Best Estimate Credibility for Link Ratios

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Common Situations Where Loss Development/Link Ratio Credibility is a Concern

- Smallish/medium sized line of busines for a company
- Individually priced but medium sized national account
- Any situation where your triangle has some credibility, but must be supplemented by a benchmark

Key Theory

- My 1992 Paper "Credibility Based on Accuracy" says that if
  - Two statistics/random variables, A, B, used to predict a third, C
  - Independence of A and B not key issue, but consider the error of A predicting C, $C - A$, and error of B predicting C, $C - B$.
    - This case requires $C - A$ and $C - B$ to be independent
  - Then the best estimate credibility for A is $\frac{E[(C-B)^2]}{E[(C-B)^2]+E[(C-A)^2]}$:It relates to B's error

Key Assumptions

- The process variances (year-to-year variations in a column) of all the link ratios are independent

- The differences between the benchmark link ratios and the true target (account?) expected link ratios are independent
  - Note is to to expected link ratios of target not observed link ratios of target
  - Will discuss sample situation where it does not hold later

Key Approach Concept

- Variance up and down columns is proxy for process variance-around true expected link ratio C
  - So $E[(C - A)^2]$ = process variance $\approx$ variance in column
- B's predictive error estimated as difference in link ratios between A and B (difference squared of course) in credibility formula

Example

Year)	Link Ratios for A	Data for B
2013	1.200	
2014	1.350	
2015	1.252	
2016	1.183	
2017	1.325	
Average*	1.262	1.350
Variance*	.005475	
Squared Difference in Means		.007744
Credibility of A	59%	
Z-Wtd LDF	1.291	

$$\frac{.007744}{.007744+.05475} = 59\%$$

* This example uses a straight average and variance for simplicity, but usually dollar weighted averages would be used for high process variance data.

Are We Done Already?

A few whistles and bells remaining

- Logarthmic version
- Very low volume columns near tail
- A caution against very low credibility data that is very different from the benchmark, with handling suggestions
- What if differences between benchmark and target link ratios are not independent

Example Using Logarithms

Year)	Link Ratios for A	Link for B	Logs of Link Ratios for A	Sqrd Diff from Mean in Logs for A	Log of Link for B
2013	1.200		.182	.00254	
2014	1.350		.300	.00454	
2015	1.252		.225	.00006	
2016	1.183		.168	.00418	
2017	1.325		.281	.00237	
Average*	1.262	1.350	.233		.3
Variance*	.005475		.00274		
Squared Difference in Means		.007744			.00454
Credibility of A	59%		62%		
Z-Wtd Log pf Link			.258		
Final Post-Z Link	1.291		1.294		

Very Low Volume Columns Near Tail

- In low volume programs, may be little development/small links near tail
- Mostly big losses possible then and "luck of the draw"
- Suggest look at parameter standard deviation as standard deviation from earlier columns, but applied to the benchmark link ratio
- Recognizing that only the "development portion" (Link Ratio-1.0) reflects claims activity, look at ratio SD to just development portion
- Use benchmark link(-1.0) times CV-type ratio to estimate process risk in target program
- Still (benchmark link-target program link)^2 for benchmark estimate error

Very Low Volume Columns Near Tail-Example

Item	Months of Development 36	48	60		108	120
A. [Data] Target Program Link Ratios	1.200	1.150	1.080		1.000	1.000
B. [Data] Process Variance of Link Ratios	0.02250	0.00640	0.00160		0.00000	0.00000
C. [Data] Benchmark Link Ratios	1.350	1.140	1.090	..	1.030	1.020
D. [= $(C\text{-}A)^2$] Sqrd Difference in Link Ratios	0.02250	0.00010	0.00010		0.00090	0.00040
E. = [D/(D+B)] Credibility of Program	50%	2%	6%			
F. = [E*A+(1-E)*C] Z-Wtd Links for Early Periods	1.275	1.140	1.089			
G. = [A-1.0] Program Development Portion	0.200	0.150	0.080			
H. [=$\sqrt{B}$] Stand Dev. Of Process Variance	0.15	0.08	0.04			
I. [=H/G] Stand Dev./Development Portion	0.75	0.53	0.50			
J. Selected Ratio [from I.] for Tail					0.5	0.5
K. [=(C-1.0)] Benchmark Devel. Portion Near Tail					0.030	0.020
L. [=($J*K.)^2$] Est. Process Variance of Benchmark					0.000225	0.0001
M. [=D/(D+L)] Credibility of Program					80%	80%
N. [=M*A+(1-M)*C] Z-Wtd Links for Tail Periods					1.006	1.004

Other—Maybe Better—Handling for Very Low Volume Columns Near Tail

- Continue last credibility that is based on reasonable volume (generally, too high credibility)
- Replace actual difference between program and benchmark link ratios with:
  - Difference from period that has reasonable volume
  - Adjusted by ratio of development portion of benchmark link low volume period to benchmark link in reasonable volume period
  - Of course, when do credibility calculation, result is squared

What if the Link Ratio Difference are not Independent

- What if say, all the development portions of programs link ratios with reasonable volume are fairly consistently twice those of the benchmark?

- Obviously, the differences are correlated

- But, there is a non-credibility solution,

  – Simply multiply all the development portions of the benchmark with medium to small credibility by 2.

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