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Isn’t it True…?

James Ely, FCAS

2019 CLRS

Question 1

Which of the following is your preferred method of loss reserving?

1. Traditional methods (link ratio, BF etc.)
2. GLMs
3. Other methods

Other Methods

We seek new perspectives that can provide clarity where the traditional perspective does not.

This new perspective can help to confirm a “gut feeling” or debunk a common misconception.

Wherever possible, we look for methods that have been proven in other disciplines.

Discrete-Time Signal Processing (DSP)

DSP methods are used in the processing of signals in electronic communications as well as in the testing of electronic circuits.

The basic formulation of the problem has three components:

1. A linear system to be tested
2. An impulse is applied to the system
3. The response of the system is recorded

Convolution

In statistics convolutions describe sums of independent distributions.

In DSP convolutions describe the system response to an input signal.

In loss reserving convolutions describe calendar year amounts:

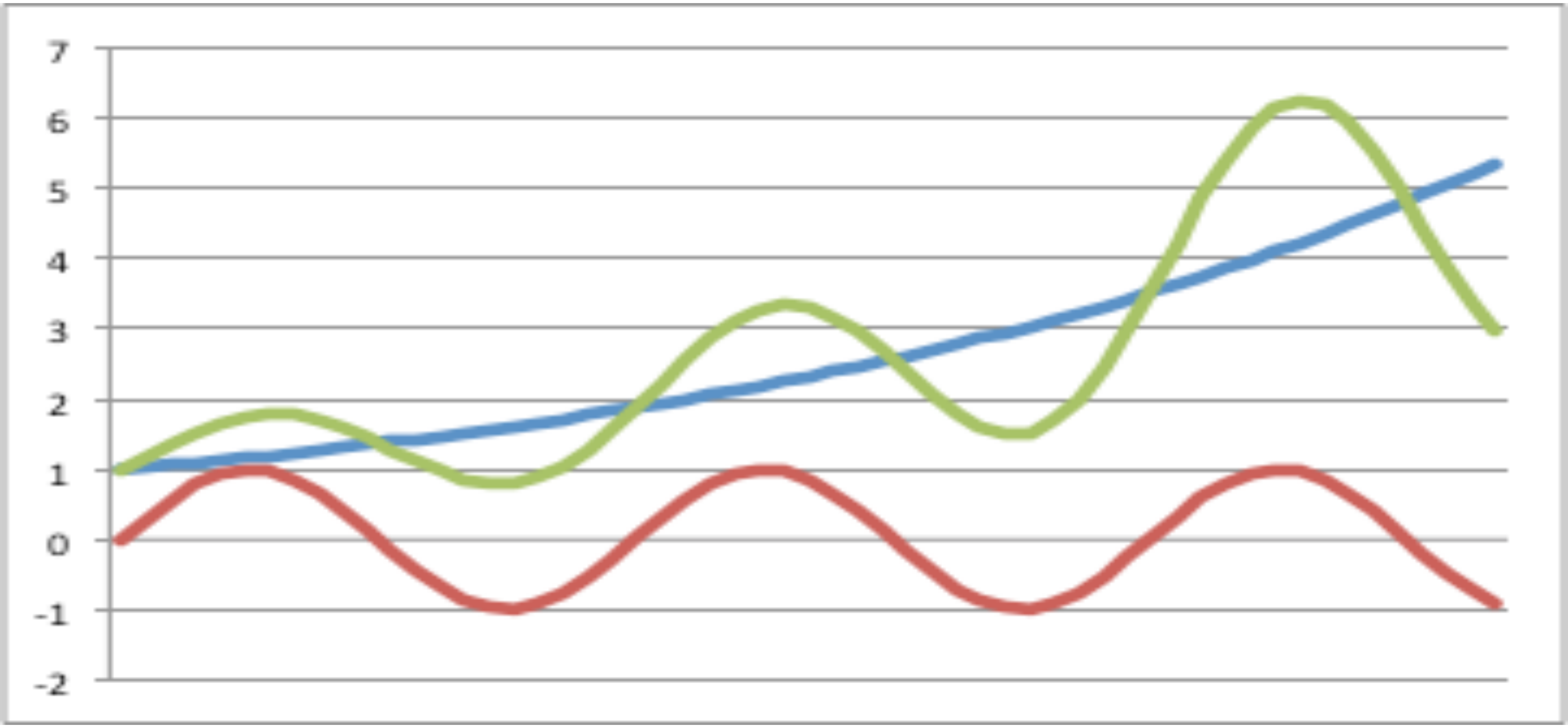
$$loss(CY) = \sum_{AY=0}^{CY} f(AY)g(CY - AY)$$

Log-Linear GLM Assumption

$$P = Incremental\ Paid(AY, CY, age) = Ae^{xAY + yCY + z(CY - AY)}$$

$$\frac{\partial^2 P}{\partial AY^2} = \frac{(x-z)^2}{(y+z)^2} \frac{\partial^2 P}{\partial CY^2}$$

Inflationary Trend-Cycle



Loss Reserving as a DSP Problem

Log-linear GLM assumptions satisfy the wave equation.

Solution of the wave equation is an eigenvalue problem.

The eigenvalues of the system are the Fourier coefficients of the solution.

Question 2

Is the BF method deterministic or stochastic?

a. Deterministic

b. Stochastic

Bornhuetter-Ferguson Method

The Incurred BF method is:

$$IBNR(CY) = \sum_{AY=0}^{CY} Expected\ Loss(AY)\ \%Unreported(CY - AY)$$

The Paid BF method is:

$$Unpaid(CY) = \sum_{AY=0}^{CY} Expected\ Loss(AY)\ \%Unpaid(CY - AY)$$

Error Terms

Additive: $f(x) + \varepsilon_f$

Multiplicative: $f(x)\varepsilon_f$

By Convolution: $f(x) * \varepsilon_f$

Error Terms of BF Inputs

Assume that the error terms are convolved with functional patterns:

$$Expected\ Loss(AY) = EL(AY) * \varepsilon_{EL}$$

$$\%Unreported(age) = \%U(age) * \varepsilon_{\%U}$$

Error Term of BF Output

$$IBNR(CY) = Expected\ Loss(AY) * \%Unreported(CY - AY)$$

$$IBNR(CY) = [EL(AY) * \varepsilon_{EL}] * [\%U(CY - AY) * \varepsilon_{\%U}]$$

Error Term of BF Output

$$IBNR(CY) = [EL(AY) * \%U(CY - AY)] * [\varepsilon_{EL} * \varepsilon_{\%U}]$$

The error term of the BF method is the convolution of the error term of the expected loss distribution and the unreported or unpaid loss distribution.

Question 3

True or false, link-ratios have no statistical meaning.

a. True

CAS Working Party on Quantifying Variability in Reserve Estimates -The Analysis and Estimation of Loss and ALAE Variability

b. False

Reserving as DSP

Reporting Pattern ↔ Development Factors

Traditionally, the BF method assumes a a reciprocal relationship between the reporting pattern and loss development factors:

$$\%Reported(age) = 1/LDF(age)$$

The BF method is a convolution, so we should consider the possibility that moment generating functions give an alternate relationship.

Convolution Theorem

$$MGF(f * g) = MGF(f)MGF(g)$$

Payment Patterns as Distributions on Intervals

Let $P(t_1,t_2)$ denote the distribution of payments on the interval (t_1,t_2).

If we assume the independence of payments on different intervals then they sum by convolution:

$$P(0,t_2) = P(0,t_1) * P(t_1,t_2)$$

Apply the Convolution Theorem

$$MGF[P(0,t_2)] = MGF[P(0,t_1)]MGF[P(t_1,t_2)]$$

Rearrange terms:

$$MGF[P(t_1,t_2)] = \frac{MGF[P(0,t_2)]}{MGF[P(0,t_1)]}$$

Interpretation of the MGF

Let P(t) denote a distributional payment stream.

$$MGF[P(t)] = \sum_{t=0}^{\infty} e^{st} P(t)$$

$$Present\ Value(P(t), s) = \sum_{t=0}^{\infty} e^{-st} P(t)$$

Constant Cost Payment Stream

We can correct the sign by revising our assumptions. Let's now assume that p(t) is a constant cost payment stream and let s denote the claim cost inflation rate:

$$Nominal\ Value(p(t), s) = \sum_{t=0}^{\infty} e^{st}\, p(t)$$

$$MGF[p(t)] = \sum_{t=0}^{\infty} e^{st}\, p(t)$$

Meaning of Link Ratios (and of MGF)

$$Link\ Ratio[(t_1, t_2), s] = MGF[p(t_1, t_2)]$$

Link ratio “distributions” are equivalent to the moment generating function of the constant cost payment pattern on the interval, where the “helper” variable s is the claim cost inflation rate.

Question 4

True or False:

$$Reserve\ Risk = Insurance\ Risk + Timing\ Risk + Inflation\ Risk$$

a. True

b. False

Decomposition of Reserve Risk

One formulation of the GLM assumptions is that incremental payments are the product of three factors:

$$Paid(AY, age) = f(AY)g(age)h(CY)$$

Decomposition of Reserve Risk

Instead of taking logs, if we simply sum over the unpaid region we get two convolutions:

$$Unpaid(SD) = f(AY) * g(age) * h(CY)$$

$$Reserve\ Risk = Insurance\ Risk + Timing\ Risk + Inflation\ Risk$$

Note: SD denotes Statement Date

Example of Decomposition Process

The example is based on nationwide WC paid data, calendar years 2004-2015.

It assumes a paid BF format. We will factor out the expected loss component using a difference equation.

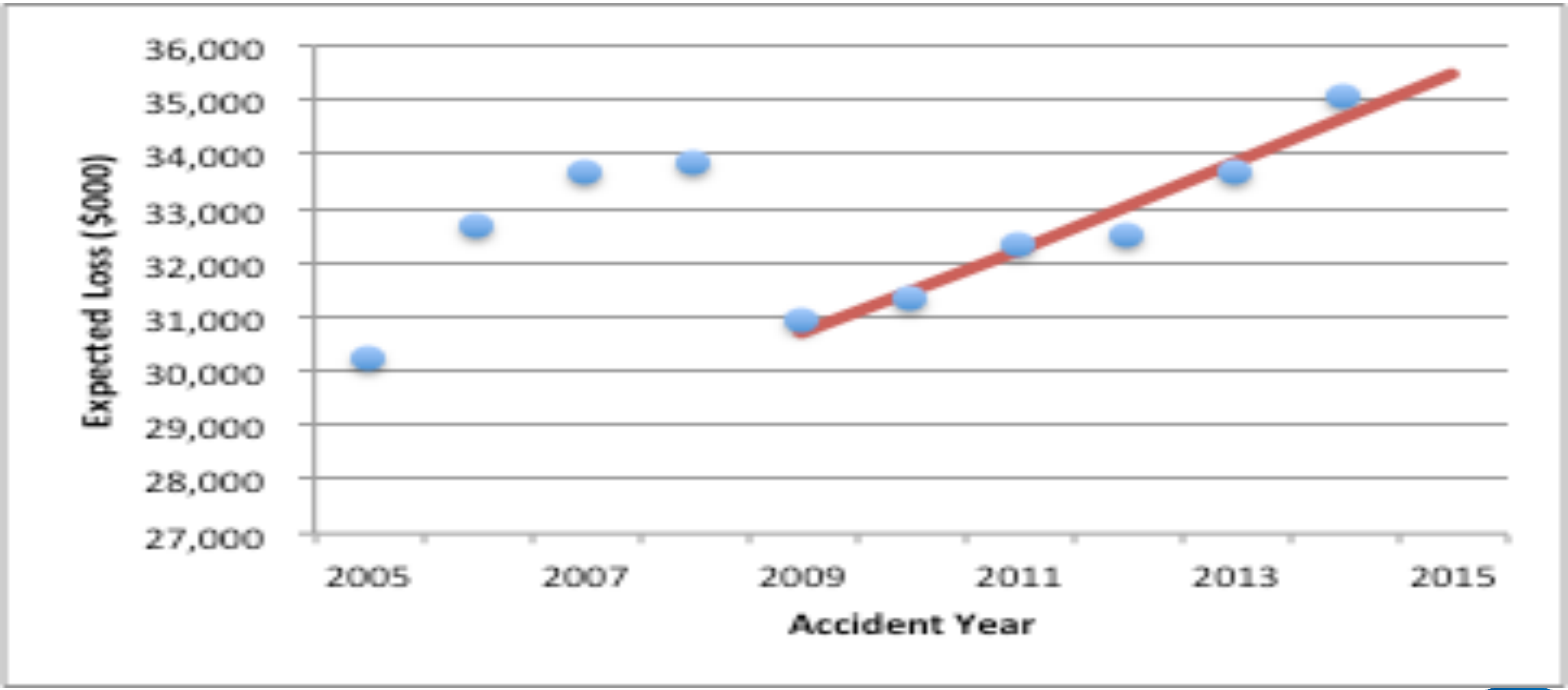
The result will be an array that exhibits patterns in two directions.

WC Net Paid - US Totals

Accident Year	1	2	3	4	5	6	7	8	9	10
1995										17,751
1996									17,917	19,394
1997								19,225	20,670	20,899
1998							20,510	21,899	22,296	22,519
1999						19,654	21,135	21,694	21,974	22,463
2000					20,918	22,333	23,150	23,596	24,083	24,942
2001				20,511	22,194	23,251	23,836	24,621	25,467	25,509
2002			17,391	20,415	22,092	23,054	24,050	25,051	25,165	26,470
2003		13,079	17,630	20,110	21,808	23,079	24,064	24,356	25,833	26,314
2004	6,516	13,612	17,161	19,533	21,181	22,475	22,799	24,513	25,147	25,697
2005	7,064	13,638	17,373	19,735	21,553	22,111	24,024	24,725	25,381	25,784
2006	7,116	14,100	18,293	21,244	22,366	24,635	25,559	26,470	27,000	27,447
2007	7,112	14,623	19,105	21,271	24,140	25,547	26,664	27,441	28,048	
2008	7,127	14,764	18,710	22,410	24,380	25,796	26,833	27,539		
2009	6,832	13,466	18,092	20,686	22,522	23,839	24,630			
2010	6,610	14,028	18,220	20,811	22,660	23,712				
2011	7,083	14,375	18,649	21,408	23,079					
2012	6,920	14,180	18,274	20,731						
2013	6,823	14,111	18,115							
2014	6,979	14,243								
2015	6,874									

Expected Losses

(Prior Year Ultimates)



Convolution Process

$h(x+y) = \Sigma f(x) g(y)$

1. Multiplication
2. Sum
3. Rotation

Inverse Process

To factor out one of the components of our loss model we invert the convolution process. We will perform three operations on a triangle:

1. Division
2. Difference
3. Rotation

Cumulative Payment Pattern

(Divide Out Expected Loss)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										0.890
1996									0.797	0.863
1997								0.792	0.852	0.861
1998							0.782	0.835	0.850	0.858
1999						0.754	0.811	0.832	0.843	0.862
2000					0.710	0.758	0.786	0.801	0.818	0.847
2001				0.654	0.708	0.741	0.760	0.785	0.812	0.813
2002			0.535	0.628	0.680	0.709	0.740	0.771	0.774	0.814
2003		0.408	0.549	0.627	0.680	0.719	0.750	0.759	0.805	0.820
2004	0.210	0.439	0.554	0.631	0.684	0.726	0.736	0.791	0.812	0.829
2005	0.229	0.442	0.563	0.640	0.699	0.717	0.779	0.802	0.823	0.836
2006	0.215	0.426	0.553	0.642	0.676	0.745	0.773	0.800	0.816	0.830
2007	0.209	0.430	0.562	0.626	0.710	0.751	0.784	0.807	0.825	
2008	0.209	0.433	0.548	0.657	0.714	0.756	0.786	0.807		
2009	0.217	0.428	0.576	0.658	0.717	0.758	0.784			
2010	0.211	0.447	0.581	0.664	0.723	0.756				
2011	0.217	0.440	0.571	0.655	0.707					
2012	0.210	0.431	0.555	0.630						
2013	0.202	0.418	0.536							
2014	0.197	0.403								
2015	0.190									

Incremental Payment Pattern

(Difference Cumulative Pattern)

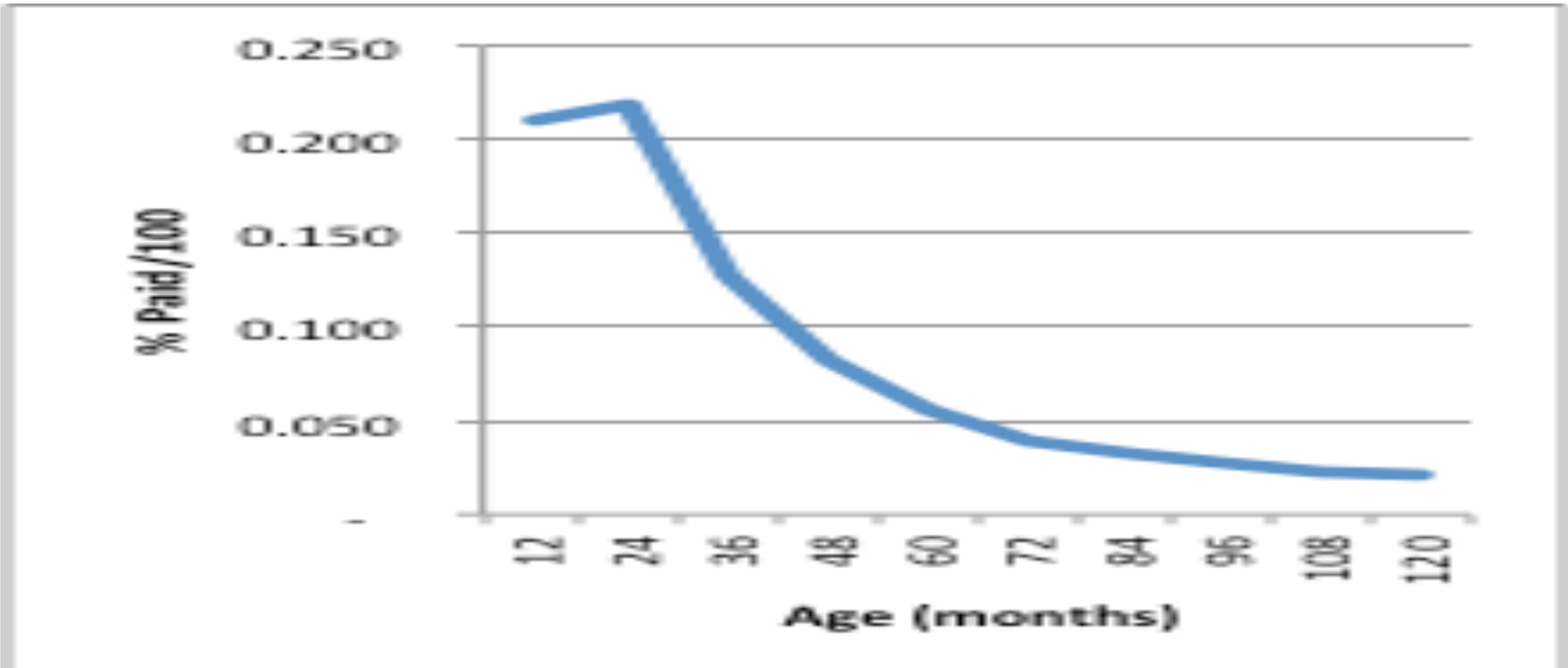
Accident Year	12	24	36	48	60	72	84	96	108	120
1995										
1996										0.066
1997									0.060	0.009
1998								0.053	0.015	0.008
1999							0.057	0.021	0.011	0.019
2000						0.048	0.028	0.015	0.017	0.029
2001					0.054	0.034	0.019	0.025	0.027	0.001
2002				0.093	0.052	0.030	0.031	0.031	0.004	0.040
2003			0.142	0.077	0.053	0.040	0.031	0.009	0.046	0.015
2004		0.229	0.115	0.077	0.053	0.042	0.010	0.055	0.020	0.018
2005	0.229	0.213	0.121	0.077	0.059	0.018	0.062	0.023	0.021	0.013
2006	0.215	0.211	0.127	0.089	0.034	0.069	0.028	0.028	0.016	0.014
2007	0.209	0.221	0.132	0.064	0.084	0.041	0.033	0.023	0.018	
2008	0.209	0.224	0.116	0.108	0.058	0.041	0.030	0.021		
2009	0.217	0.211	0.147	0.083	0.058	0.042	0.025			
2010	0.211	0.237	0.134	0.083	0.059	0.034				
2011	0.217	0.223	0.131	0.084	0.051					
2012	0.210	0.221	0.124	0.075						
2013	0.202	0.216	0.119							
2014	0.197	0.205								
2015	0.190									

CY Incremental Payment Pattern

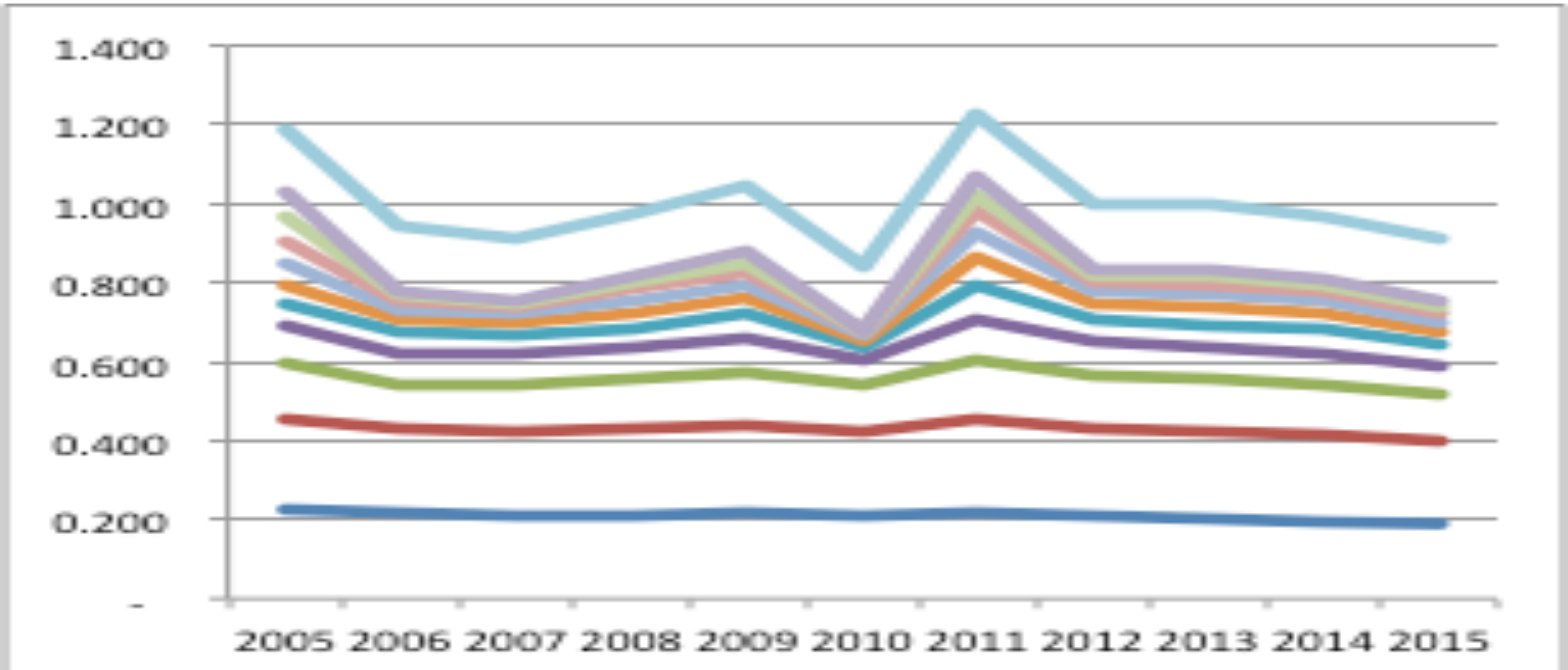
(Rotate Incremental Pattern)

Calendar Year	12	24	36	48	60	72	84	96	108	120
2005	0.229	0.229	0.142	0.093	0.054	0.048	0.057	0.053	0.060	0.066
2006	0.215	0.213	0.115	0.077	0.052	0.034	0.028	0.021	0.015	0.009
2007	0.209	0.211	0.121	0.077	0.053	0.030	0.019	0.015	0.011	0.008
2008	0.209	0.221	0.127	0.077	0.053	0.040	0.031	0.025	0.017	0.019
2009	0.217	0.224	0.132	0.089	0.059	0.042	0.031	0.031	0.027	0.029
2010	0.211	0.211	0.116	0.064	0.034	0.018	0.010	0.009	0.004	0.001
2011	0.217	0.237	0.147	0.108	0.084	0.069	0.062	0.055	0.046	0.040
2012	0.210	0.223	0.134	0.083	0.058	0.041	0.028	0.023	0.020	0.015
2013	0.202	0.221	0.131	0.083	0.058	0.041	0.033	0.028	0.021	0.018
2014	0.197	0.216	0.124	0.084	0.059	0.042	0.030	0.023	0.016	0.013
2015	0.190	0.205	0.119	0.075	0.051	0.034	0.025	0.021	0.018	0.014

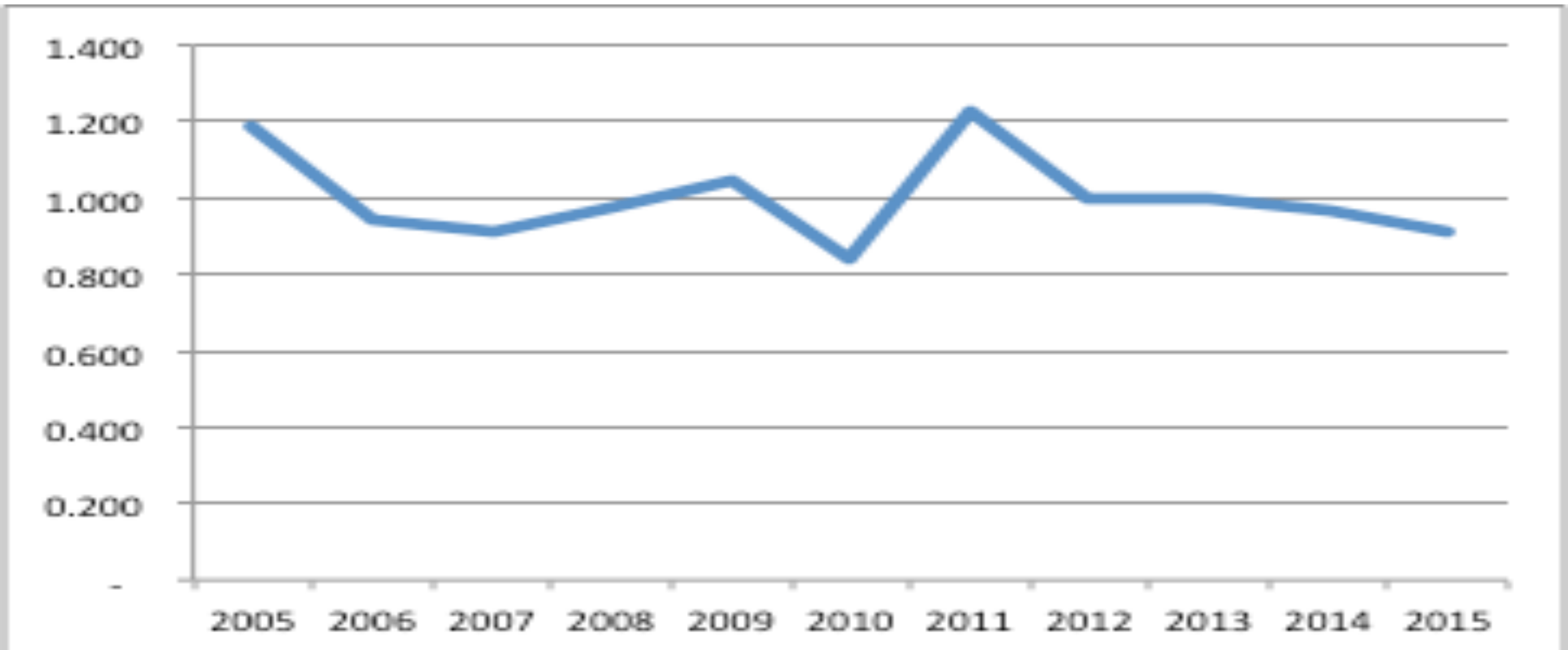
Incremental Payment Pattern



CY Cycle


1.400
1.200
1.000
0.800
0.600
0.400
0.200
-
2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015

CY Cycle



1.400
1.200
1.000
0.800
0.600
0.400
0.200
-
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015

Conclusions

The BF method and its error term are convolutions.

Link-ratios and the LDF method arise from transforming and rearranging convolutions.

Reserve risk can be viewed as the convolution of insurance risk, timing risk and inflation risk components.

Partial Severities

(Divide by Ult. Claim Counts)

Accident Year	12	24	36	48	60	72	84	96	108	120
1995										2,519
1996									2,659	2,878
1997								2,966	3,188	3,224
1998							3,019	3,224	3,282	3,315
1999						2,985	3,210	3,295	3,338	3,412
2000					3,059	3,266	3,385	3,451	3,522	3,648
2001				3,215	3,479	3,645	3,737	3,860	3,992	3,999
2002			2,966	3,482	3,768	3,932	4,102	4,273	4,292	4,515
2003		2,504	3,375	3,850	4,175	4,418	4,606	4,662	4,945	5,037
2004	1,280	2,675	3,372	3,839	4,162	4,417	4,480	4,817	4,942	5,050
2005	1,433	2,767	3,524	4,003	4,372	4,485	4,873	5,015	5,149	5,230
2006	1,472	2,917	3,784	4,395	4,627	5,096	5,287	5,476	5,586	5,678
2007	1,557	3,202	4,183	4,657	5,285	5,593	5,838	6,008	6,141	
2008	1,684	3,489	4,422	5,297	5,762	6,097	6,342	6,509		
2009	1,824	3,595	4,831	5,523	6,014	6,365	6,576			
2010	1,756	3,726	4,839	5,527	6,018	6,298				
2011	1,828	3,709	4,812	5,524	5,955					
2012	1,783	3,654	4,709	5,342						
2013	1,780	3,681	4,725							
2014	1,752	3,576								
2015	1,685									

Incremental Partial Severities

(Difference)

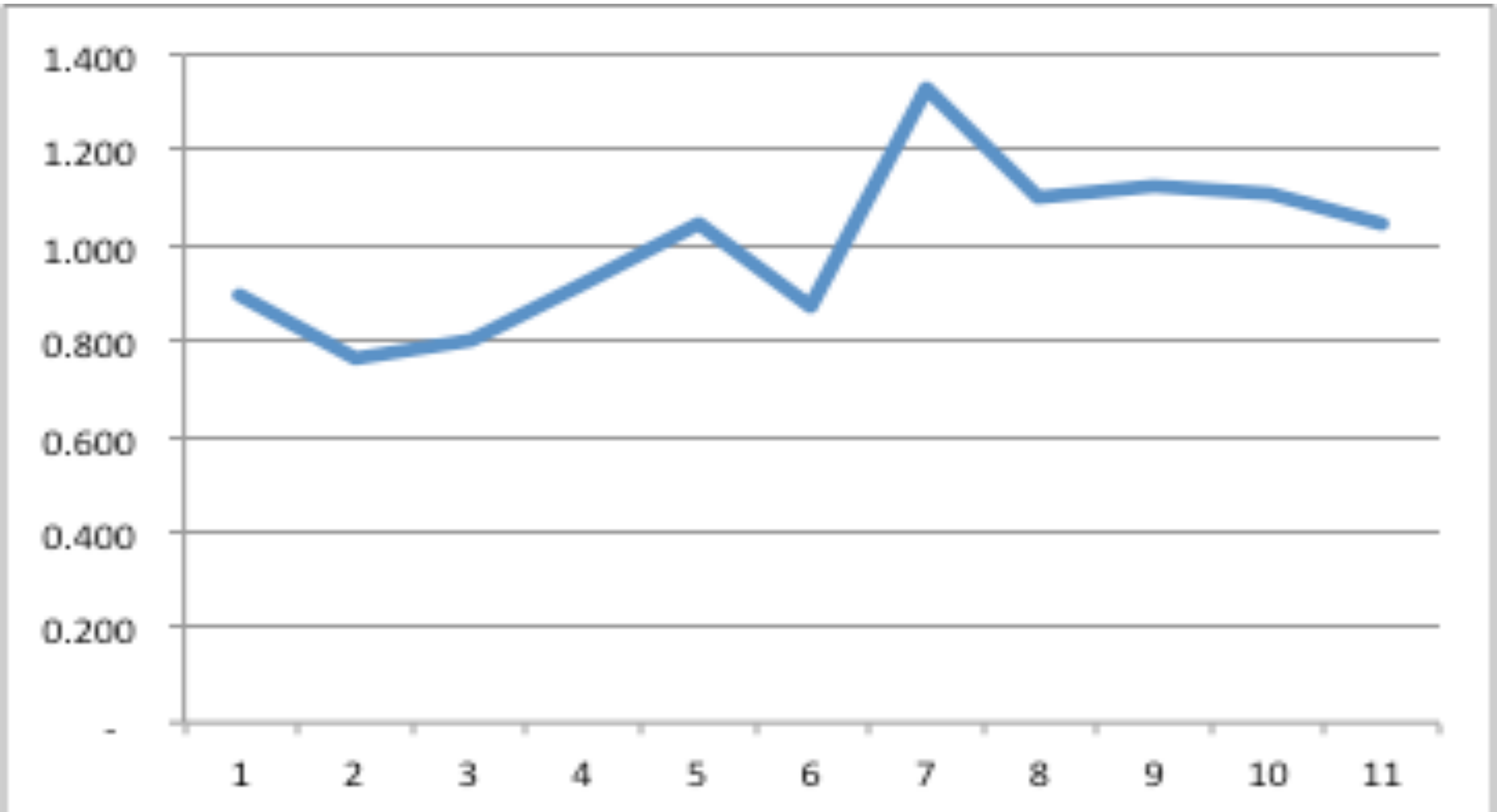
Accident Year	12	24	36	48	60	72	84	96	108	120
1995										
1996										219
1997									223	35
1998								204	58	33
1999							225	85	43	74
2000						207	119	65	71	126
2001					264	166	92	123	133	7
2002				516	286	164	170	171	19	223
2003			871	475	325	243	189	56	283	92
2004		1,395	697	466	324	254	64	337	125	108
2005	1,433	1,334	758	479	369	113	388	142	133	82
2006	1,472	1,445	867	611	232	469	191	188	110	92
2007	1,557	1,644	981	474	628	308	245	170	133	
2008	1,684	1,805	933	875	466	335	245	167		
2009	1,824	1,771	1,235	693	490	352	211			
2010	1,756	1,970	1,114	688	491	279				
2011	1,828	1,882	1,103	712	431					
2012	1,783	1,871	1,055	633						
2013	1,780	1,901	1,044							
2014	1,752	1,824								
2015	1,685									

CY Incremental Partial Severities

(Rotate)

Calendar Year	12	24	36	48	60	72	84	96	108	120
2005	1,433	1,395	871	516	264	207	225	204	223	219
2006	1,472	1,334	697	475	286	166	119	85	58	35
2007	1,557	1,445	758	466	325	164	92	65	43	33
2008	1,684	1,644	867	479	324	243	170	123	71	74
2009	1,824	1,805	981	611	369	254	189	171	133	126
2010	1,756	1,771	933	474	232	113	64	56	19	7
2011	1,828	1,970	1,235	875	628	469	388	337	283	223
2012	1,783	1,882	1,114	693	466	308	191	142	125	92
2013	1,780	1,871	1,103	688	490	335	245	188	133	108
2014	1,752	1,901	1,055	712	491	352	245	170	110	82
2015	1,685	1,824	1,044	633	431	279	211	167	133	92

Severity Trend-Cycle



Cycle vs. Trend-Cycle

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