Introduction

Instructional Approach to Mack and ODP Bootstrap Models

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Overview

- Introductions

- Notation

- Ranges vs. Distributions

- Mack Model

- ODP Bootstrap Model

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Logistics Overview

- Facilities
- Time / Breaks
- Language, Terminology & Currency
- Your view blocked? / Can't See?
- Feedback forms
- Don't Skip Ahead
- Have Fun / Ask Questions
- Examples NOT Commercial Software

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Notation

w	– denotes an Accident (or Policy) period ("when").
d	– denotes a development age ("delay").
k	– denotes a diagonal or constant $w + d$.
$c(w,d)$	– denotes cumulative losses for accident (or policy) period w, and development age d.
$c(w,n)$	– denotes ultimate losses for accident (or policy) period w [also denoted as $U(w)$].
$R(w,d)$	– denotes future development for accident (or policy) period w, and development age d.
	Note: $R(w,d) = U(w) - c(w,d)$

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Notation

$q(w,d)$	– denotes incremental losses for accident (or policy) period w, and development age d.
$f(d)$	– denotes the factor applied to $c(w,d)$ to estimate $q(w,d+1)$.
$F(d)$	– denotes the factor applied to $c(w,d)$ to estimate $c(w,d+1)$. For example, the d to $d+1$ age-to-age factor.
	[Note: WP Report uses $c(w,n)$]

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Notation

$G(w)$	– denotes a factor relating to accident (or policy) year w.
$h(w+d)$	– denotes a factor relating to diagonal k (or calendar year) in which $w+d$ is constant.
$e(w,d)$	– denotes the mean of zero random fluctuation which occurs in cell w, d.

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Notation

$E(x)$ — denotes the expectation of the random variable x. (Also μ_x)

$Var(x)$ — denotes the variance of the random variable x. (Also σ_x^2 or $Sigma_x^2$)

W — Weight

N — Total number of Accident (Policy) periods

n — Total number of Development periods

$\hat{x}$ — Estimate of x.

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Notation Example

Cumulative Losses

AY	0	1	2
1981	5,012	8,269	10,907
1982	106	4,285	
1983	3,410		

w Can be Year (i.e., 1981) or Index (i.e., 1)

Notation

AY	0	1	2
1981	$c(1981,0)$	$c(1981,1)$	$c(1981,2)$
1982	$c(1982,0)$	$c(1982,1)$	
1983	$c(1983,0)$		

d

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Notation Example

Incremental Losses

AY	0	1	2
1981	5,012	3,257	2,638
1982	106	4,179	
1983	3,410		

Calendar Effect denoted $h(1982+1)$ or $h(1983)$ or $h(3)$

Notation

AY	0	1	2
1981	$q(1981,0)$	$q(1981,1)$	$q(1981,2)$
1982	$q(1982,0)$	$q(1982,1)$	
1983	$q(1983,0)$		

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Notation Example

Age-to-Age Factors				Notation		
AY	1	2		AY	1	2
1981	1.650	1.319		1981	$F(1981,1)$	$F(1981,2)$
1982	40.425			1982	$F(1982,1)$	
Mean	21.037	1.319		Mean	$F(1)$	$F(2)$

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Ranges vs. Distributions

- A **Range** is not the same as a **Distribution**

- *A Range of Reasonable Estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable.

- A *Distribution* is a statistical function that attempts to quantify probabilities of all possible outcomes.

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Ranges vs. Distributions

- A **Range** is generally considered to be either a subset of "*central estimates*" or a subset of the "*possible outcomes*".

- For a "*central estimate*" the incremental values will essentially have the random movements "*averaged*" or "*smoothed*" out.

- A "*possible outcome*" will generally include random movements in the incremental values (e.g., calendar period payments within each accident period).

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Ranges vs. Distributions

Range of <u>Reasonable</u> Estimates

Range of <u>Possible</u> Estimates

"Best" Estimate

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Ranges vs. Distributions

Distribution of <u>Statistical</u> Outcomes

"Best" Estimate

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Ranges vs. Distributions

Distributions of <u>Possible</u> Outcomes

Estimated Unpaid Claims

With multiple models:
You can evaluate the relative strengths of each model!

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Ranges vs. Distributions

"Best Estimate" of a Distribution of <u>Possible</u> Outcomes

Range of <u>Mean</u> Estimates

"Best Estimate" of the Mean

Estimated Unpaid Claims

With multiple models:
You can use credibility weights to get your "best estimate"!

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Ranges vs. Distributions

"Best Estimate" of a Distribution of <u>Possible</u> Outcomes

Confidence Interval

"Best Estimate" of the Mean

Estimated Unpaid Claims

With multiple models:
You can calculate confidence intervals.

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Basic Models

- And Now for Something Completely Different...

    Mack Model

    ODP Bootstrap Model

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Final Questions?

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Suggested Bibliography

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