deviation)

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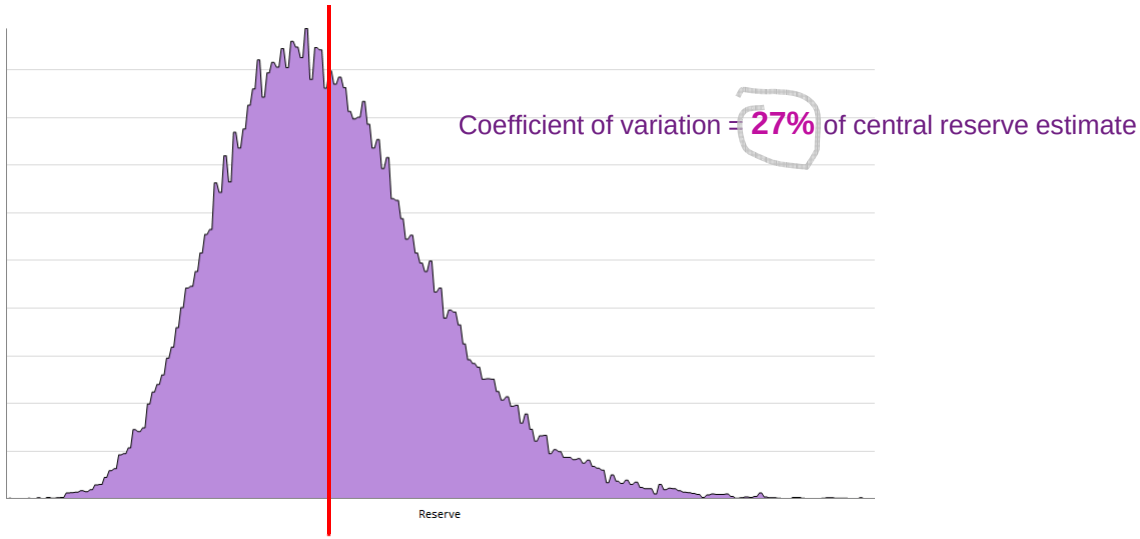


The **coefficient-of-variation** provides similar information about the characteristics of the distributions but is relative to the size of the reserve

Although more descriptive than a general ‘high’ or ‘low’, such measures don’t provide insight into the probability of any one outcome

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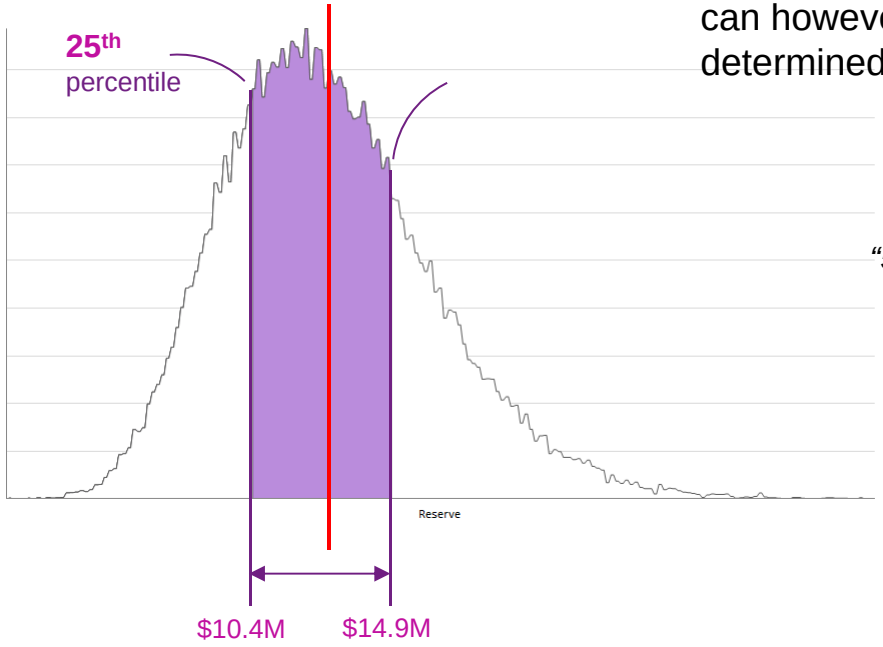


- As we have a full distribution, we can however, identify the reserves at determined percentiles:

*"50:50 chance that eventual payments will lie between $10.4M and $14.9M"*

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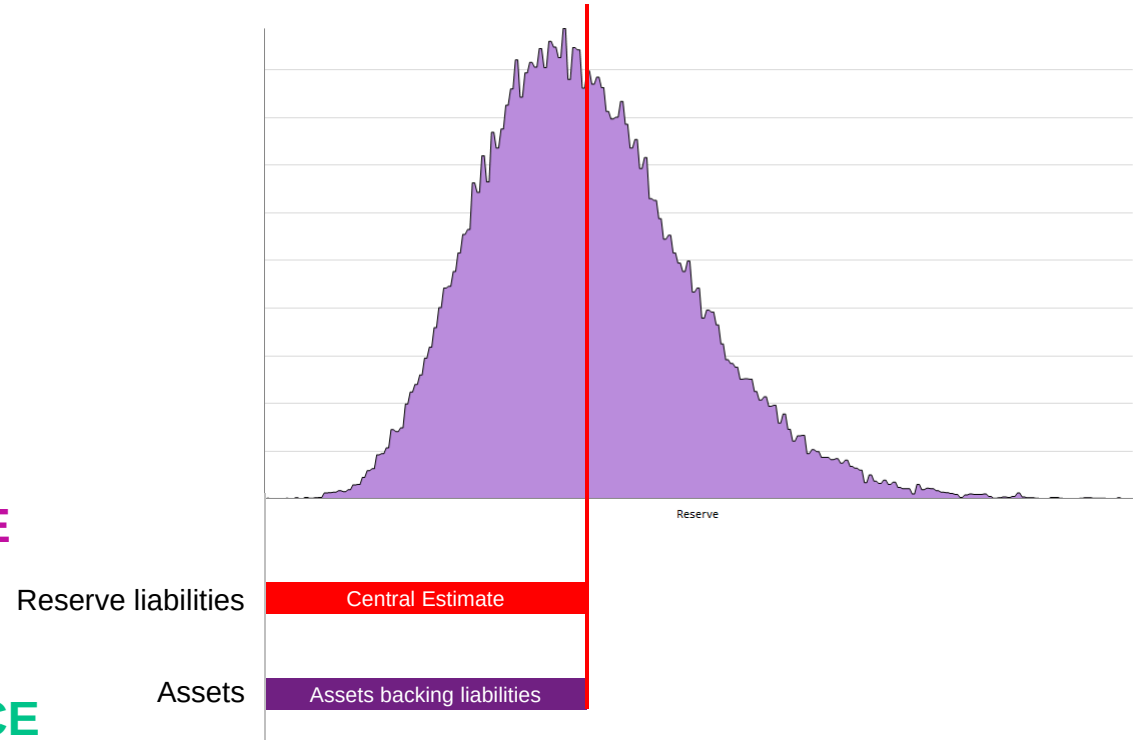

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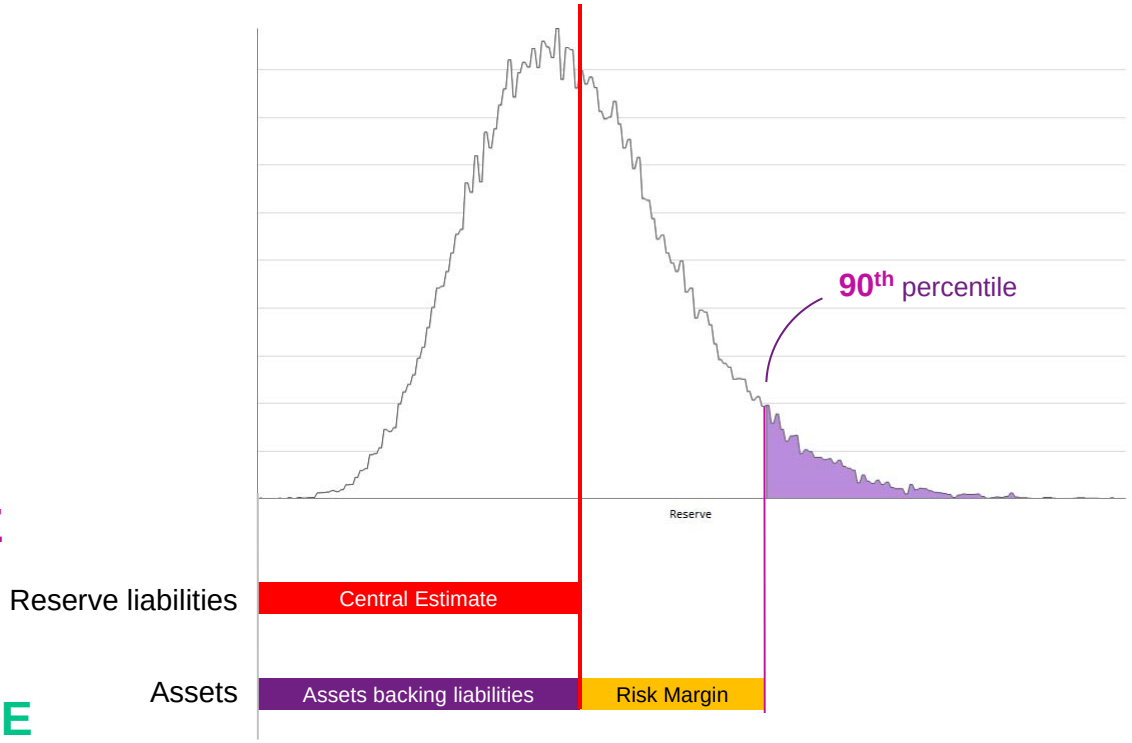

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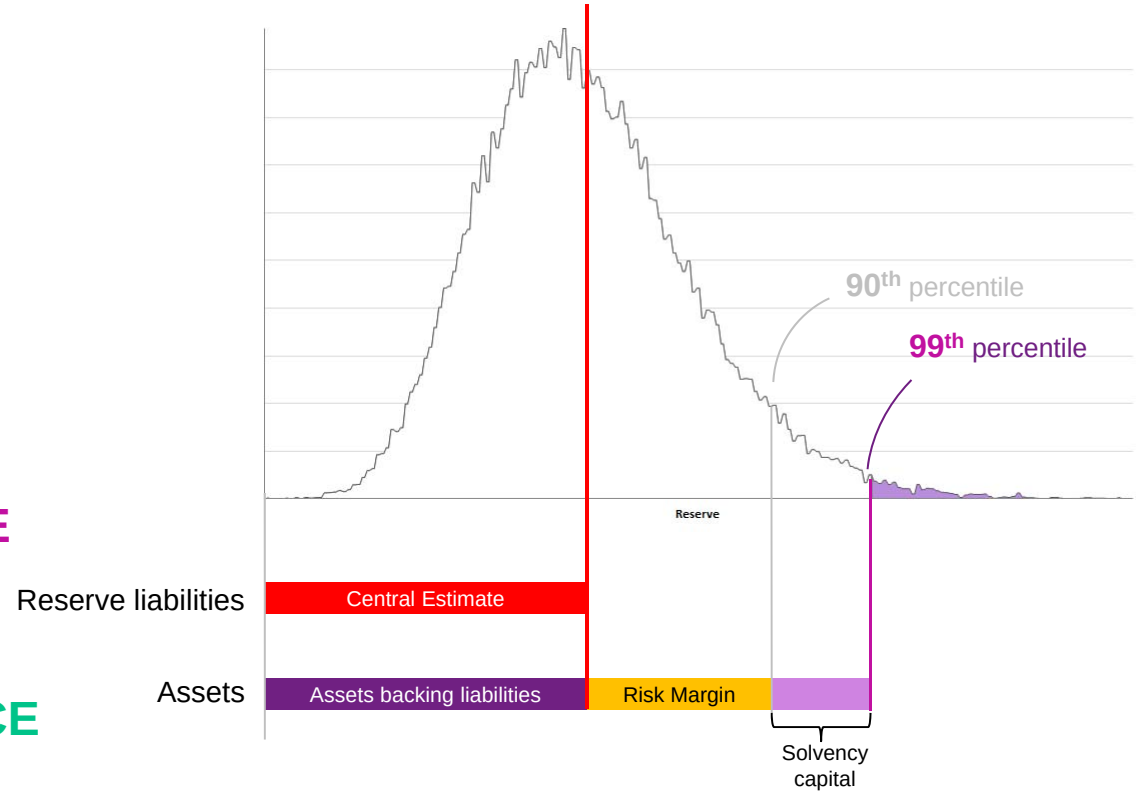

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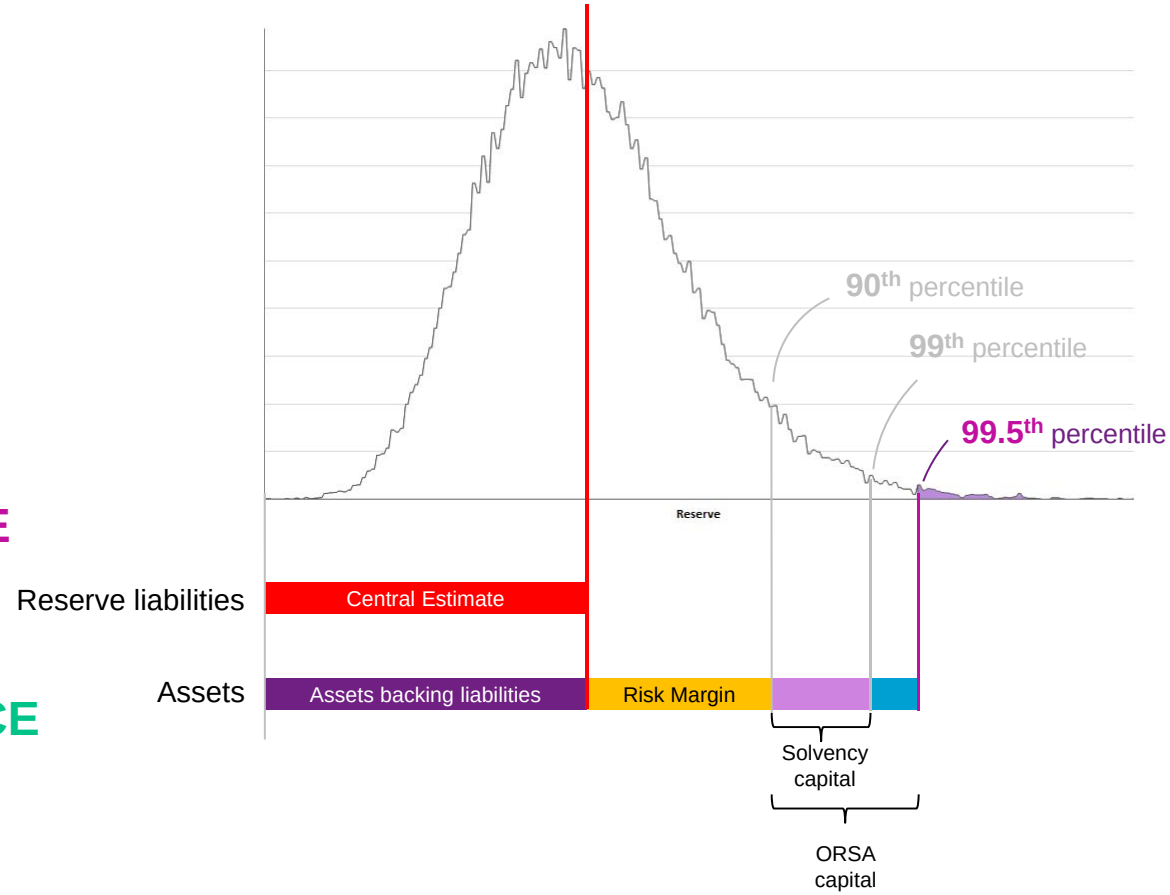

# Shaped by your requirements

# Shaped by the method

- So, what approaches should I use?

**Range of estimates?** *or* **Range of outcomes?**

**Sensitivity Testing**

**Scenario Testing**

**Alternative methods**

**Mack**

**Bootstrapping**

**Re-reserving**

**Merz & Wuthrich**

**Weighted sampling of simulations**

**Markov Chain Monte Carlo**

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**Central estimate is LDF-based?** *or* **GLM-based?**

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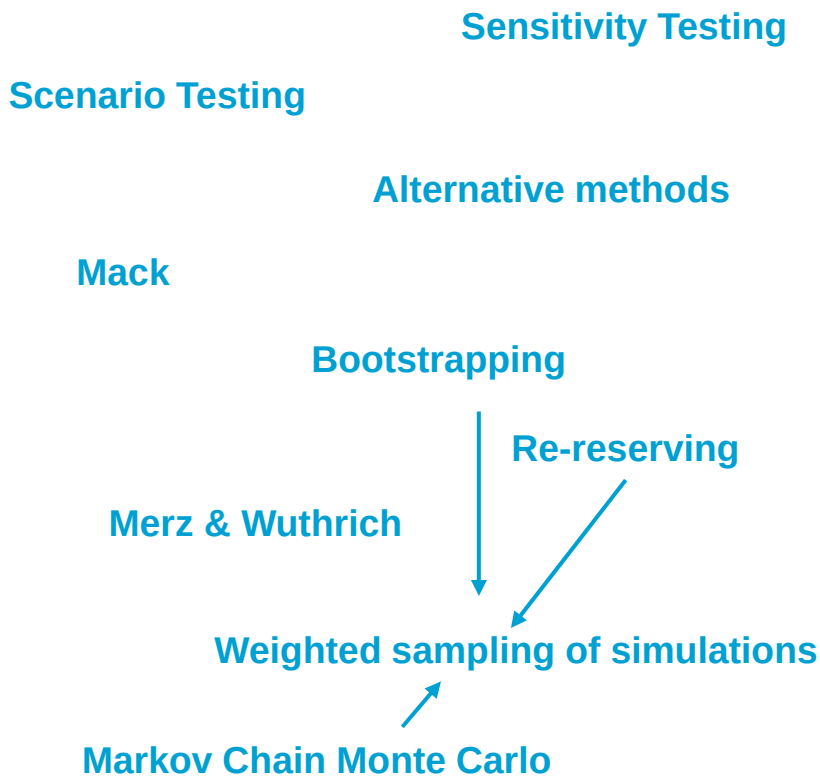
**Parameter error?** **Process error?** **Model uncertainty?**



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Variability over one year time-horizon? *or* Variability of eventual outcome?

Central estimate is LDF-based? *or* GLM-based?

Parameter error? Process error? Model uncertainty?

**Practical to aggregate across LoBs?**

**Sensitivity Testing**

**Scenario Testing**

**Alternative methods**

**Mack**

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# Shaped by the line of business

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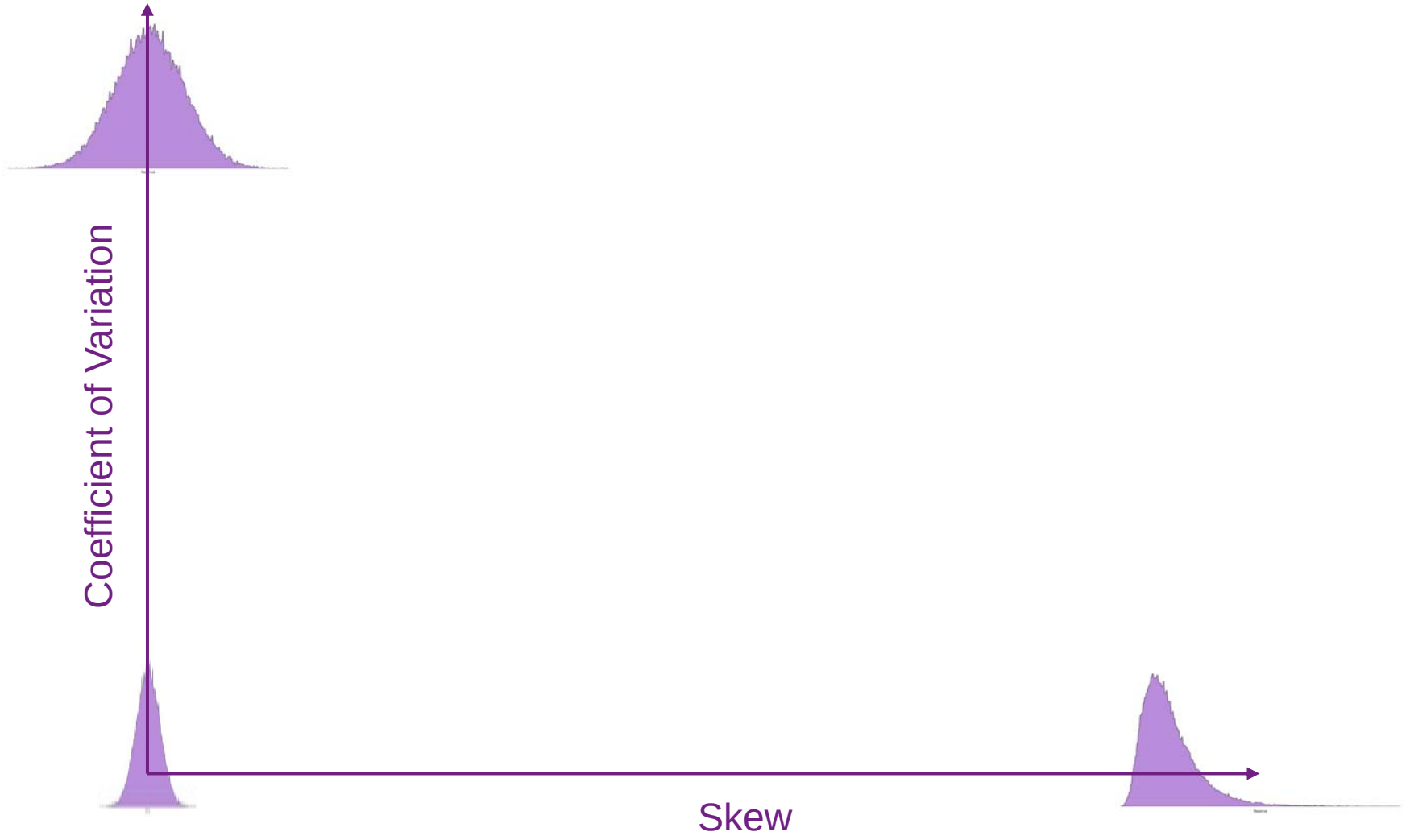

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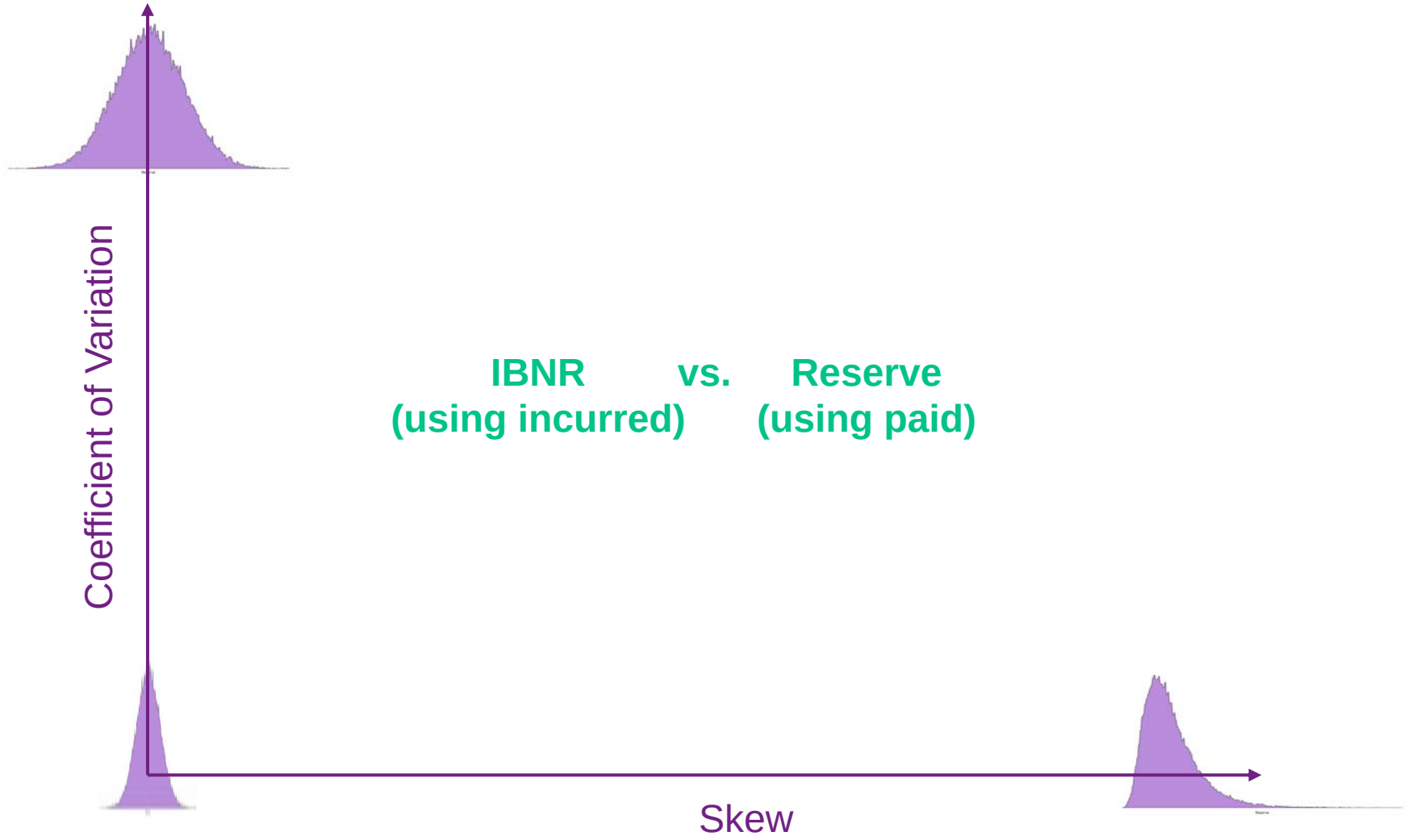

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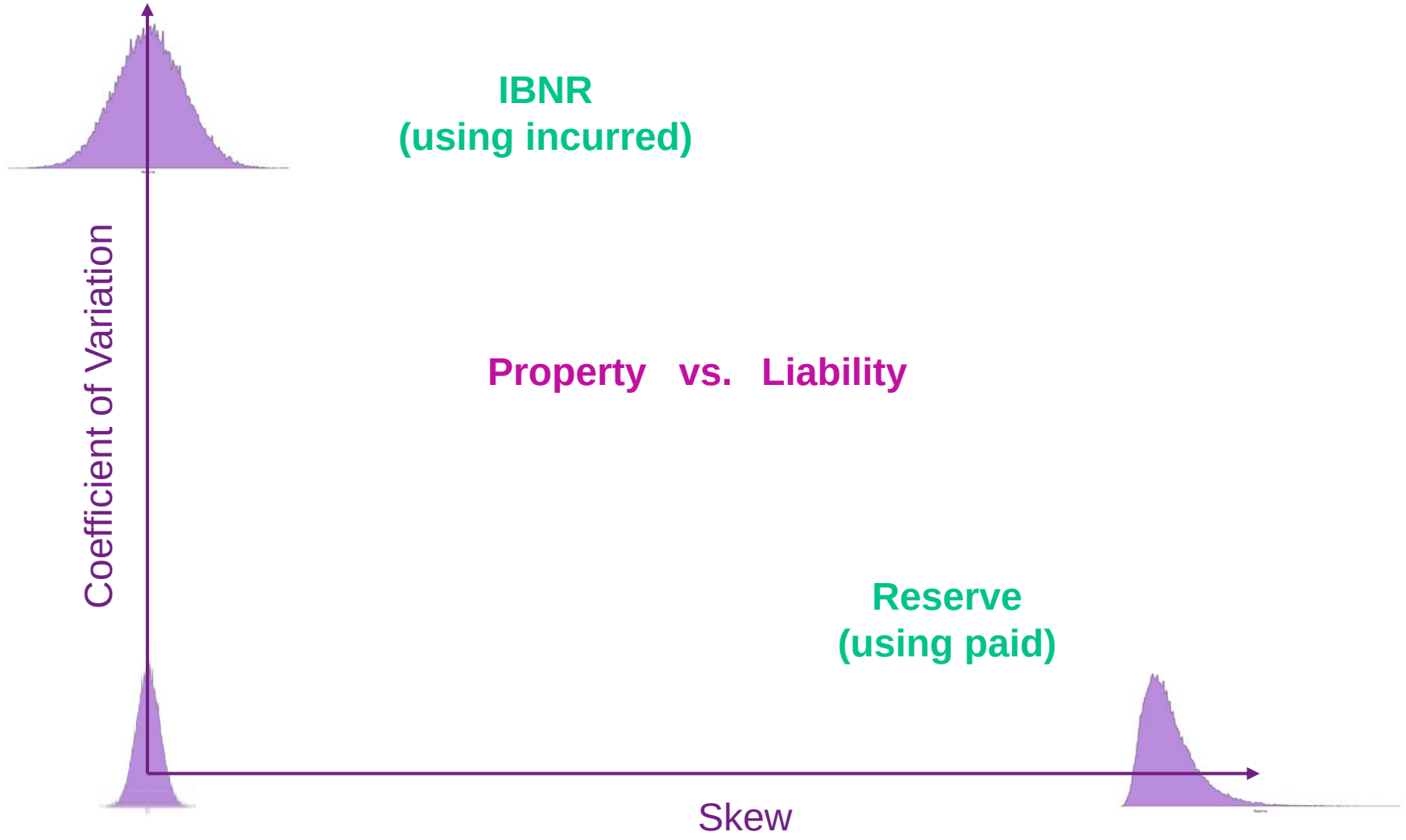

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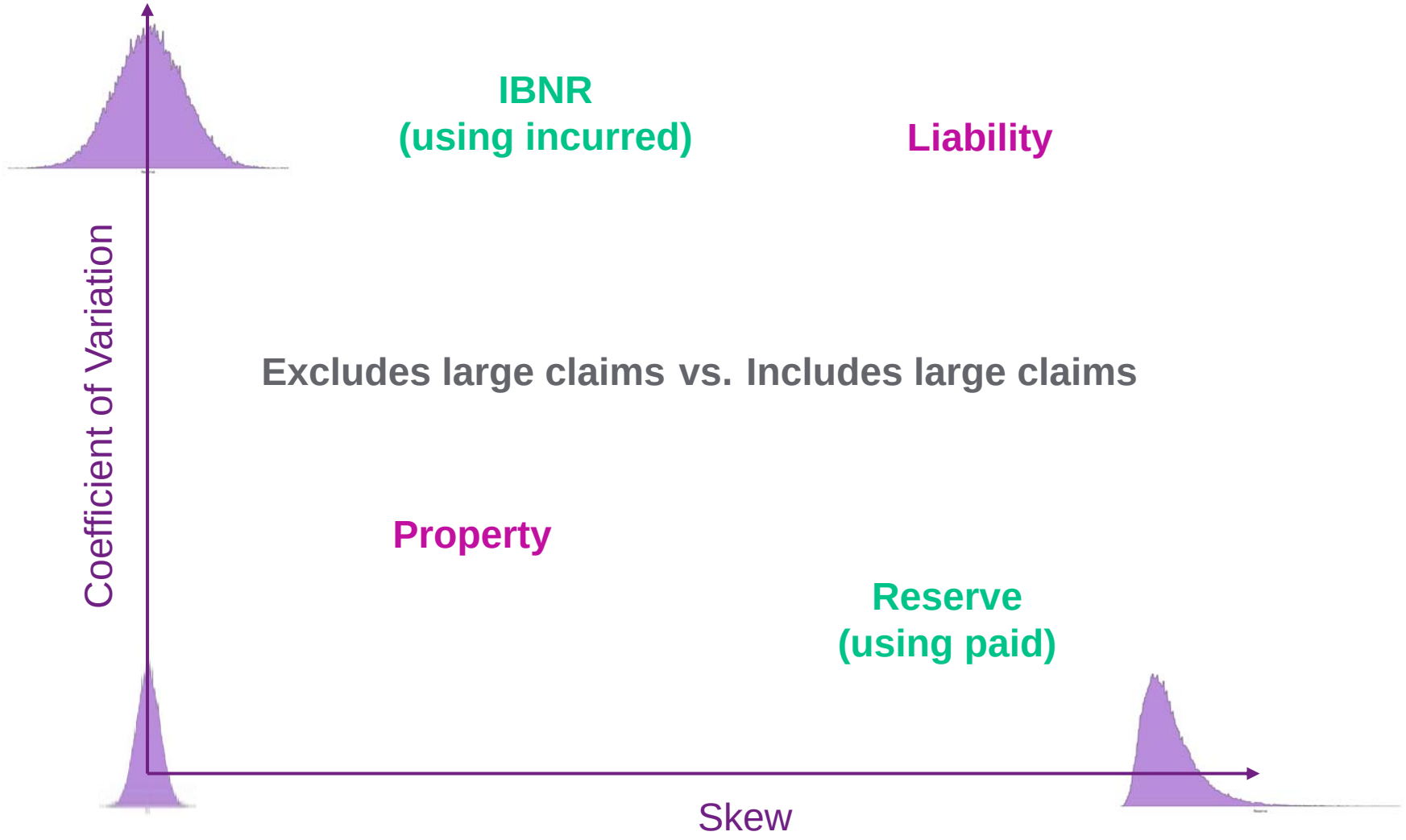
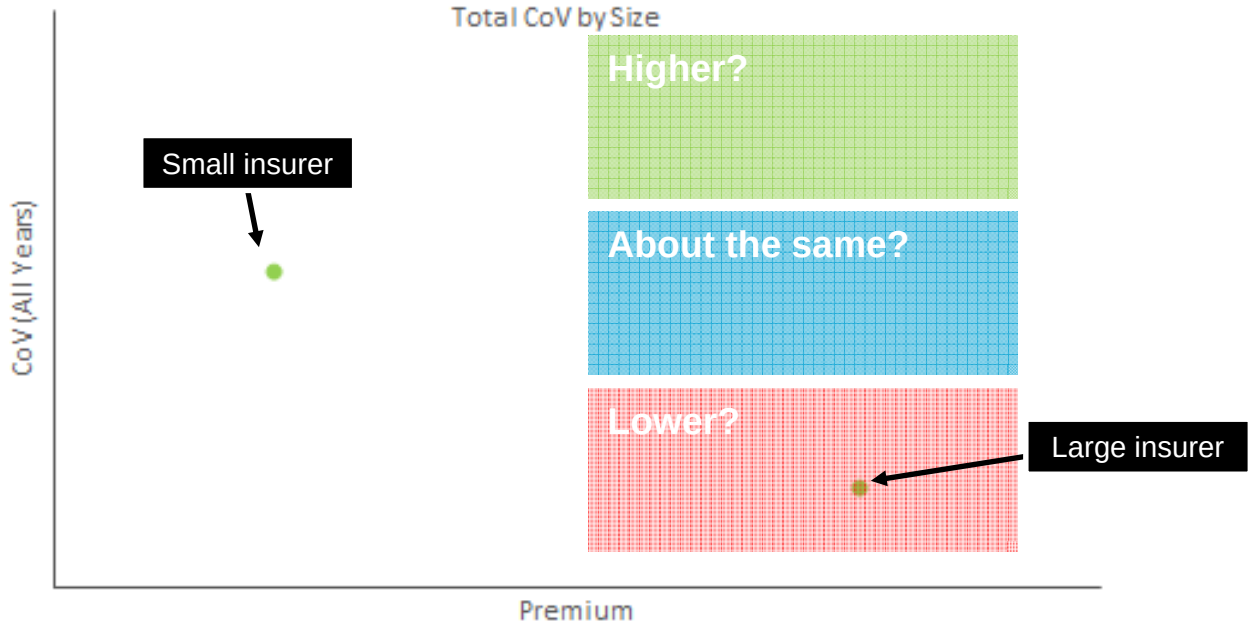


# Shaped by the line of business

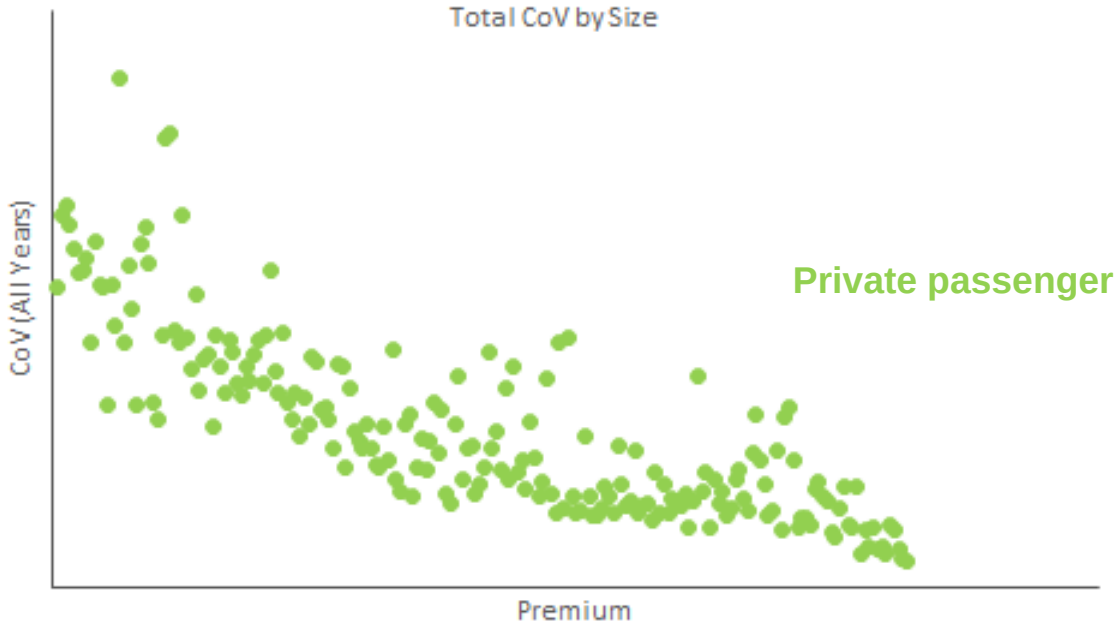
- Understanding how the shape of the distribution varies relative to the underlying business is important information when reviewing our degree of confidence around our estimate
- The probability of a -/+10% outcome is very different prospect for personal auto non-liability as it would be for umbrella coverage
- Understanding how the shapes of the distribution can vary across a business can also help understand and communicate how diversification affects the stability of a book

# Shaped by your industry

# Shaped by YOUR line of business

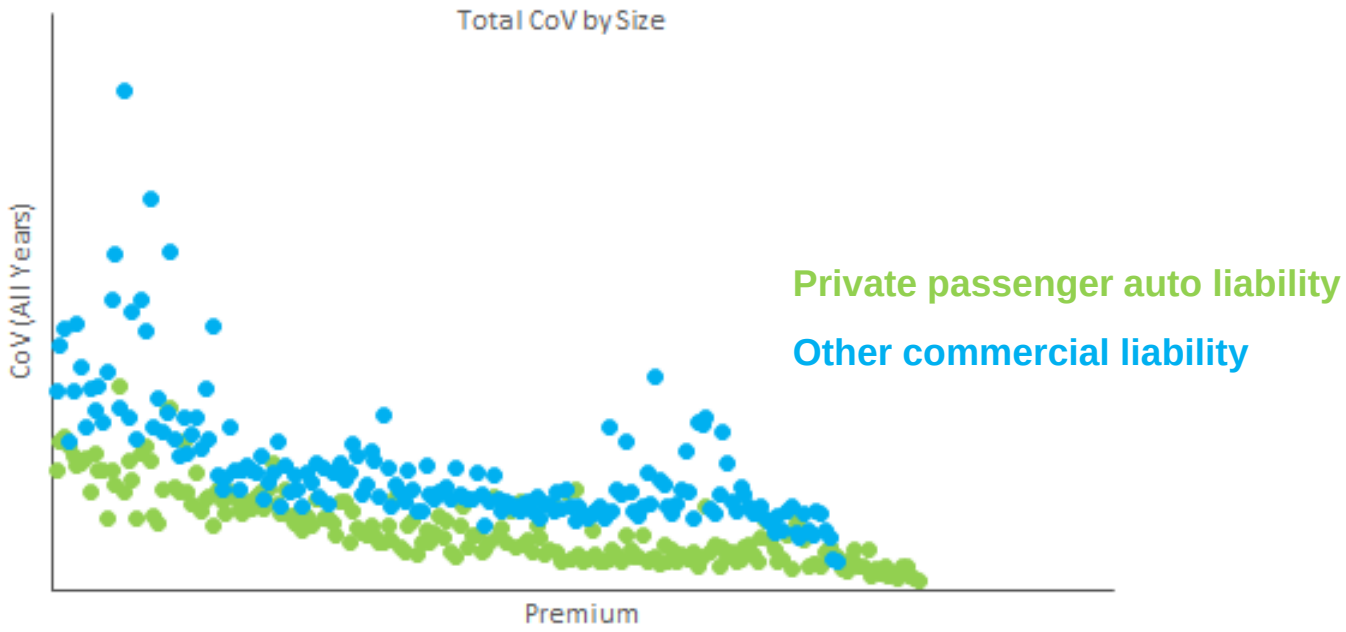


# Shaped by YOUR line of business




**Private passenger auto liability**

# Shaped by YOUR line of business



# Shaped by YOUR line of business

- Just because we may compare two lines of business that are similar, we may expect very different shapes to our uncertainty. E.g.
  - Small volume vs. large?
  - Stable volume vs. growing or contracting?
  - Gross or net?
  - Minimum vs. high limits?
  - Impact / inclusion or other non-RI recoveries?
  - CATs are included vs. excluded?
  - etc

# Shaped by YOUR line of business

- Furthermore, although we're modeling a range of expected *outcomes*, those outcomes are dependent on the validity of the underlying model and the data included in that model. For example:
  - Bad selections in the underling chain-ladder methods (for bootstrap-based approaches) or GLM (for MCMC-based approaches)
  - Volatile claim payment history due to changing processing systems
  - Shock or sudden change in case reserves as a result of a file-review or new claims manager
- We therefore need to be very clear when interpreting and communicating our results. Our estimate of the eventual outcome is not just based on the exact business being written, but also the:
  - Specific nature of *your* business (size, policy types, etc)
  - The way that the data has been affected by internal or external processes or changes to those processes
  - How the data has been projected to ultimate

# The shape of uncertainty

## Summary

The range that we produce around our central estimate is going to be influenced by a number of factors. Our uncertainty will be:

- Shaped by our **analytical requirements**
  - What ***time horizon*** are we interested in?
  - What do we intend to include within our ***risk profile***?
  - What ***risk metric*** do we wish to use?
  - What value of that metric determines our ***risk tolerance***?
- Shaped by the **method** or approach that we use
  - Deterministic vs. analytical vs. simulated
  - Development factor vs. incremental amounts
  - Type of error included
- Shaped by the **lines of business** being analyzed
- Shaped by the **characteristics** of your data and projection models

# The shape of uncertainty

*We are now going to look at a* ***risk metric*** *that may help understand and communicate the variability associated with events in the "tail" of a distribution*

# Fat Tails in Risk Models

David Ingram CERA, FRM, PRM

CLRS
2018

# Fat Tails

Many risks taken by insurers have Fat Tails

## Fat Tails

So Why is that a Problem?

1. We model risks
2. We have no data to fit to tails
3. So we extrapolate
4. And we validate our models by validating our extrapolation process
5. We also explain our models with a process description

6. That leaves non-modelers in the dust

7. Which may be a problem

## Today's Talk

"Chasing our Tails with Risk Models"

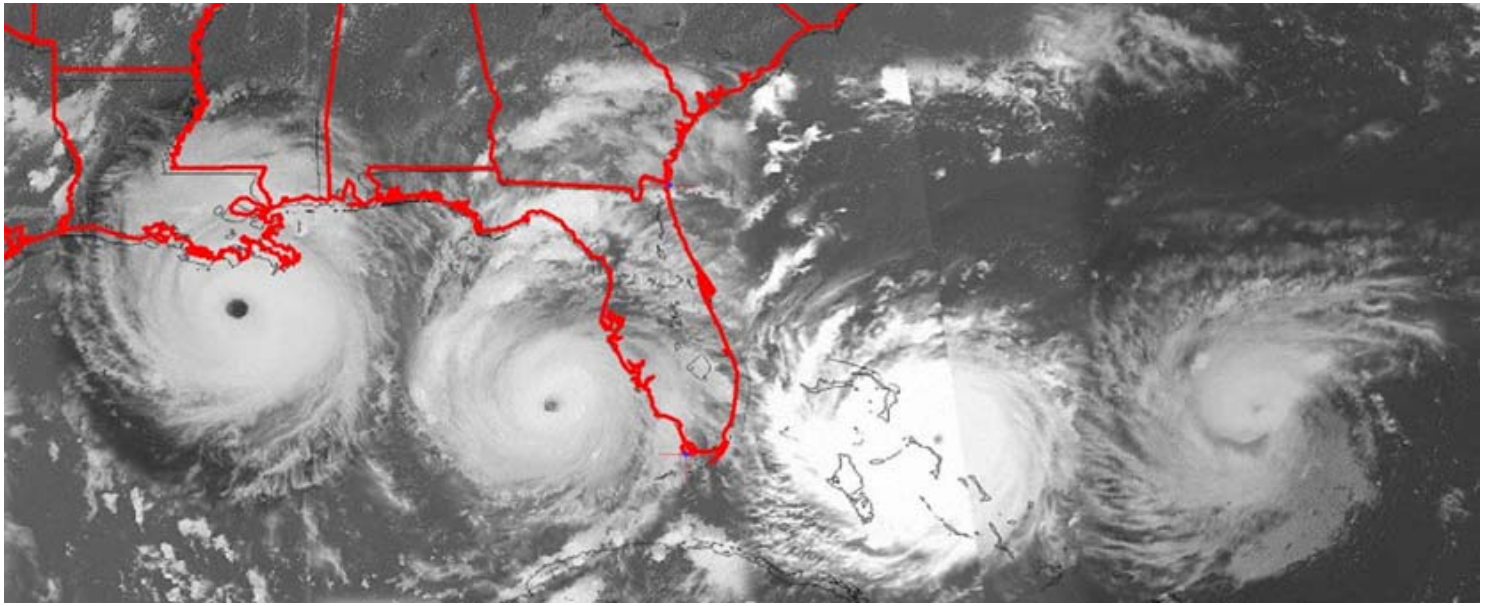
How we might bridge the gap between modelers and non-modelers regarding Fat Tails

- Suggest using a new/old metric
  - **Coefficient of Risk (COR)**
- **Provide a variety of examples of COR values and use**
  - **Underwriting Risk Models**
  - **Reserve Risk Models**

# Natural Catastrophes

- Earthquakes, Hurricanes, Typhoons, Tsunamis, Floods are all the end stage of a system that has exceeded its capacity

- When capacity is exceeded, things are thrown into a different system where great deals of energy are released, rather than being dampened within the system.



## Why do big complex systems fail

A **Bias** of many systems analysts

- Many believe that complex systems are inherently fragile
- Natural systems usually develop natural control systems
- Big complicated human systems are sometimes fragile

Ashby's Law a.k.a.

**The Law of Requisite Variety**

# COEFFICIENT OF RISK

# Fat Tails

- **Definition:**
  - A Fat Tail means that high severity/low probability events are more severe/more likely than would be predicted by a Gaussian distribution
- **Why is this an issue?**
  - Many risk models had assumed Gaussian distribution of one or all risk drivers
  - Many risks actually have Fat Tails
- **Solution:**
  - Use Fat Tailed Model

# Fat Tails

- So are we done with this talk already?
  - Perhaps not.

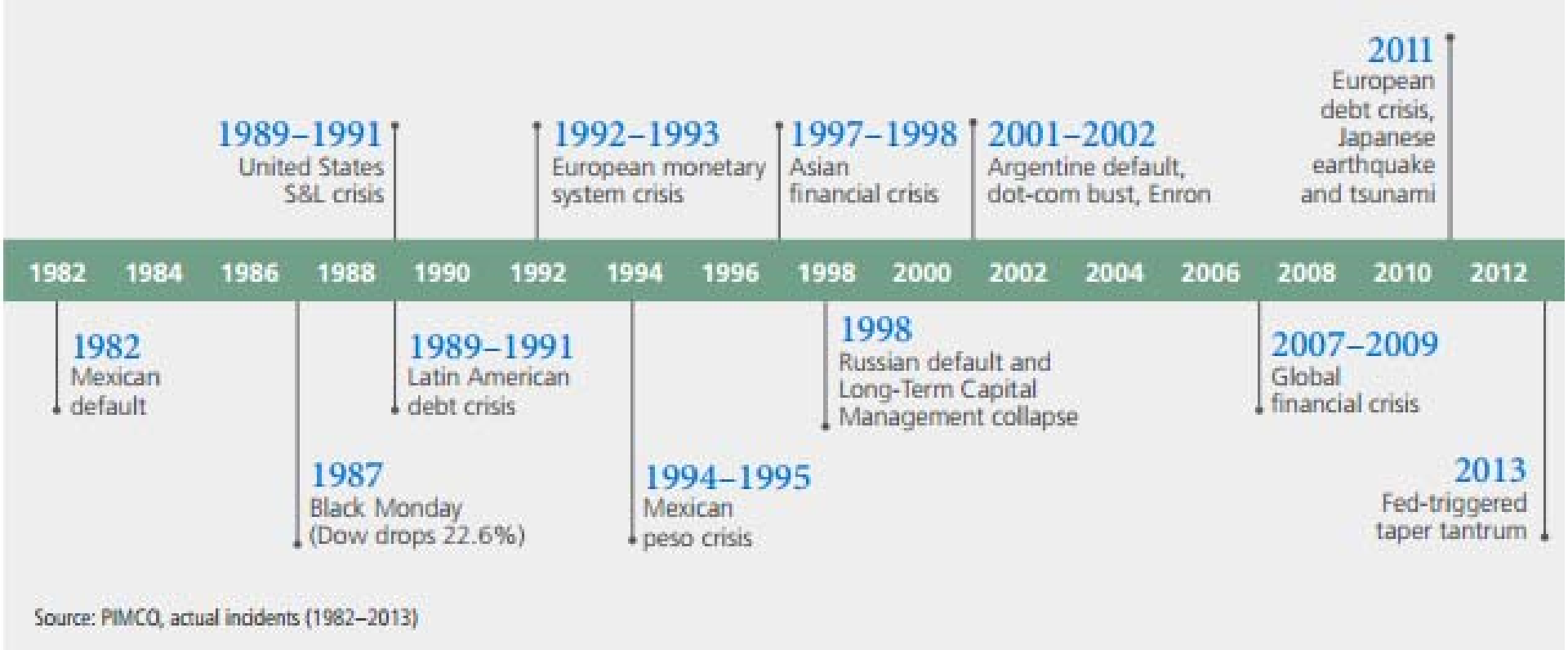
- Questions:
  - How Fat are the Tails of your Model?
  - Why should anyone believe what your model says about the tail values?
  - Are they Fat enough? Or Too Fat?
  - How do they compare with the Tails of other Models?
  - How Fat should the Tails be?
  - Who should be involved in deciding?
  - Can you explain your answer to any of the above questions to anyone who is not a modeler?

## Four Models

How do they each see the world?

| | |
|---|---|
| **Natural Decision Making**<br>*From the Gut* | **Newtonian**<br>*Logical* |
| **Statistical**<br>*Future as Multiverse* | **Systems Analysis**<br>*Complex Independencies* |

# Fat Tail Incidents



# Coefficient of Risk$_{1000}$

- Use 1 in 1000 loss as a proxy for the tail of the distribution of gains and losses
- With CLT assumed Extreme Loss is quick and easy to determine
- Tail is 3.09 standard deviations worse than the mean
  - For simplicity, round to 3
- Call that the **Coefficient of Risk (CoR$_{1000}$)**

$$CoR = \frac{V_{999} - \mu}{\sigma}$$

# Chebyshev's Inequality

- CoR is the **k factor** in *Chebyshev's Inequality*

$$\Pr(\,|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

| k | Percentile |
|---|---|
| 10.00 | 99.00% |
| 14.14 | 99.50% |
| 15.81 | 99.60% |
| 22.36 | 99.80% |
| 31.62 | 99.90% |

# Preliminary Tests of COR

- The following slides show some preliminary tests of the COR calculation applied to many insurance risk models that were developed by Willis Re actuaries for our clients

- These tests show that in many cases the insurance blocks have much higher COR's than 3.09

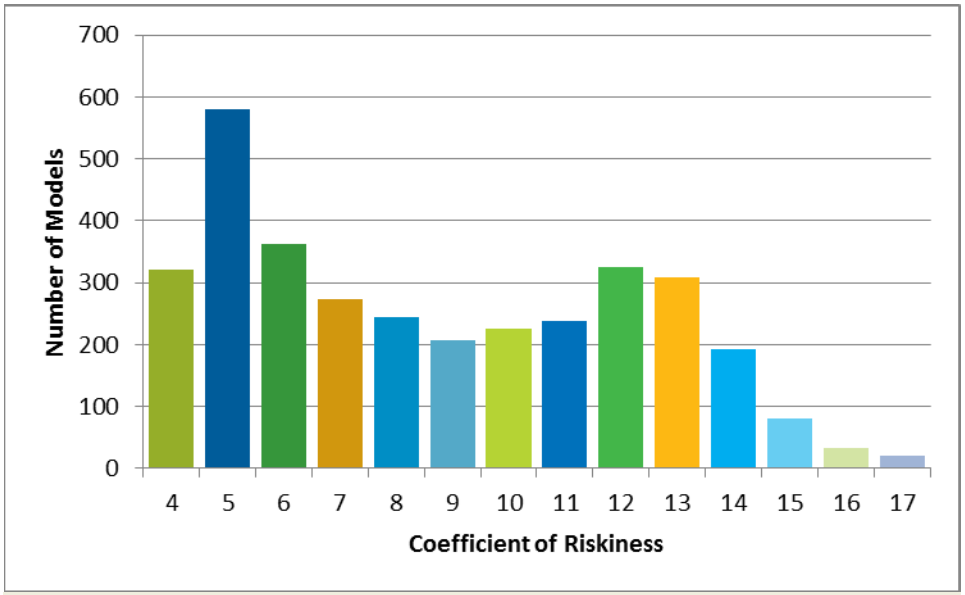
- Will also use the ratio of COR from a model to COR for a Gaussian Model

  - COR1000/3.09
  - COR200/2.58

  And call that Tail Fatness

# Test of Coefficient of Riskiness

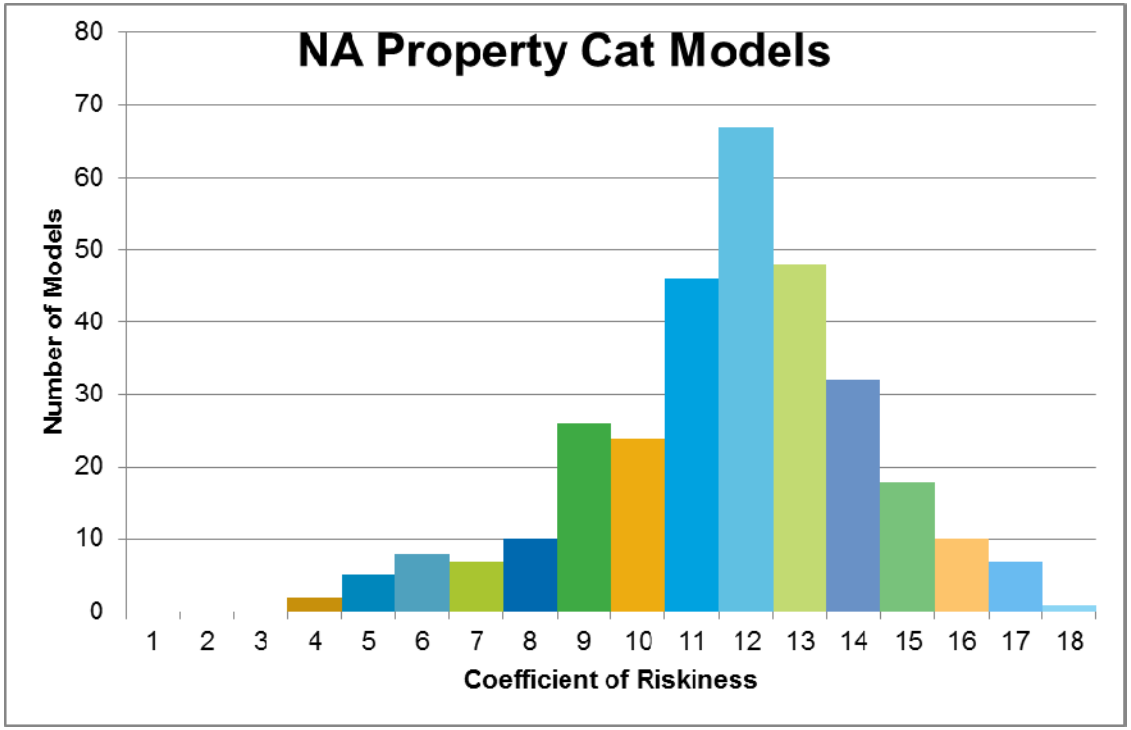
- COR was calculated for 3400 insurance models that were created by Willis Re actuaries over 2011-2014
- This is a plot of all of those 3400 mixed insurance risk models.
- Next step will be to stratify those 3400 models by type.
- For instance, we note that the model with the highest COR is a Homeowner only model for a single state company in a Nat Cat zone.




**Note:** COR 4 indicates value is 3 – 4, etc

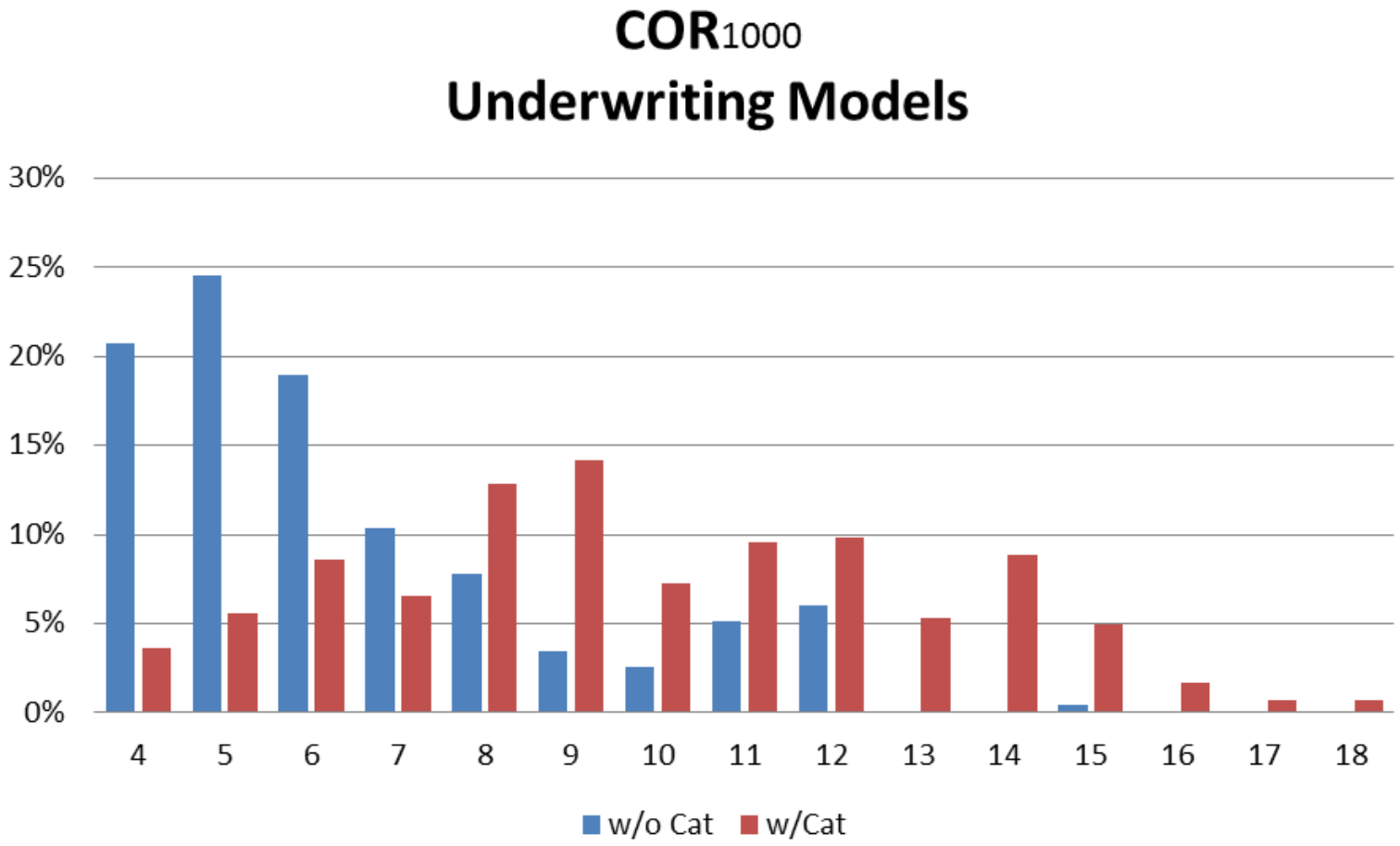
# Stratification of Models

- This plot looks at 400 models of Property Risk Natural Catastrophe (Windstorm &/or Earthquake) losses

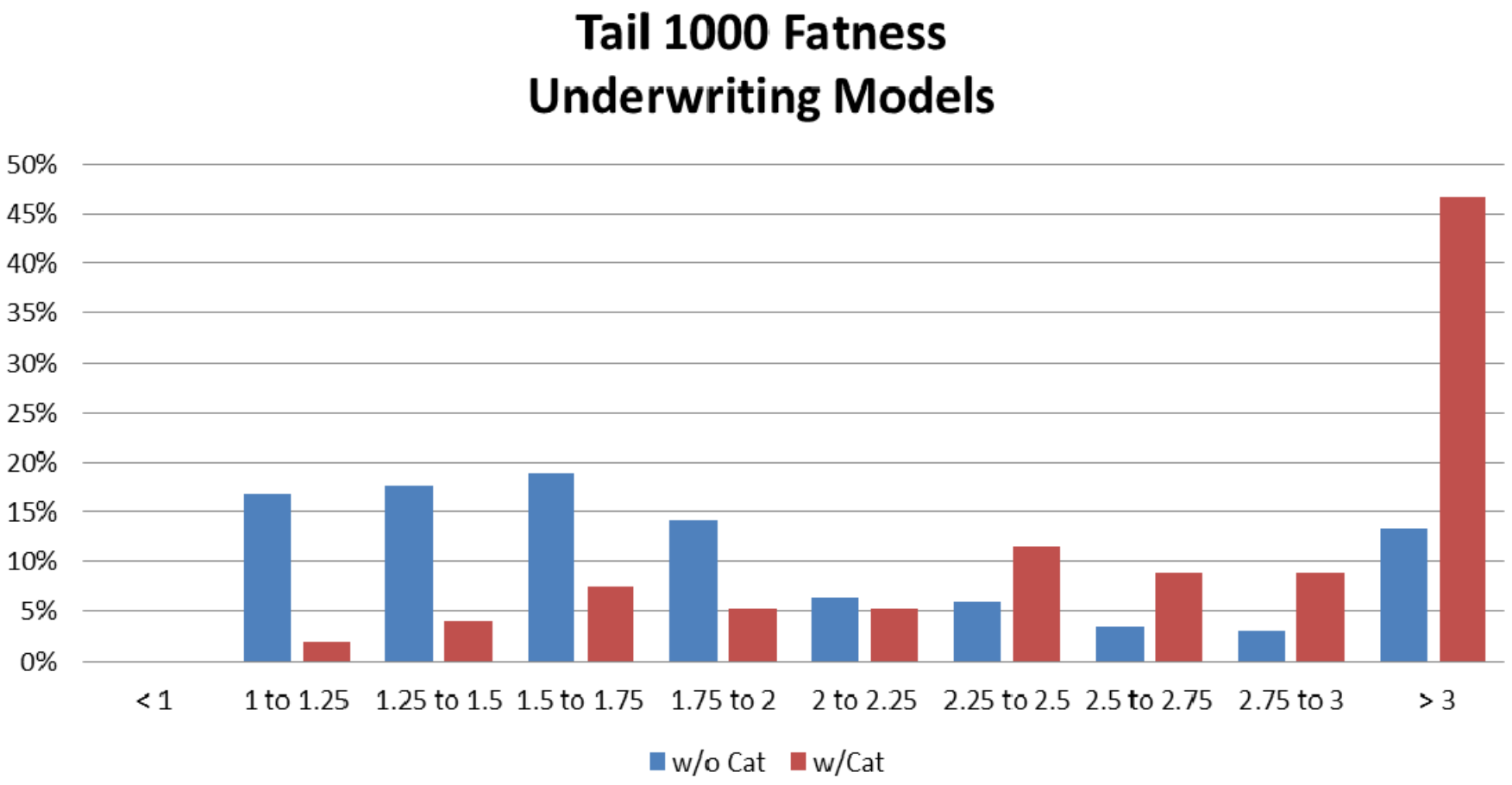


# Underwriting Models

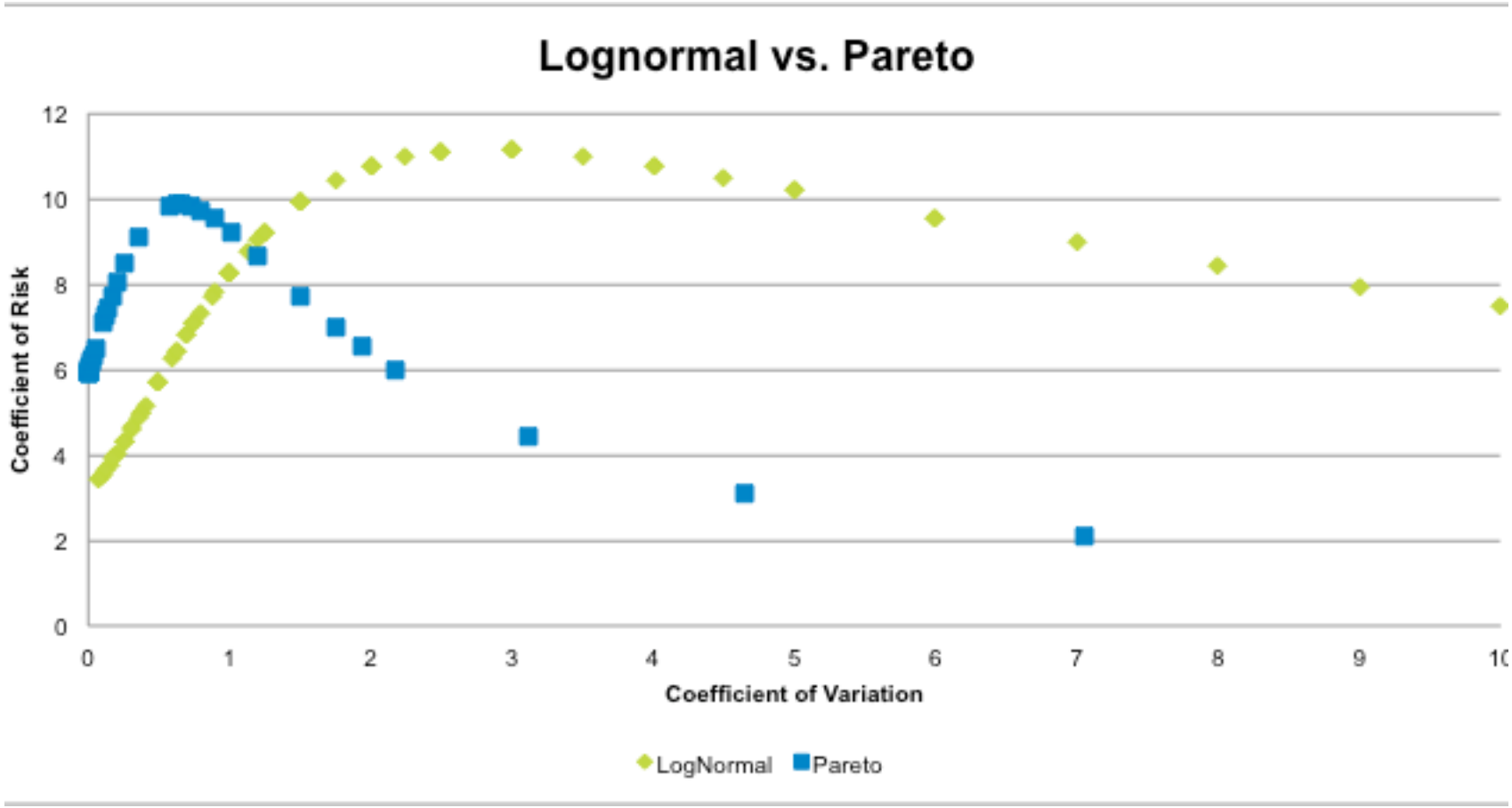
COR 1000 with and without cat risk



# Tail Fatness at 99.9%tile – Underwriting Models

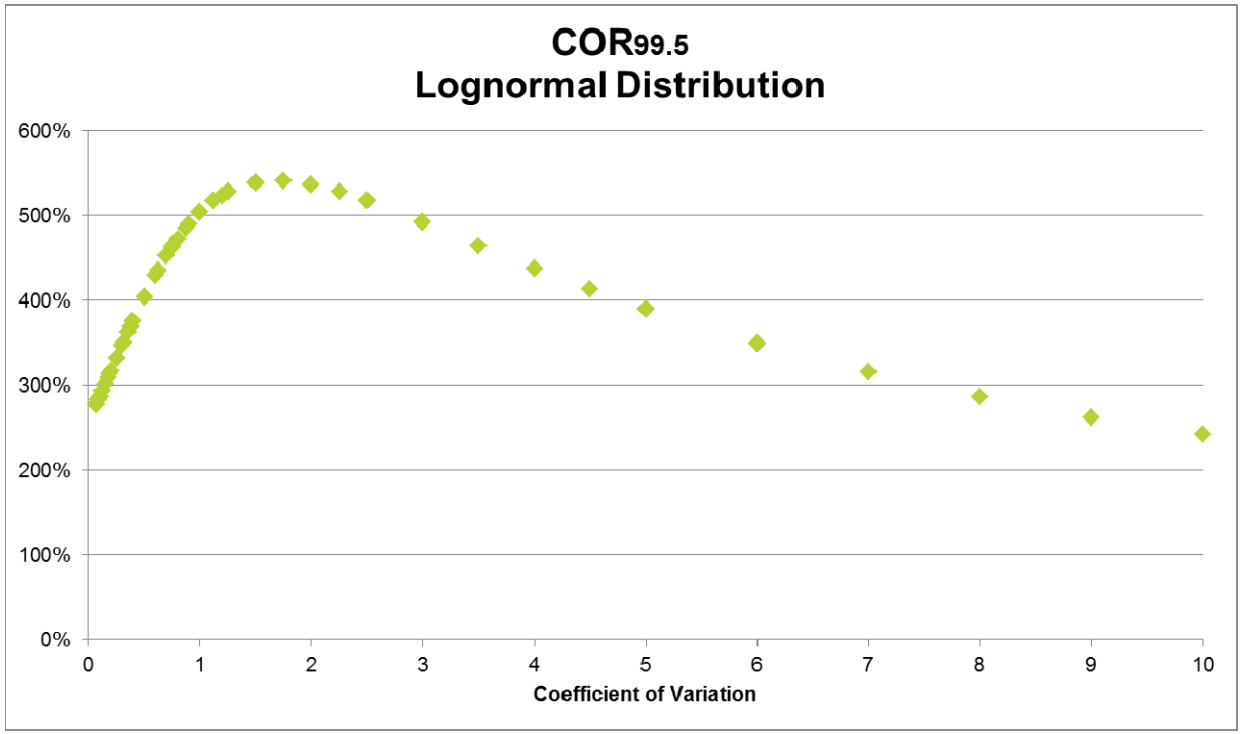


# Distributions 99.9%tile

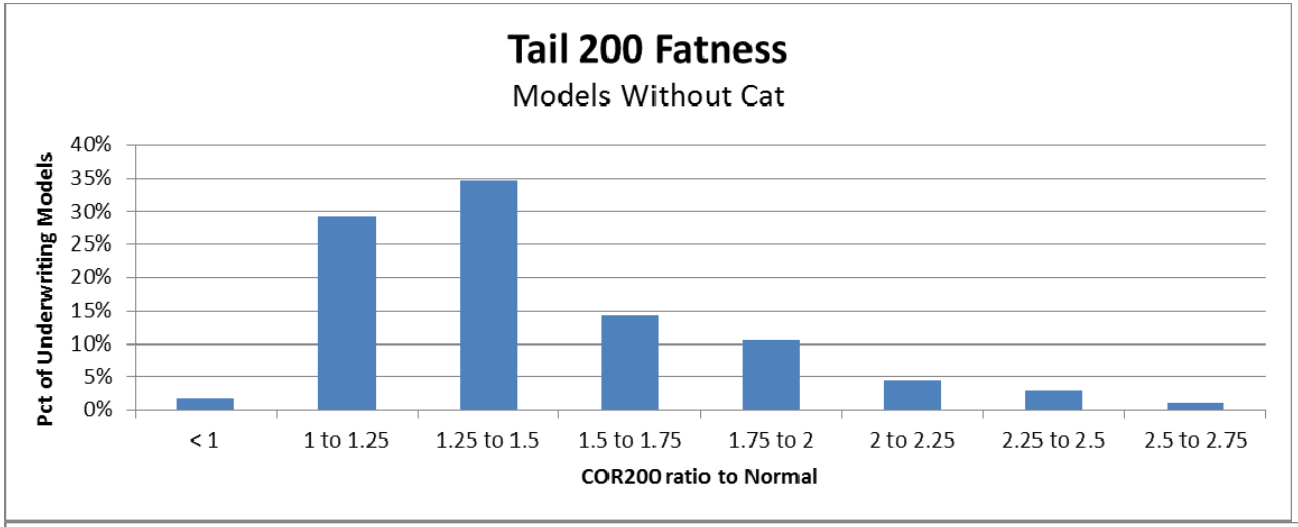


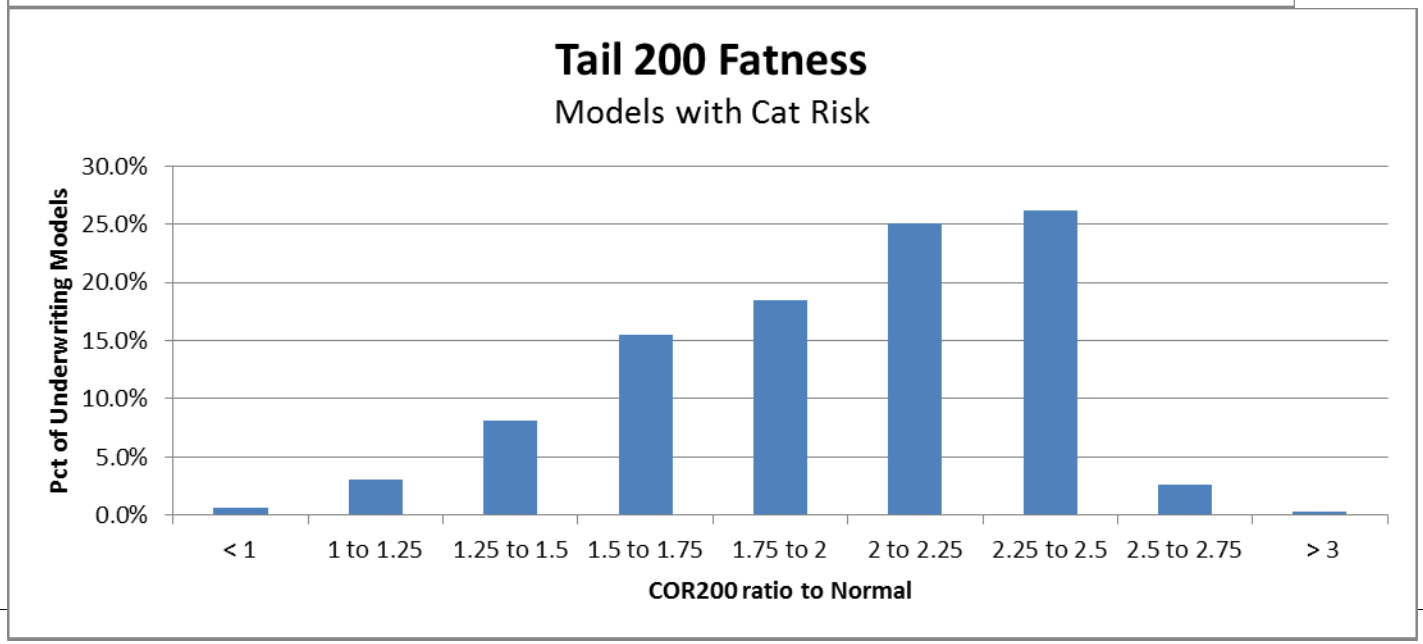
# What about 99.5%tile?

- All of this discussion applies equally to 99.5%tile

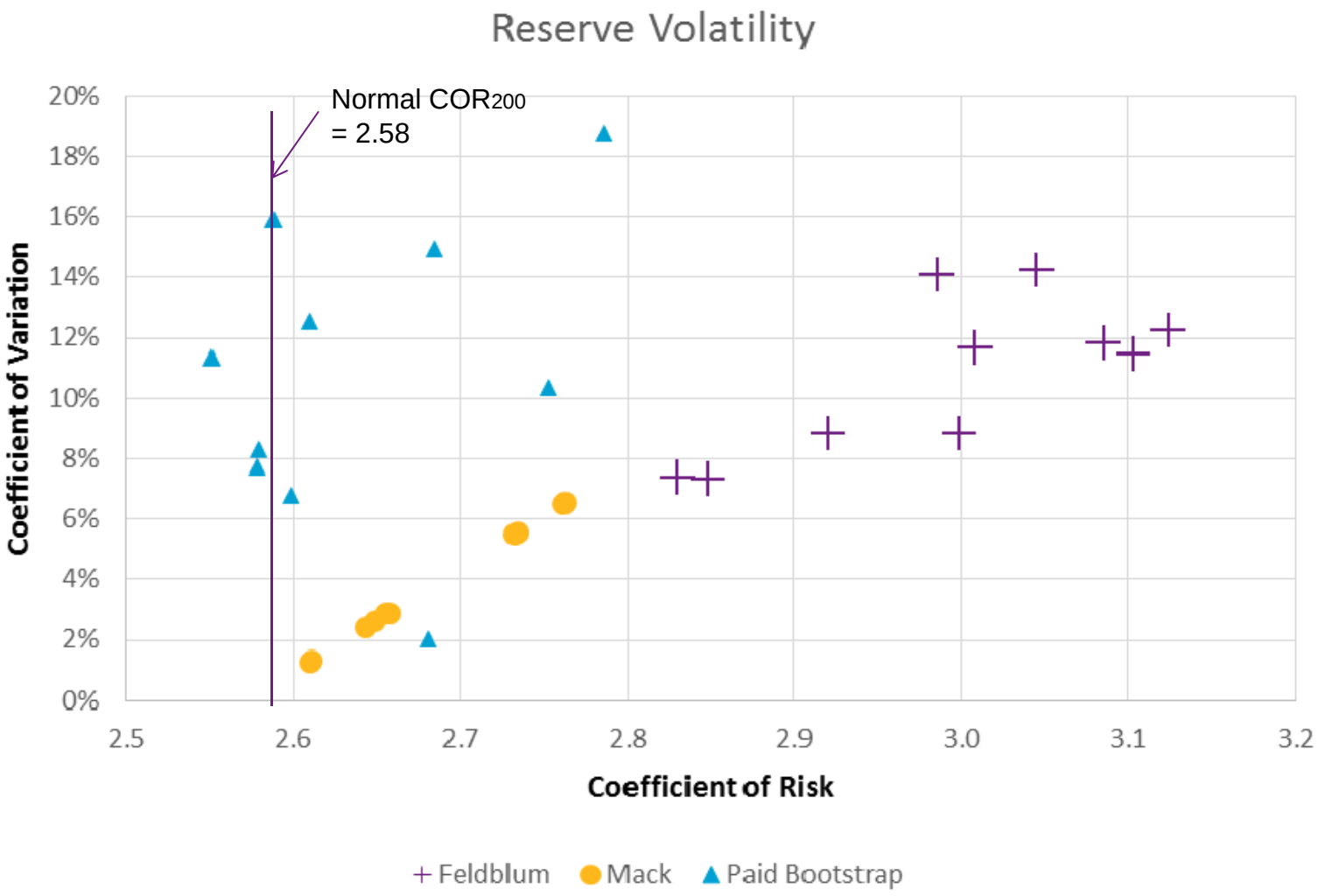


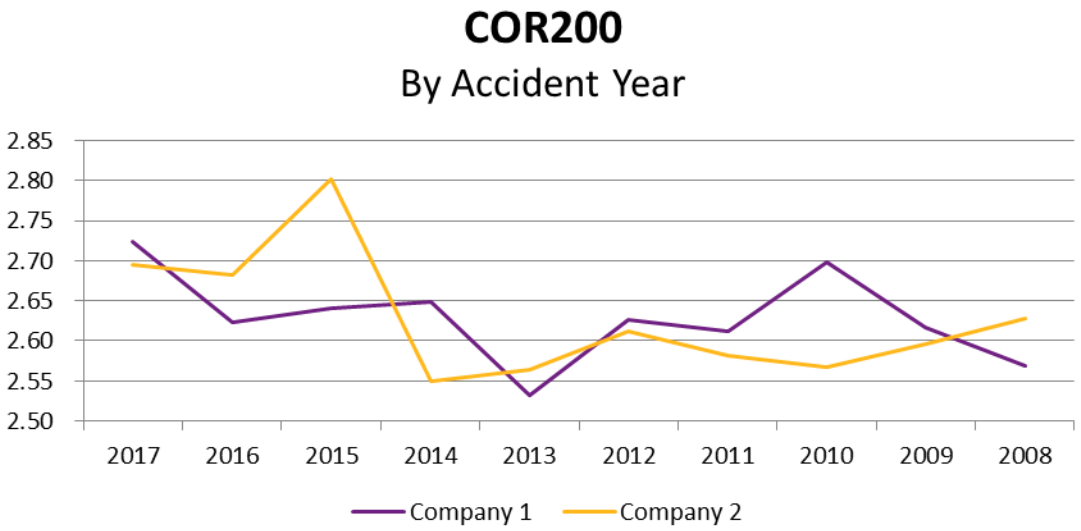
# Tail Fatness at 99.5%tile – Underwriting models

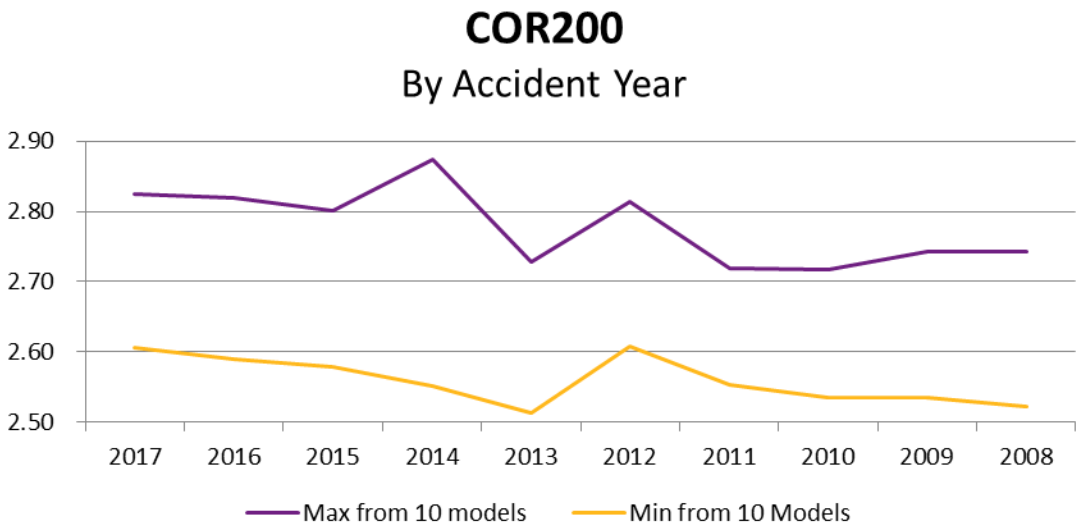


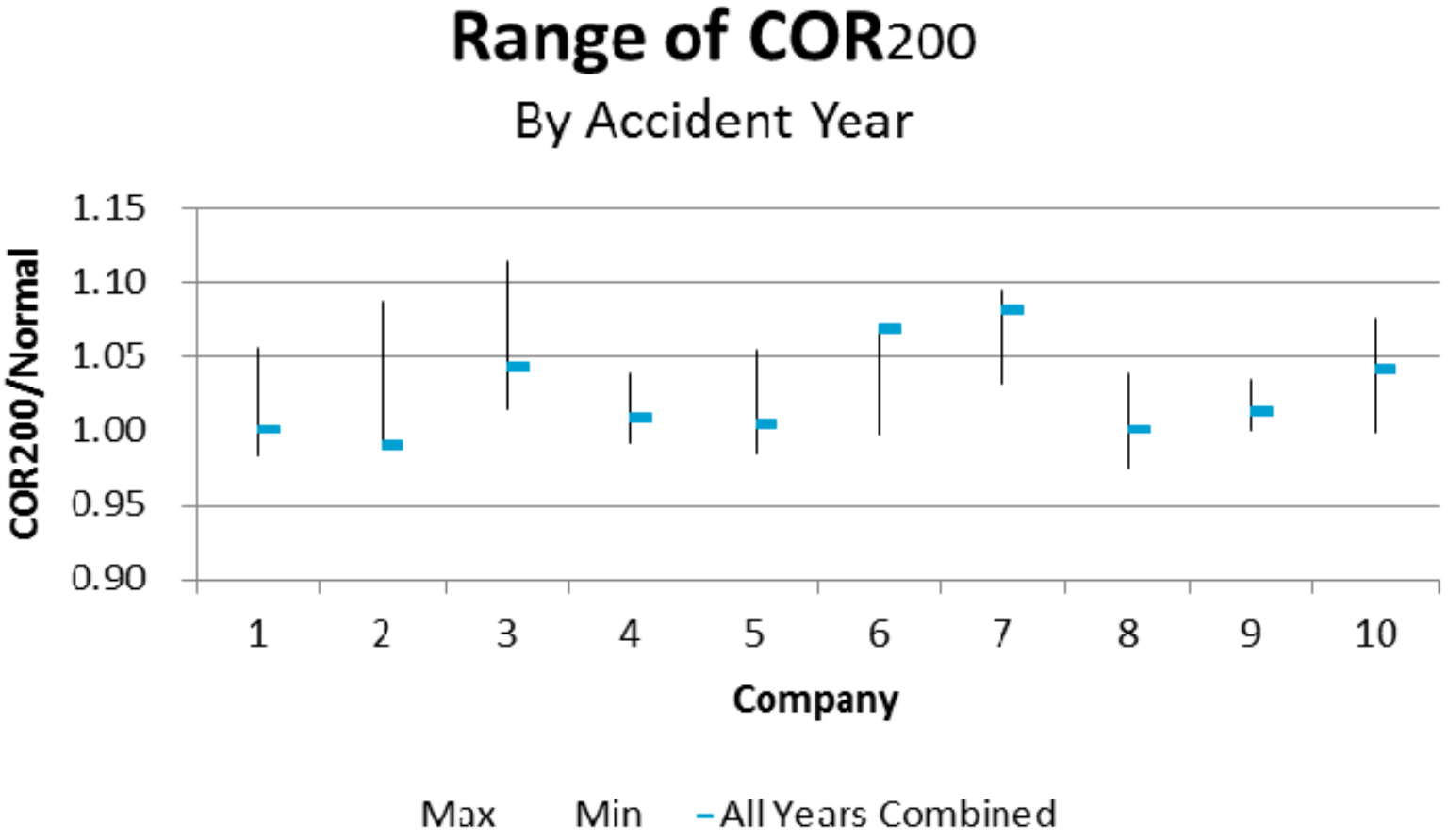
# Reserve Volatility – 99.5%tile COR





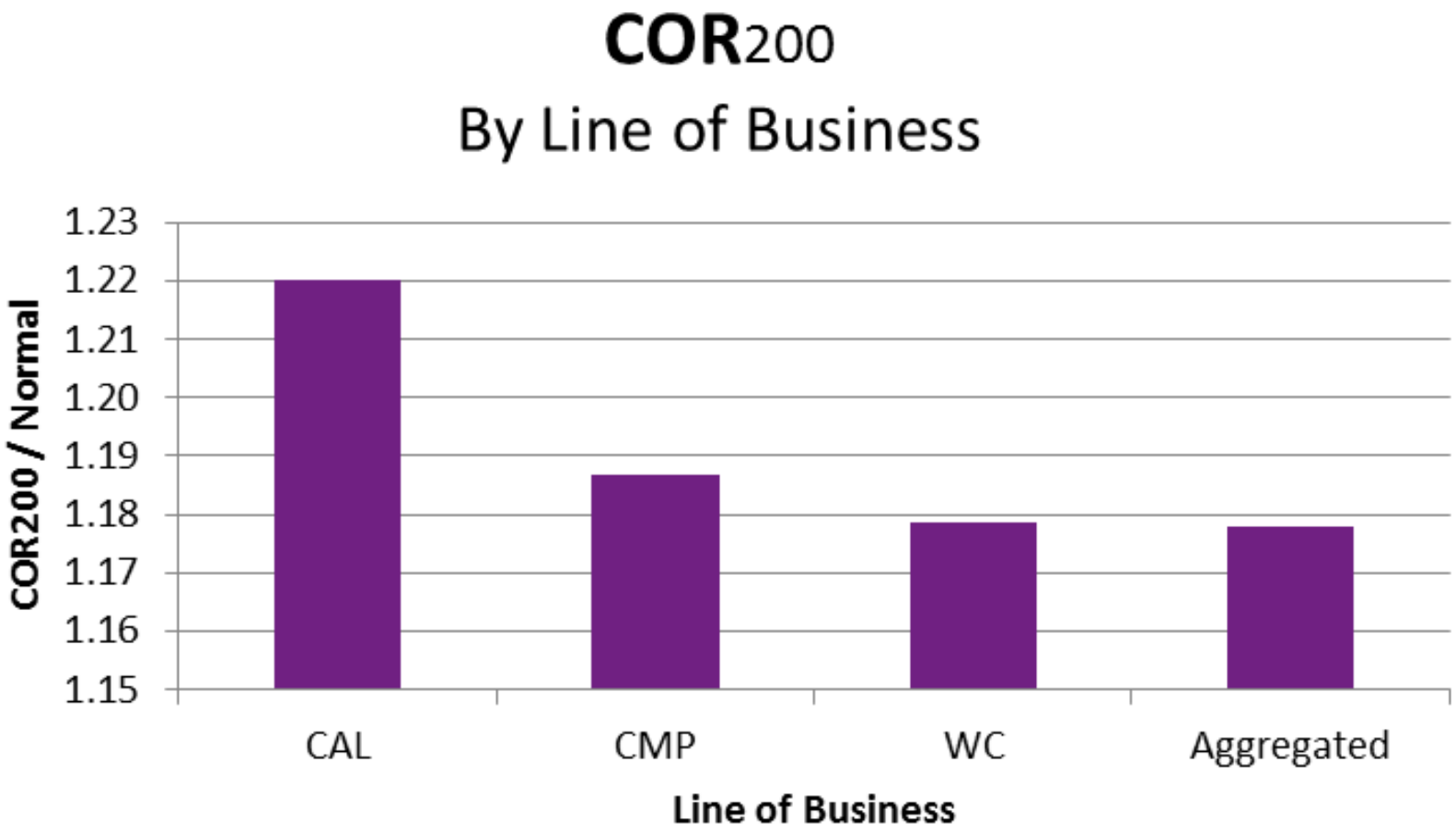



# Individual Accident Years are not so Fat

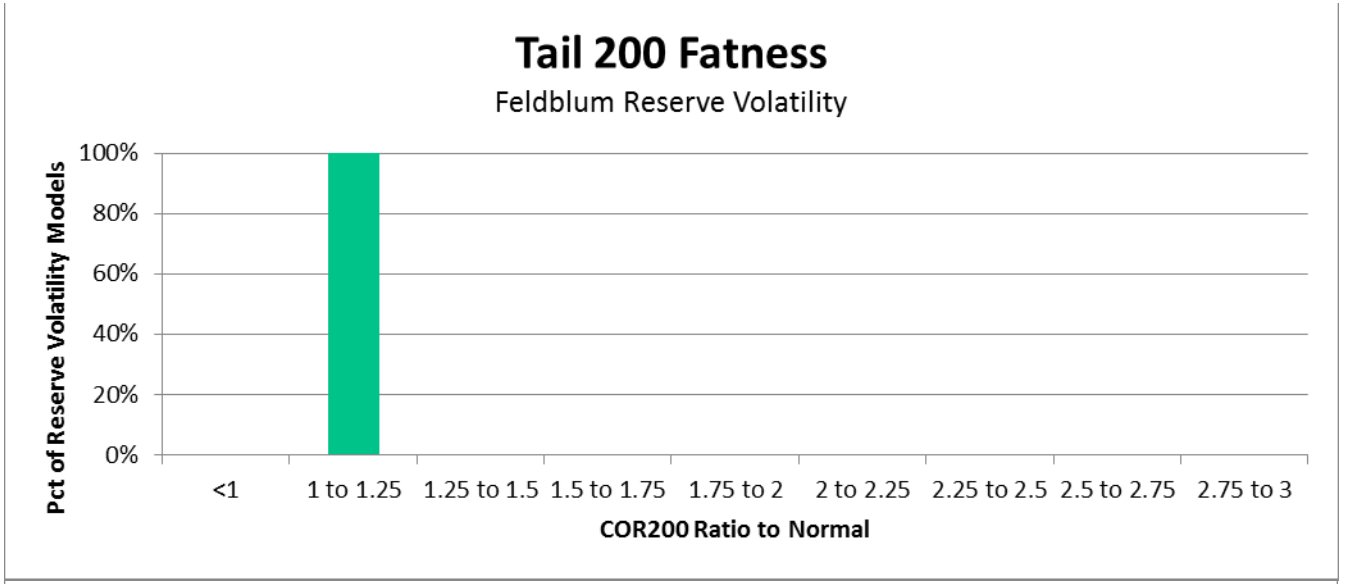


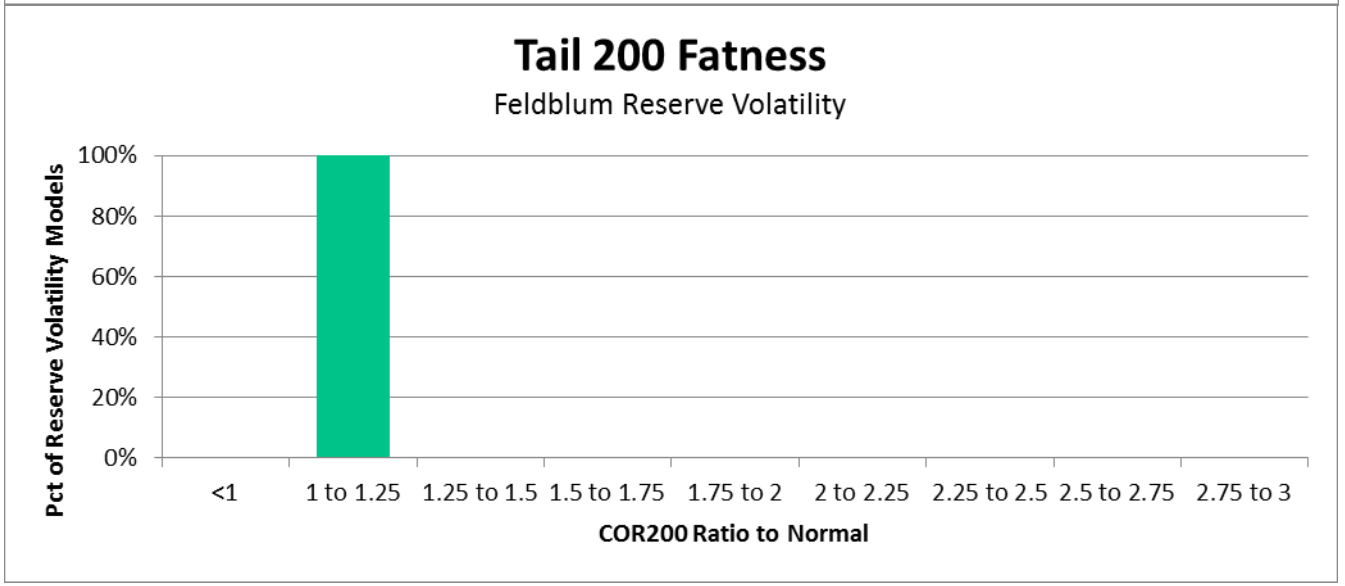
## By Line of business

Single company example

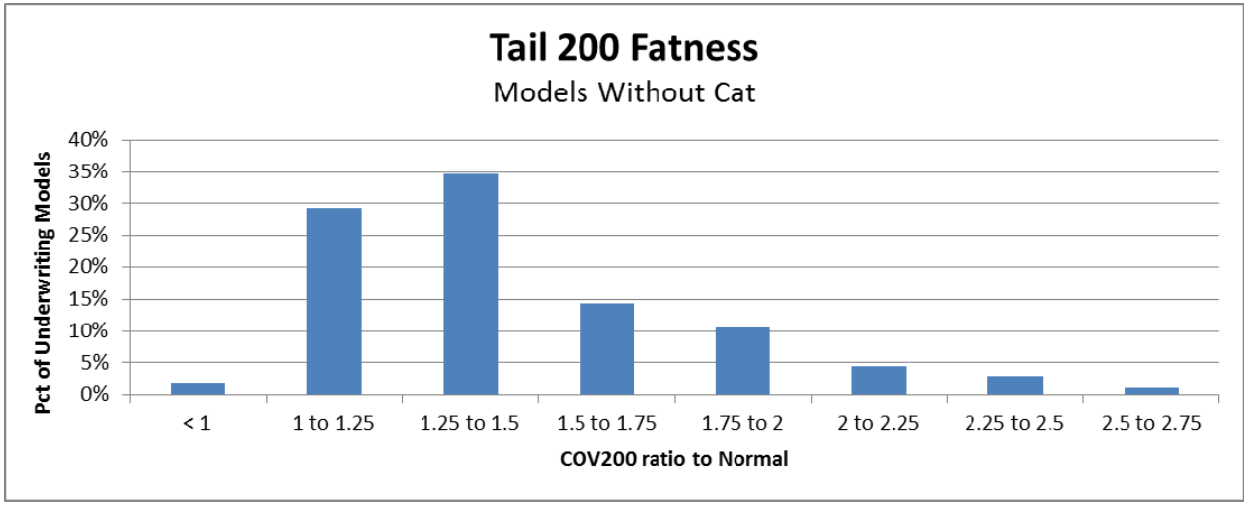


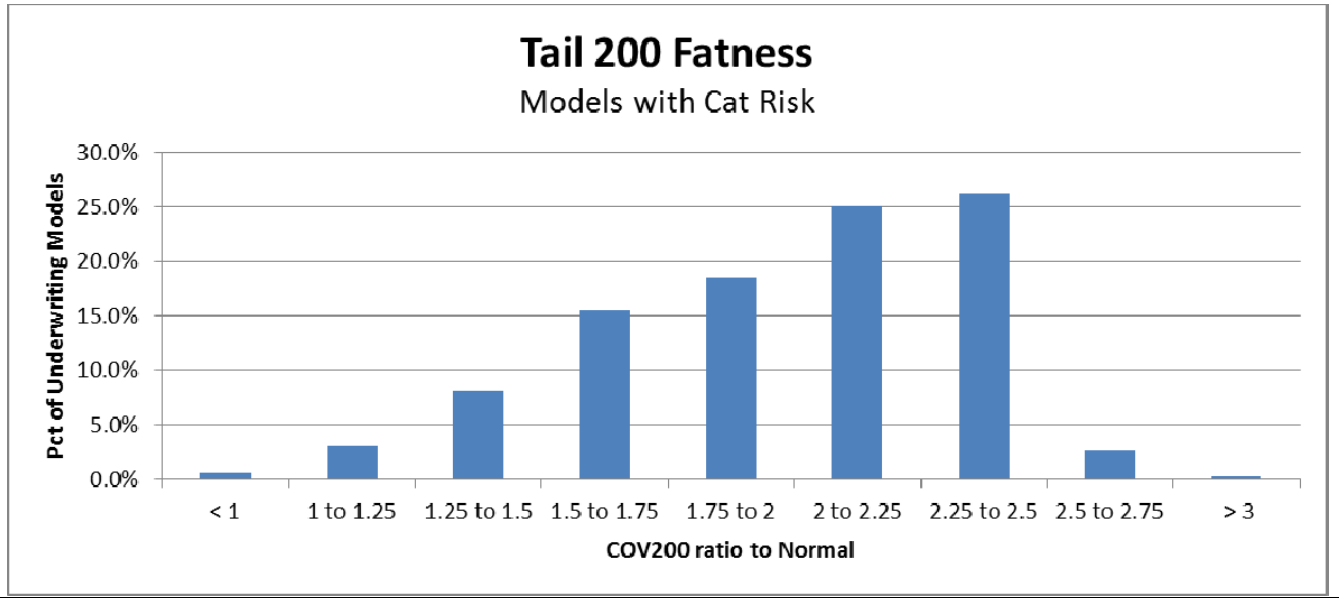
# Tail Fatness at 99.5%tile – Reserve Models



# Tail Fatness at 99.5%tile – Underwriting models



## Why are Tails of Reserve Volatility model so skinny?

Especially compared to Underwriting models

1. The underwriting models are for reinsurance programs.

2. Underwriting models are of individual blocks of business
   - reserve models are developed from whole company triangles.

3. More claims volatility occurs before setting initial reserves

4. Reserve model has some time diversification - representing many years of business.

5. Skinny tails of reserve volatility model is a design feature
   - That may not be documented or communicated

# Parting Suggestion

Someone ought to . . .

- Reconcile the Tail Fatness of the Underwriting and Reserve models within Economic Capital Models of a firm

# Conclusions

- COR provides a simple way to assess tail risk
  - *Willis Re has used with clients when choosing model for a new risk analysis*
- Today, you saw how actuarial models have a fairly wide range of “Tail Fatness”
  - And we raised a question

    *Why so much less Tail Fatness in Reserve Volatility Models?*

- We believe that with COR, we can start to develop:
  - Language for discussing model tail risk
  - Processes for using it to validate models
  - Procedure for estimating risk capital using company’s own risk volatilities
    - Which can lead to more people having a realistic “Gut Feel” for Tail Fatness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