CAS Casualty Loss Reserves Seminar
September, 2019

Beyond Numerology- Practical LDF Interpolation

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Quarterly LDF Interpolation - Agenda

- Introduction: Numerology vs Actuarial Utility
- From Quarterly LDF to Quarterly IBNR
  - Prior AY – IBNR Survival Factors
  - Current AY – BF IBNR
- Exercises deriving IBNR
- Why Interpolate LDF?
  - Alternative: derive LDF by Q from Qtrly triangles
- Interpolation Properties
  - Inherited Monotonicity
  - Equilibrium IBNR Stability
  - Monotonic IBNR Run-off Property

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LDF INTERPOLATION OVERVIEW

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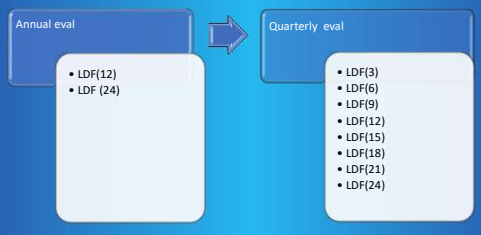
LDF Interpolation: Annual to Qtrly



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Fundamental Question: What makes one LDF interpolation scheme better than another?

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General Answer: Mathematically Nice

Actuarial Answer: Leads to well-behaves quarterly IBNR

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Conceptual Hurdle
Relating Properties of Interpolates to Properties of IBNR

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Quarterly IBNR and Quarterly LDF

- Need quarterly IBNR to generate quarterly results.
- Quarterly projection of IBNR needed for planning
- Need to start with year-end AY IBNR for all prior AY
- Quarterly LDF can be used to derive Quarterly IBNR

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Quarterly IBNR

- Need quarterly IBNR to generate quarterly results.
- Quarterly projection of IBNR needed for planning
- Need to start with year-end AY IBNR for all prior AY
- Quarterly LDF can be used to derive Quarterly IBNR

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IBNR by Quarter

	Age	E[L]	IBNR	IBNR at Quarter End				
AY			Year End	Q1	Q2	Q3	Q4	Q5
2019		1,000		235	441	627	800	719
2018	12		800	719	640	566	500	385
2017	24		300	231	175	132	100	64
2016	36		200	129	81	50	0	0
2015	48		0	0	0	0	0	0
Current AY		IBNR	-	235	441	627	800	719
Prior AY		IBNR	1,300	1,079	896	748	600	449
All AY Total		IBNR	1,300	1,313	1,337	1,375	1,400	1,169

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Change in IBNR by Quarter

	Age			Change in IBNR by Quarter				
AY			Year End	Q1	Q2	Q3	Q4	Q5
2019				235	207	186	173	(81)
2018	12			(81)	(79)	(74)	(66)	(115)
2017	24			(69)	(56)	(43)	(32)	(36)
2016	36			(71)	(48)	(31)	(50)	0
2015	48			0	0	0	0	0
Current AY	Change in IBNR		-	235	207	186	173	(81)
Prior AY	Change in IBNR		-	(221)	(183)	(148)	(148)	(151)
All AY Total	Change in IBNR		-	13	24	38	25	(231)

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Why interpolate at all? Why not just derive quarterly LDF from quarterly evaluation triangles?

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Derivation from Quarterly Triangles

- Quarterly data leads directly to quarterly LDF
- More oscillations – greater need for smoothing
- Four times as many selections to be made
- Each cell has ¼ the volume of incremental loss on average
- Each cell is less credible
- No need to interpolate

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Perspective on Interpolation

- Interpolated Quarterly LDF from Annual triangles are useful to have even if only as a standard of comparison.
- Interpolates should obey reasonable properties.
- Aesthetic appeal of interpolation formulas is not enough.
- Big Idea: Defining desirable properties of LDF Interpolation Methods by examining behavior of the resulting IBNR.

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From Quarterly LDF to Quarterly IBNR

Prior AY – IBNR Survival Factor Method
Current AY – BF Formula
Exercises with a few different interpolation formulas

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Prior AY IBNR Derivation

- Start with separate AY IBNR at year end.
- Survival Factor Method
- Apply Qtrly Survival Factors to compute how much IBNR "survives" by qtr.
- Use LDF to derive Qtrly IBNR Survival Factors

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Quarterly IBNR Survival Factor Formula

$$IBNR(\text{as of Qtr end}) =$$

$$S \cdot IBNR(\text{as of Yr end})$$
$$\text{where}$$
$$S = \text{Survival Factor}$$

$$= \frac{1 - 1/ATU(\text{Qtr end})}{1 - 1/ATU(\text{Yr end})}$$

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Current AY - BF IBNR by Q

$$IBNR(\text{as of Qtr end})$$

$$= EP \cdot ELR \cdot \left(1 - \frac{1}{ATU(\text{Qtr end})}\right)$$

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LDF INTERPOLATE EXAMPLES

- Linear
- Geometric
- Inverse Power
- Linear on PCT of ULT

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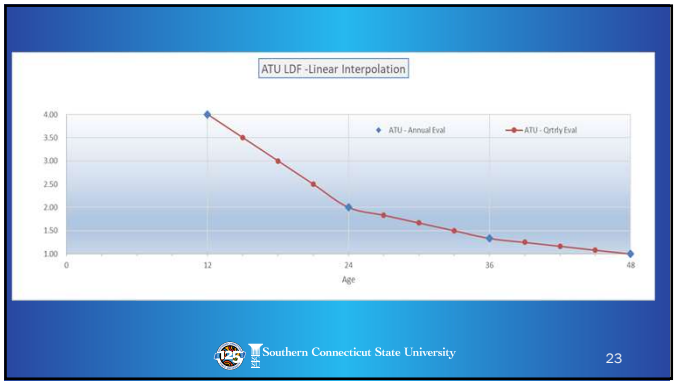
ATU LDF -Linear Interpolation
ATU - Annual Eval
ATU - Qrtrly Eval
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0
12
24
36
48
Age
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23

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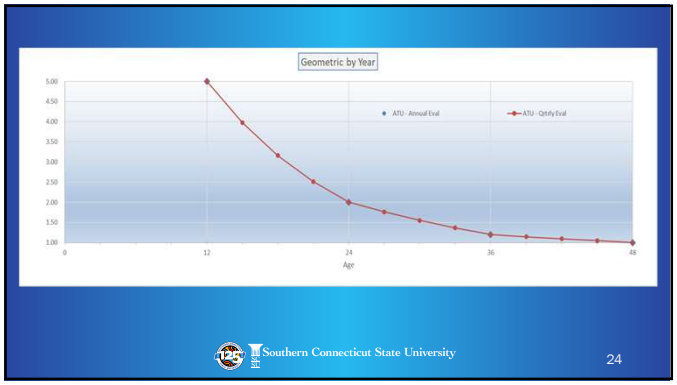
Geometric by Year
ATU - Annual Eval
ATU - Qrtrly Eval
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0
12
24
36
48
Age
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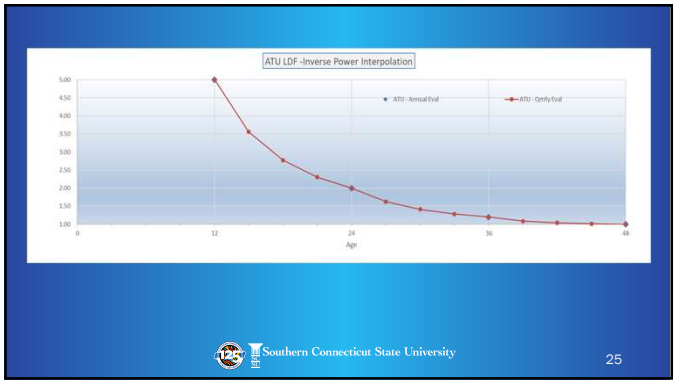
ATU LDF -Inverse Power Interpolation
ATU - Annual Eval
ATU - Qrtrly Eval
5.00
4.50
4.00
3.50
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2.00
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12
24
36
48
Age
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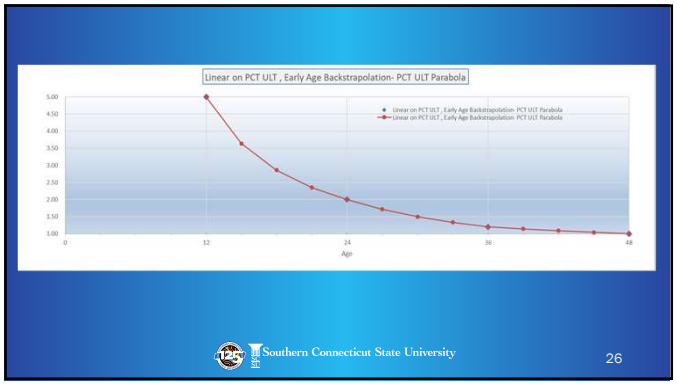

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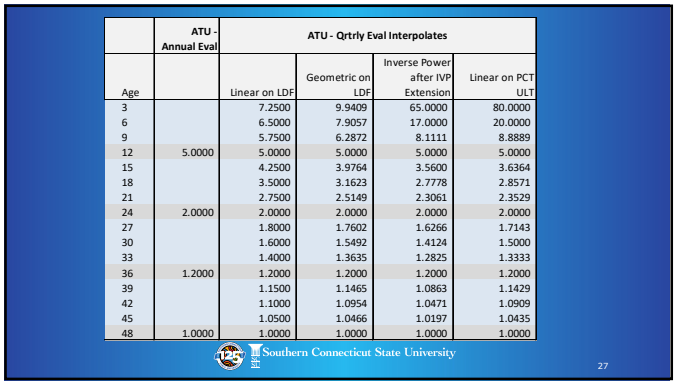

Age	ATU - Annual Eval	ATU - Qrtrly Eval Interpolates			
		Linear on LDF	Geometric on LDF	Inverse Power after IVP Extension	Linear on PCT ULT
3		7.2500	9.9409	65.0000	80.0000
6		6.5000	7.9057	17.0000	20.0000
9		5.7500	6.2872	8.1111	8.8889
12	5.0000	5.0000	5.0000	5.0000	5.0000
15		4.2500	3.9764	3.5600	3.6364
18		3.5000	3.1623	2.7778	2.8571
21		2.7500	2.5149	2.3061	2.3529
24	2.0000	2.0000	2.0000	2.0000	2.0000
27		1.8000	1.7602	1.6266	1.7143
30		1.6000	1.5492	1.4124	1.5000
33		1.4000	1.3635	1.2825	1.3333
36	1.2000	1.2000	1.2000	1.2000	1.2000
39		1.1500	1.1465	1.0863	1.1429
42		1.1000	1.0954	1.0471	1.0909
45		1.0500	1.0466	1.0197	1.0435
48	1.0000	1.0000	1.0000	1.0000	1.0000

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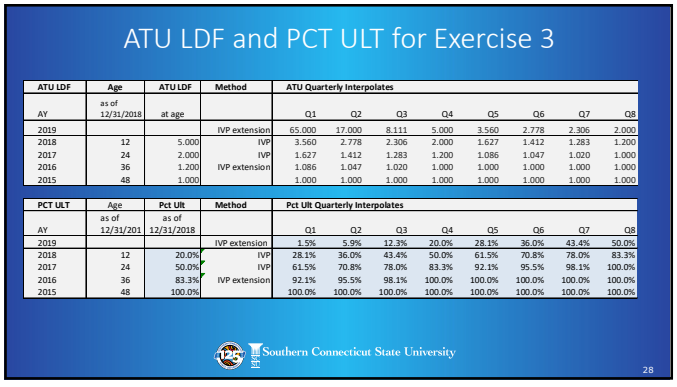

ATU LDF and PCT ULT for Exercise 3

ATU LDF	Age	ATU LDF	Method	ATU Quarterly Interpolates							
AY	as of 12/31/2018	at age		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
2019			IVP extension	65.000	17.000	8.111	5.000	3.560	2.778	2.306	2.000
2018	12	5.000	IVP	3.560	2.778	2.306	2.000	1.627	1.412	1.283	1.200
2017	24	2.000	IVP	1.627	1.412	1.283	1.200	1.086	1.047	1.020	1.000
2016	36	1.200	IVP extension	1.086	1.047	1.020	1.000	1.000	1.000	1.000	1.000
2015	48	1.000		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

PCT ULT	Age	Pct Ult	Method	Pct Ult Quarterly Interpolates							
AY	as of 12/31/201	as of 12/31/2018		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
2019			IVP extension	1.5%	5.9%	12.3%	20.0%	28.1%	36.0%	43.4%	50.0%
2018	12	20.0%	IVP	28.1%	36.0%	43.4%	50.0%	61.5%	70.8%	78.0%	83.3%
2017	24	50.0%	IVP	61.5%	70.8%	78.0%	83.3%	92.1%	95.5%	98.1%	100.0%
2016	36	83.3%	IVP extension	92.1%	95.5%	98.1%	100.0%	100.0%	100.0%	100.0%	100.0%
2015	48	100.0%		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

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Tail and Survival Factors for Exercise 3

Tail %	Age	Tail %		Tail % Quarterly Interpolates							
AY	as of 12/31/2	as of 12/31/2018		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
2019											
2018	12	80.00%		71.91%	64.00%	56.64%	50.00%	38.52%	29.20%	22.03%	16.67%
2017	24	50.00%		38.52%	29.20%	22.03%	16.67%	7.95%	4.50%	1.93%	0.00%
2016	36	16.67%		7.95%	4.50%	1.93%	0.00%	0.00%	0.00%	0.00%	0.00%
2015	48	0.00%		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Survival Factors	Age	IBNR Survival Factor		IBNR Survival Factor Quarterly Interpolates							
AY	as of 12/31/2	as of 12/31/2018		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
2019								89.89%	80.00%	70.80%	62.50%
2018	12	100.00%		89.89%	80.00%	70.80%	62.50%	48.15%	36.50%	27.53%	20.83%
2017	24	100.00%		77.04%	58.40%	44.06%	33.33%	15.89%	9.00%	3.86%	0.00%
2016	36	100.00%		47.67%	27.01%	11.58%	0.00%	0.00%	0.00%	0.00%	0.00%
2015	48	0.00%		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

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IBNR for Exercise 3

IBNR	Age		IBNR	IBNR at Quarter End							
AY	as of 12/31/201	E[L]	Year End	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
2019		1,200	0	282	529	752	960	863	768	680	600
2018	12		800	719	640	566	500	385	292	220	167
2017	24		500	385	292	220	167	79	45	19	0
2016	36		100	48	27	12	0	0	0	0	0
2015	48		0	0	0	0	0	0	0	0	0
Current AY		IBNR	-	282	529	752	960	*863*	*768*	*680*	*600*
Prior AY		IBNR	1,400	1,152	959	798	667	*465*	*337*	*240*	*167*
All AY Total		IBNR	1,400	1,434	1,488	1,550	1,627	*1,328*	*1,105*	*919*	*767*

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Calculation WalkThru

- AY 2018 Q2: IBNR= IBNR YE * Survival factor=
  = 800 * 80% = 640
- AY 2019 Q2: IBNR = Ult Loss * (Earned Exposure PCT- Loss PCT ULT)
  = 1200*(50.0% -5.9%) = 529

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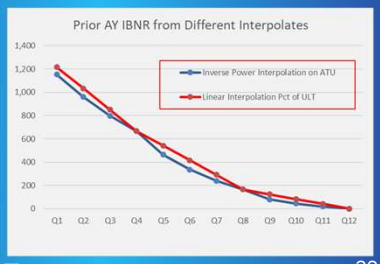
Prior AY IBNR

CQ	Prior AY IBNR: Inverse Power Interpolation on ATU	Prior AY IBNR: Linear Interpolation Pct of ULT
Q1	1,152	1,217
Q2	959	1,033
Q3	798	850
Q4	667	667
Q5	465	542
Q6	337	417
Q7	240	292
Q8	167	167
Q9	79	125
Q10	45	83
Q11	19	42
Q12	0	0



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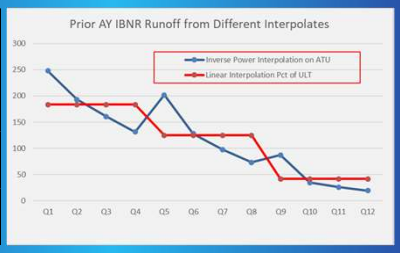
Prior AY IBNR Runoff

CQ	Prior AY IBNR Runoff: Inverse Power Interpolation on ATU	Prior AY IBNR Runoff: Linear Interpolation Pct of ULT
Q1	248	183
Q2	193	183
Q3	161	183
Q4	132	183
Q5	202	125
Q6	128	125
Q7	97	125
Q8	73	125
Q9	87	42
Q10	34	42
Q11	26	42
Q12	19	42



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Review of Example

- IVP Method:
  - Is the spike up in IBNR runoff each 1st quarter versus the 4th quarter of the prior year a concern?
  - A possible example of algorithmic induced seasonality
- PCT ULT linear interpolation
  - Large jumps each year
  - Always decreasing runoff.

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Properties of Interpolates

Inherited Monotonicity –"No extra bumps"
Equilibrium IBNR Stability
Monotonic Declining Prior Year IBNR Runoff in Equilibrium

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Inherited Monotonicity of ATA LDF

- Interpolated Quarterly ATA LDF do not violate the property of inherited montonicity if they don't oscillate any more often than the original Annual Evaluation ATA LDF.
- "No extra bumps!" rule.

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Inherited Monotonicity Violation

Annual Evaluation ATA LDF	
12 - 24	24 - 36
1.5000	1.2000

Quarterly Evaluation ATA LDF							
12 - 15	15 - 18	18 - 21	21 - 24	24 - 27	27 - 30	30 - 33	33 - 36
1.1855	1.1200	1.0800	1.0460	1.0700	1.0500	1.0400	1.0270

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Inherited Monotonicity Violation

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Equilibrium IBNR Stability

- Once in equilibrium, a book of business with same loss ratio and same earned premium each Quarter should have stable IBNR.
- No reason for bouncing around in equilibrium.
- Violation produces algorithmic-induced seasonality.

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Violation of Equilibrium IBNR Stability

	IBNR by Qtr in Equilibrium				
Evaluation	12/31/y-1	3/31/y	6/30/y	9/30/y	12/31/y
Current AY	-	180	500	600	700
Prior AY	1,000	800	560	360	300
All AY Total	1,000	980	1,060	960	1,000

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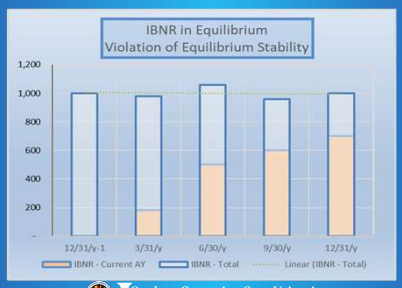
Violation of Equilibrium IBNR Stability



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Monotonic Declining Prior AY Equilibrium IBNR Runoff

- Once in equilibrium, prior AY IBNR Run-off should decline each quarter
- Assumes LDF pattern with no expected negative development.

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Violation of Monotonically Declining Prior AY Equilibrium IBNR Run-off

	IBNR by Qtr in Equilibrium				
Evaluation	12/31/y-1	3/31/y	6/30/y	9/30/y	12/31/y
IBNR - Prior AY	1,000	800	560	360	300
Prior AY IBNR Runoff		200	240	200	60

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Violation of Montonically Declining Prior AY Equilibrium IBNR Run-off

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Conclusions and Questions

- For actuarial purposes, the merit of an interpolation method should be judged on whether it leads to well-behaved IBNR.
- Interpolation Algorithms can give rise to algorithmic induced seasonality.
- Acceptable interpolation methods should satisfy the three properties
  - Inherited Montonicity
  - Equilibrium IBNR Stability
  - Montonically Declining Equilibrium IBNR Runoff
- Deriving interpolates for each year separately and ignore neighboring blocks is generally not sufficient.

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