RELATIVE UNPAID CLAIMS LOSS RESERVING

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Outline

- Basic Triangle LDF Methods
- Relative Unpaid Claims Loss Reserving Model
- Illustrative Method
- Estimating Model Parameters
- Empirical Results
- Conclusion, Future Research

All dollars $000 Omitted

Examples use actual data from CAS Loss Reserve Data Base

Table Numbering from Actual Paper

Basic Triangle Methods – Rooted in LDFs

Payment Development Triangle

CUMULATIVE LOSS PAYMENTS

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1988	3,962	26,287	53,000	92,637	106,336	116,677	120,616	123,777	125,856	127,920
1989	6,066	28,297	60,135	84,025	103,086	117,532	122,380	126,114	131,199	
1990	3,751	31,503	68,116	100,424	112,788	123,077	129,081	132,513		
1991	3,336	46,188	95,800	135,163	153,146	165,430	178,462			
1992	6,647	49,319	94,860	129,306	148,235	165,476				
1993	8,056	81,087	147,732	193,275	217,199					
1994	9,720	65,339	130,303	186,750						
1995	7,171	82,822	160,302							
1996	16,696	88,800								
1997	21,098									

	1-2 Years	2-3 Years	3-4 Years	4-5 Years	5-6 Years	6-7 Years	7-8 Years	8-9 Years	9-10 Years
3 Yr. Wt'd LDF (a)	7.055	1.912	1.366	1.133	1.096	1.059	1.028	1.029	1.016

(a) 8-9 Years = 2 Yr. Wt'd LDF; 9-10 Years 1 Year LDF

Basic Triangle Methods – Rooted in LDFs

Method	**Requirements**
Payment Development	Payment Development Triangle Select Payment Development LDFs
Incurred Development	Payment Development Triangle + Case Reserves Triangle Select Incurred Development LDFs
Bornhuetter-Ferguson	Incurred (Payment) Development Triangle Select Expected Losses Select LDFs
Cape Cod	Incurred (Payment) Development Triangle Expected Losses derived from experience Select LDFs
Frequency/Severity (Counts/Averages)	Frequency (Claim Count) Development Triangle Severity (Averages) Development Triangle Select LDFs, disposal rates, and/or trend factors

Relative Unpaid Claims Model - Concept

INCREMENTAL LOSS PAYMENTS

Accident Year	During Year 1	During Year 2	During Year 3	During Year 4	During Year 5	During Year 6	During Year 7	During Year 8	During Year 9	During Year 10
1988										2,064
1989									5,085	
1990								3,432		
1991							13,032			
1992						17,241				
1993					23,924					
1994				56,447						
1995			77,480							
1996		72,104								
1997	21,098									

Relative Unpaid Claims Model – Concept

d = accounting date year-end 1997
$U_{1988,1997}$ = accident year 1988 unpaid loss as of d

INCREMENTAL LOSS PAYMENTS

Accident Year	During Year 1	During Year 2	During Year 3	During Year 4	During Year 5	During Year 6	During Year 7	During Year 8	During Year 9	During Year 10
1988										2,064
1989									5,085	
1990								3,432		
1991							13,032			
1992						17,241				
1993					23,924					
1994				56,447						
1995			77,480							
1996		72,104								
1997	21,098									

$U_{1988,1997}$

$$r_i = \frac{\text{accident year } i \text{ unpaid loss as of } d}{\text{accident year } i-1 \text{ unpaid loss as of } d-1}$$

If $U_{1988,1997}$ known and all r_i known, then all $U_{i,1997}$ are known

Relative Unpaid Claims Model – Formally

Definitions: For consecutive accident years m through n (n>m), define:

$U_{i,j}$ = accident year i unpaid losses as of year-end j j $\geq i$

$p_{i,j}$ = accident year i payments during calendar year j j $\geq i$

As of accounting date year-end d, define the ratio of unpaid loss at common maturities:

$$r_i = \frac{U_{i,d}}{U_{i-1,d-1}} \quad m+1 \leq i \leq n, \quad d \geq n, \quad U_{i-1,d-1} \neq 0$$

r_i equals the **relativity** of accident year i unpaid losses as of accounting date year-end d to accident year i-1 unpaid losses as of accounting date year-end d-1.

Model: Beginning with initial value $U_{m,d}$, each $U_{i,d}$ (m + 1 ≤ i ≤ n) may be computed using the recursive algorithm:

$$U_{m+1,d} = r_{m+1}[U_{m,d} + p_{m,d}]$$
$$U_{m+2,d} = r_{m+2}[U_{m+1,d} + p_{m+1,d}]$$
$$\cdots\cdots\cdots$$
$$U_{n,d} = r_n[U_{n-1,d} + p_{n-1,d}]$$

Relative Unpaid Claims Model – Formally

Model proof relies upon $U_{i-1,d-1} = U_{i-1,d} + p_{i-1,d}$

Model exact representation of unpaid losses

To Apply Model in a Method:

$p_{i,d}$ typically known as part of the historical data base

r_i typically unknown; estimated

$U_{m,d}$ typically unknown; estimated analogous to tail factor development methods

Case Reserve Ratios – Candidate for Estimated r_i

CASE RESERVES

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1988									1,588	116
1989								2,838	1,419	
1990							4,883	1,436		
1991						7,016	3,282			
1992					23,466	11,991				
1993				31,248	15,482					
1994			56,994	46,505						
1995		66,826	55,399							
1996	54,941	70,761								
1997	61,839									

$$r_{1997} = \frac{61{,}839}{54{,}941} = 1.1255529$$

$$r_{1992} = \frac{11{,}991}{7{,}016} = 1.7090935$$

Estimate r_i Using Case Reserve Ratios

TABLE 3.1

ASSUMPTIONS SELECTION

(1)	(2)	(3)	(4)= (3)/[Prior(2)]	(5)
Accident Year	Case Reserves as of 12/31/96	Case Reserves as of 12/31/97	Selected Ratio Unpaid Loss	Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i			Selected r_i	Selected $U_{1988,1997}$
1988	1,588	116		1,048
1989	2,838	1,419	0.8935768	
1990	4,883	1,436	0.5059901	
1991	7,016	3,282	0.6721278	
1992	23,466	11,991	1.7090935	
1993	31,248	15,482	0.6597631	
1994	56,994	46,505	1.4882552	
1995	66,826	55,399	0.9720146	
1996	54,941	70,761	1.0588843	
1997		61,839	1.1255529	

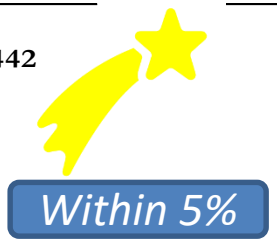
(1) m=1988; n=1997
(2), (3) CAS Loss Reserve Data Base [7]
(5) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Estimate Unpaid Losses Using Table 3.1 Assumptions

TABLE 3.2

INDICATED UNPAID LOSSES

(1) Accident Year i	(2) Case Reserve Ratio Unpaid Selected r_i	(3) Incremental Paid Loss During 1997 $p_{i,1997}$	(4) Indicated Unpaid Loss as of 12/31/97 Indicated $U_{i,1997}$	(5) Actual Emergence
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.5059901	3,432	3,980	4,875
1991	0.6721278	13,032	4,982	8,939
1992	1.7090935	17,241	30,787	27,175
1993	0.6597631	23,924	31,687	38,236
1994	1.4882552	56,447	82,764	75,947
1995	0.9720146	77,480	135,315	130,558
1996	1.0588843	72,104	225,325	216,789
1997	1.1255529	21,098	334,772	309,458
Total			**853,442**	815,254



(1) m=1988; n=1997
(2) Table 3.1, Column (4)
(3) CAS Loss Reserve Data Base [7]
(4) d=1997
For i = m = 1988: Table 3.1, Column (5)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year
+ company filed loss reserves (including IBNR) nine years subsequent to accident year

Initial Observations Using Rudimentary Table 3.1 Assumptions

Table 3.2 Method vs. Basic Triangle LDF Methods:

Direct vs. indirect

More efficient to apply

Only requires experience from most recent calendar year; no triangle; no LDFs

To effectively apply, need only – calendar year d payments by accident year; each case reserve ratio at common maturities reasonably correspond to the respective ratio of total unpaid (including IBNR) losses; reasonable estimate of unpaid losses for the oldest included accident year

Potential Distortions

Though Table 3.2 Method uses the ratio of case reserves to estimate r_i, several potential distortions in any measure or proxy for r_i:

Internal (e.g., mix of business shifts, changing claim procedures or case reserve adequacy)
External (e.g., law changes, inflation, social influences)
Credibility (i.e., randomness, sparseness- unrepresentative of the future)

Estimating Relative Unpaid Losses r_i

Reproduction of Actual Emergence

Table 4.1.1 and 4.1.2 of Paper illustrate

Case Reserve Ratios as per Table 3.1, 3.2 Example

Advantages

- Typically readily available
- Reflects actual loss experience
- If claims personnel have behaved consistently, case ratio as measure of the ratio of all unpaid losses (including IBNR) intuitively appealing

Potential Distortions

- Non-homogenous mix of business
- Different levels of adequacy - changing conditions (e.g., claims personnel practices) external conditions (e.g., inflation).
- Common time maturity could correspond to different stages of development
- Relativity of IBNR losses may be different than the corresponding case reserve ratio
- Low credibility, sparse case reserve experience

Calendar Year d Reported Emergence

Using Calendar Year d Reported Emergence to Estimate r_i

APPENDIX B, SHEET 1

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL INCREMENTAL PAID LOSSES AND CASE RESERVES

Accident Year	**Paid During Year 1**	*Case Reserves as of Year 1*	**Paid During Year 2**	*Case Reserves as of Year 2*	**Paid During Year 3**	*Case Reserves as of Year 3*	**Paid During Year 4**	*Case Reserves as of Year 4*	**Paid During Year 5**	*Case Reserves as of Year 5*	**Paid During Year 6**	*Case Reserves as of Year 6*	**Paid During Year 7**	*Case Reserves as of Year 7*	**Paid During Year 8**	*Case Reserves as of Year 8*	**Paid During Year 9**	*Case Reserves as of Year 9*	**Paid During Years 10**	*Case Reserves as of Year 10*
1988	3,962	*18,455*	22,325	*32,519*	26,713	*24,536*	39,637	*10,366*	13,699	*6,640*	10,341	*3,393*	3,939	*4,025*	3,161	*3,177*	2,079	*1,588*	2,064	*116*
1989	6,066	*18,674*	22,231	*27,084*	31,838	*16,408*	23,890	*13,583*	19,061	*10,691*	14,446	*6,809*	4,848	*3,791*	3,734	*2,838*	5,085	*1,419*		
1990	3,751	*15,681*	27,752	*32,388*	36,613	*26,127*	32,308	*19,254*	12,364	*12,150*	10,289	*6,913*	6,004	*4,883*	3,432	*1,436*				
1991	3,336	*22,485*	42,852	*38,265*	49,612	*40,475*	39,363	*24,041*	17,983	*16,674*	12,284	*7,016*	13,032	*3,282*						
1992	6,647	*31,730*	42,672	*48,726*	45,541	*43,345*	34,446	*25,248*	18,929	*23,466*	17,241	*11,991*								
1993	8,056	*44,945*	73,031	*69,391*	66,645	*48,541*	45,543	*31,248*	23,924	*15,482*										
1994	9,720	*41,128*	55,619	*62,428*	64,964	*56,994*	56,447	*46,505*												
1995	7,171	*51,969*	75,651	*66,826*	77,480	*55,399*														
1996	16,696	*54,941*	72,104	*70,761*																
1997	21,098	*61,839*																		

CAS Loss Reserve Data Base [7]

Using Calendar Year d Reported Emergence to Estimate r_i

APPENDIX B, SHEET 2

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL ONE YEAR REPORTED EMERGENCE

Accident Year		*Case Reserves as of Year 1*	**One Year Reported as of Year 2**	*Case Reserves as of Year 2*	**One Year Reported as of Year 3**	*Case Reserves as of Year 3*	**One Year Reported as of Year 4**	*Case Reserves as of Year 4*	**One Year Reported as of Year 5**	*Case Reserves as of Year 5*	**One Year Reported as of Year 6**	*Case Reserves as of Year 6*	**One Year Reported as of Year 7**	*Case Reserves as of Year 7*	**One Year Reported as of Year 8**	*Case Reserves as of Year 8*	**One Year Reported as of Year 9**	*Case Reserves as of Year 9*	**One Year Reported as of Year 10**
1988		18,455	54,844	32,519	51,249	24,536	50,003	10,366	20,339	6,640	13,734	3,393	7,964	4,025	6,338	3,177	3,667	1,588	2,180
	1 Year LDF		2.972		1.576		2.038		1.962		2.068		2.347		1.575		1.154		1.373
1989		18,674	49,315	27,084	48,246	16,408	37,473	13,583	29,752	10,691	21,255	6,809	8,639	3,791	6,572	2,838	6,504	1,419	
	1 Year LDF		2.641		1.781		2.284		2.190		1.988		1.269		1.734		2.292		
1990		15,681	60,140	32,388	62,740	26,127	51,562	19,254	24,514	12,150	17,202	6,913	10,887	4,883	4,868	1,436			
	1 Year LDF		3.835		1.937		1.974		1.273		1.416		1.575		0.997				
1991		22,485	81,117	38,265	90,087	40,475	63,404	24,041	34,657	16,674	19,300	7,016	16,314	3,282					
	1 Year LDF		3.608		2.354		1.566		1.442		1.157		2.325						
1992		31,730	91,398	48,726	88,886	43,345	59,694	25,248	42,395	23,466	29,232	11,991							
	1 Year LDF		2.880		1.824		1.377		1.679		1.246								
1993		44,945	142,422	69,391	115,186	48,541	76,791	31,248	39,406	15,482									
	1 Year LDF		3.169		1.660		1.582		1.261										
1994		41,128	118,047	62,428	121,958	56,994	102,952	46,505											
	1 Year LDF		2.870		1.954		1.806												
1995		51,969	142,477	66,826	132,879	55,399													
	1 Year LDF		2.742		1.988														
1996		54,941	142,865	70,761															
	1 Year LDF		2.600																
1997		61,839 `																	
W'td Avg. Dev. Factor			2.7249017		1.8627350		1.6082550		1.4460186		1.2571046		1.7282284		1.3999528		1.6909393		1.3727960

One Year Reported as of Year x = Appendix B, Sheet 1: Paid During Year x + Case Reserves as of Year x
Wt'd Avg. Dev. Factor equals dollar weighted average of (up to 3) most recent years underlined 1 Year LDFs

Using Calendar Year d Reported Emergence to Estimate r_i

TABLE 4.3.1

ASSUMPTIONS SELECTION

(1) Accident Year i	(2) Unpaid Loss as of 12/31/96 Reported as of 12/31/97	(3) Case Reserves as of 12/31/97	(4) Selected One Year Development Factor	(5)= (3)x(4) Estimated Unpaid Loss as of 12/31/97 Reported as of 12/31/98	(6)= (5)/[Prior (2)] Selected Ratio Unpaid Loss Selected r_i	(7) Selected Unpaid Loss of Oldest Accident Year as of 12/31/97 Selected $U_{1988,1997}$
1988	2,180					1,048
1989	6,504	1,419	1.3727960	1,948	0.8935768	
1990	4,868	1,436	1.6909393	2,428	0.3733378	
1991	16,314	3,282	1.3999528	4,595	0.9438465	
1992	29,232	11,991	1.7282284	20,723	1.2702701	
1993	39,406	15,482	1.2571046	19,462	0.6657941	
1994	102,952	46,505	1.4460186	67,247	1.7065192	
1995	132,879	55,399	1.6082550	89,096	0.8654103	
1996	142,865	70,761	1.8627350	131,809	0.9919475	
1997		61,839	2.7249017	168,505	1.1794715	

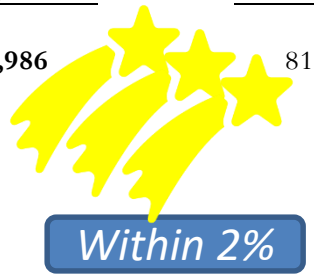
(1) m=1988; n=1997
(2) Appendix B, Sheet 2; One Year Reported final diagonal
(3) Appendix B, Sheet 2; final diagonal
(4) Appendix B, Sheet 2; final underlined row
(7) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Using Calendar Year d Reported Emergence to Estimate r_i

TABLE 4.3.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year i	Selected Ratio Unpaid Loss Selected r_i	Incremental Paid Loss During 1997 $p_{i,1997}$	Indicated Unpaid Loss as of 12/31/97 Indicated $U_{i,1997}$	Actual Emergence
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.3733378	3,432	2,937	4,875
1991	0.9438465	13,032	6,011	8,939
1992	1.2702701	17,241	24,190	27,175
1993	0.6657941	23,924	27,584	38,236
1994	1.7065192	56,447	87,900	75,947
1995	0.8654103	77,480	124,919	130,558
1996	0.9919475	72,104	200,770	216,789
1997	1.1794715	21,098	321,847	309,458
Total			**799,986**	815,254



(1) m=1988; n=1997
(2) Table 4.3.1, Column (6)
(3) Table 3.2, Column (3)
(4) d=1997
For i = m = 1988: Table 4.3.1, Column (7)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year
+ company filed loss reserves (including IBNR) nine years subsequent to accident year

Estimating Relative Unpaid Losses r_i

Steady State Value for r_i = 1 + trend rate

Steady State = same real (without consideration of unpaid loss frequency trend and severity trend) unpaid claim exposure as of common maturities for each accident year

Earned Premium

TABLE 4.5

(1) Accident Year i	(2) Earned Premium	(3)= (2)/[Prior (2)] Indicated Ratio Unpaid Loss Indicated r_i
1988	138,743	
1989	163,183	1.1761530
1990	162,184	0.9938780
1991	177,393	1.0937762
1992	197,770	1.1148692
1993	225,434	1.1398797
1994	267,578	1.1869461
1995	318,426	1.1900306
1996	363,402	1.1412447
1997	400,300	1.1015349

(2) CAS Loss Reserve Data Base [7]

Strength:

Ratio of successive accident year earned premium may provide stability and credibility to corresponding r_i indications

Potential weaknesses:

Change in premium adequacy

Measure relative total accident year exposure rather than relative unpaid loss exposure

Actual loss experience not directly reflected

Appropriate to complement with other r_i measures since estimated r_i based solely upon earned premium ignores impact of recent loss experience through valuation date

Estimating Relative Unpaid Losses r_i

Unpaid Claim Counts and Severity Indices

$$Estimated\ r_i = \frac{Estimated\ C_{i,d}}{Estimated\ C_{i-1,d-1}}\ x\ \frac{Estimated\ S_{i,d}}{Estimated\ S_{i-1,d-1}}$$

Where $C_{i,j}$= accident year i number of claims unpaid as of year-end j j $\geq i$

$S_{i,j}$= accident year i unpaid severity as of year-end j j $\geq i$

Other Measures and Adjustments

Other Measure Examples: payroll; number of vehicles; miles driven; operating expenditures; square footage; average occupied beds; outpatient visits; number of employees

Claim analytics

Potential Adjustments: policy limits and deductibles; reinsurance provisions; and law changes

Optimal Application of Model

Optimal Estimated Relative Unpaid Claims r_i

Cannot be universally prescribed

Requires investigation and care

Sensitivity analysis

Credibility

Optimal Oldest Included Accident Year Unpaid Losses $U_{m,d}$

Analogous to tail development factor

Adapt methods from CAS Committee on Reserves "The Estimation of Loss Development Tail Factors: A Summary Report"

Credibility

Exclude oldest accident years?

Empirical Evidence – CAS Loss Reserve Data Base

Qualifying Criteria

- Actual emergence of at least $25,000
- Positive earned premium for each calendar year 1989 through 1997
- Calendar year 1997 loss payments $\geq$ 0 for each accident year 1988 through 1997
- Each accident year 1988 - 1996 case reserve as of 12/31/96 $\geq$ $25 and each accident year 1989 through 1997 case reserve as of 12/31/97 $\geq$ $25
- No division by zero in working through any of the seven methods

Seven Unpaid Claim Methods

Basic	Relative Unpaid Claim Model
Payment Development Method (a)	Relative Unpaid Claims Method 1 (d); r_i per Table 3.2
Incurred Development Method (b)	Relative Unpaid Claims Method 2 (e); r_i per Table 4.3.2
Bornhuetter-Ferguson Method (c) (Expected Losses = A/Y 1988-90 Incurred Development Loss Ratio)	Relative Unpaid Claims Method 3 (f) r_i = .75(r_i per Table 3.2) + .25(r_i per Table 4.5)
	Relative Unpaid Claims Method 4 (g) r_i = .75(r_i per Table 4.3.2) + .25(r_i per Table 4.5)

Empirical Evidence – CAS Loss Reserve Data Base

TABLE 6

RETROSPECTIVE ACCURACY TEST OF 12/31/97 UNPAID CLAIM ESTIMATES:
46 Qualifying CAS Loss Reserve Data Base U.S. Property/Casualty Business Segments

(1) Loss Reserving Method	(2) Number of Business Segments where Estimate Falls Within 20% of Actual Emergence	(3) Number of Business Segments where Estimate Falls Within 10% of Actual Emergence
Payment Development (a)	19	13
Incurred Development (b)	26	17
Bornhuetter-Ferguson (c)	32	21
Relative Unpaid Claims 1 (d)	30	16
Relative Unpaid Claims 2 (e)	27	18
Relative Unpaid Claims 3 (f)	38	21
Relative Unpaid Claims 4 (g)	33	23

(2) Number of 46 Business Segments where $1/1.2 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.2
(3) Number of 46 Business Segments where $1/1.1 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.1

Heuristic Comparison Indicates Relative Unpaid Claim Methods Perform Well Versus Comparable Basic Methods

Model Conclusion; Future Research

Conclusion

- Straightforward exact Relative Unpaid Claims Loss Reserving Model
- Generally requires less data and fewer assumptions than basic triangle LDF methods
- Guidance provided on selecting appropriate estimates for r_i and $U_{m,d}$
- Generally results in unpaid claim estimates as accurate as comparable basic reserving methods
- Practical, efficient and powerful reserving tool

Future Research

Develop techniques to further improve estimated parameter accuracy – alternative parameter estimate weightings, incorporate steady state properties

Cast Model in stochastic setting