Cash Flow and Unpaid Claim Runoff Estimates Using Mack and Merz-Wüthrich Models

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Paper Outline

- Mack Model
  - Unpaid Claim Uncertainty
  - Unpaid Claim Runoff Uncertainty
  - Cash Flow Uncertainty
- Merz-Wüthrich Model
  - Unpaid Claim Uncertainty
  - Unpaid Claim Runoff Uncertainty
  - Cash Flow Uncertainty
- Alternative Model
- Conclusions

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Notation

$c(w,d)$	cumulative loss from accident year w as of age d.
$q(w,d)$	incremental loss for accident year w from d-1 to d.
$c(w,n)=U(w)$	total loss from accident year w when claims are at ultimate values at time n.
$R(w)$	future development after age n-w+1 for accident year w, i.e., = $U(w)$-$c(w,n-w+1)$.
$F(d)$	factor applied to $c(w,d)$ to estimate $c(w,d+1)$.
$e(w,d)$	a random fluctuation, or error, which occurs at the w, d cell.
$E(x)$	the expectation of the random variable x.
$Var(x)$	the variance of the random variable x. Or, alternatively σ_x^2.
σ_x	the standard deviation of the random variable x.
N	the total number of accident years.

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Notation

t	"at time" is equivalent to the valuation date used for financial accounting.
T	"time horizon" is the period for which the full distribution, including both process and parameter variance, is estimated.
T'	"time window" is the period between the valuation date and the time when the process variance and only a *portion* of the parameter variance is estimated.

Companion Excel files illustrate all formulas in the paper.

Free software is also available.

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Mack Model

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Mack Model

- Unpaid Claim Uncertainty
  - Variance of Development Periods

$$\hat{\sigma}_d^2 = \frac{1}{N-d-1} \times \sum_{j=1}^{N-d} c(j,d) \times \left\{ \frac{c(j,d+1)}{c(j,d)} - \hat{F}(d) \right\}^2 ; \; 1 \le d \le n-2$$

  - Variance of Accident Year

$$Var[\hat{R}(w)] = \hat{c}(w,n)^2 \times \sum_{d=n+1-w}^{n-1} \frac{\hat{\sigma}_d^2}{\hat{F}(d)^2} \times \left\{ \frac{1}{\hat{c}(w,d)} + \frac{1}{\sum_{j=1}^{N-d} c(j,d)} \right\}$$

  - Variance of Total of All Accident Years

$$Var[\hat{R}(tot)] = \sum_{w=2}^{N} \left\{ Var[\hat{R}(w)] + 2\hat{c}(w,n) \left(\sum_{i=w+1}^{N} c(i,n) \right) \sum_{d=n+1-w}^{n-1} \left(\frac{\hat{\sigma}_d^2}{\hat{F}(d)^2} \times \frac{1}{\sum_{j=1}^{N-d} c(j,d)} \right) \right\}$$

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Mack Model

- Sample Data

Sample Data Triangle Development Period (d)										
Year (w)	1	2	3	4	5	6	7	8	9	10
1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
7	440,832	1,288,463	2,419,861	3,483,130						
8	359,480	1,421,128	2,864,498							
9	376,686	1,363,294								
10	344,014									
$\hat{f}(d)$	*3.491*	*1.747*	*1.457*	*1.174*	*1.104*	*1.086*	*1.054*	*1.077*	*1.018*	*1.000*
$\hat{\sigma}_d$	400.35	194.26	204.85	123.22	117.18	90.48	21.13	33.87	21.13	-

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Mack Model

- Unpaid Claim Uncertainty

Mack Model			
Year	Unpaid Claims	Standard Deviation	CoV
1	-	-	0.0%
2	94,634	75,535	79.8%
3	469,511	121,699	25.9%
4	709,638	133,549	18.8%
5	984,889	261,406	26.5%
6	1,419,459	411,010	29.0%
7	2,177,641	558,317	25.6%
8	3,920,301	875,328	22.3%
9	4,278,972	971,258	22.7%
10	4,625,811	1,363,155	29.5%
CVA		*1,353,961*	
Total	18,680,856	2,447,095	13.1%

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Mack Model

- Unpaid Claim Runoff Uncertainty
  - Runoff of Mean Unpaid
    - Roll forward using development factors
  - Runoff of Accident Year Uncertainty
    - Roll forward to remove *t* diagonals
  - Runoff of Total of All Accident Year Uncertainty
    - Roll forward to remove *t* diagonals

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Mack Model

- Unpaid Claim Mean Runoff

Runoff of Mack Model Mean Values by Valuation Period (t)									
Year (w)	0	1	2	3	4	5	6	7	8
1	-	-	-	-	-	-	-	-	-
2	94,634	-	-	-	-	-	-	-	-
3	469,511	93,678	-	-	-	-	-	-	-
4	709,638	462,448	92,268	-	-	-	-	-	-
5	984,889	650,741	424,066	84,611	-	-	-	-	-
6	1,419,459	1,036,173	684,625	446,148	89,016	-	-	-	-
7	2,177,641	1,572,093	1,147,592	758,242	494,122	98,588	-	-	-
8	3,920,301	2,610,043	1,884,254	1,375,463	908,802	592,237	118,164	-	-
9	4,278,972	3,260,138	2,170,522	1,566,954	1,143,840	755,764	492,507	98,266	-
10	4,625,811	3,769,007	2,871,597	1,911,841	1,380,205	1,007,518	665,692	433,810	86,555
Total	18,680,856	13,454,320	9,274,925	6,143,258	4,015,986	2,454,107	1,276,363	532,076	86,555

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Mack Model

- Unpaid Claim Runoff Uncertainty

Runoff of Mack Model Standard Deviations by Valuation Period									
Year	0	1	2	3	4	5	6	7	8
1	-	-	-	-	-	-	-	-	-
2	75,535	-	-	-	-	-	-	-	-
3	121,699	74,931	-	-	-	-	-	-	-
4	133,549	120,373	74,041	-	-	-	-	-	-
5	261,406	125,695	113,131	69,186	-	-	-	-	-
6	411,010	269,797	130,224	117,306	71,982	-	-	-	-
7	558,317	437,273	287,714	139,969	126,301	78,029	-	-	-
8	875,328	623,100	489,142	323,291	159,581	144,441	90,307	-	-
9	971,258	785,070	557,224	436,400	287,117	139,643	125,999	77,826	-
10	1,363,155	903,373	729,436	516,796	404,139	265,121	127,697	114,976	70,421
CVA	*1,353,961*	*1,039,055*	*773,477*	*556,945*	*384,712*	*263,965*	*170,358*	*79,424*	
Total	2,447,099	1,788,912	1,340,940	954,131	663,602	431,762	263,362	159,952	70,421

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Mack Model

- Cash Flow Uncertainty
  - Cash Flow Mean Unpaid
    - Sum the mean along the diagonal
  - Cash Flow Uncertainty
    - Calculate standard deviation along the diagonal
  - Cash Flow Total Uncertainty
    - Calculate standard deviation for all diagonals
    - More covariance since need to include development period correlation

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Cash Flow and Unpaid Claim Runoff Estimates Using Mack and Merz-Wüthrich Models

Mack Model

- Cash Flow Uncertainty

Mack Model Cash Flow Uncertainty			
k	Unpaid Claims	Standard Deviation	CoV
11	5,226,536	665,562	12.7%
12	4,179,394	609,716	14.6%
13	3,131,668	558,467	17.8%
14	2,127,272	445,167	20.9%
15	1,561,879	353,389	22.6%
16	1,177,744	248,729	21.1%
17	744,287	142,151	19.1%
18	445,521	118,457	26.6%
19	86,555	70,421	81.4%
CVA		*2,106,547*	
Total	**18,680,856**	**2,447,095**	**13.1%**

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Mack Model

- Mean Cash Flow Runoff

Runoff of Mack Model — Mean Cash Flow Values by Valuation Period (t)										
k	0	1	2	3	4	5	6	7	8	
11	5,226,536	-	-	-	-	-	-	-	-	
12	4,179,394	4,179,394	-	-	-	-	-	-	-	
13	3,131,668	3,131,668	3,131,668	-	-	-	-	-	-	
14	2,127,272	2,127,272	2,127,272	2,127,272	-	-	-	-	-	
15	1,561,879	1,561,879	1,561,879	1,561,879	1,561,879	-	-	-	-	
16	1,177,744	1,177,744	1,177,744	1,177,744	1,177,744	1,177,744	-	-	-	
17	744,287	744,287	744,287	744,287	744,287	744,287	744,287	-	-	
18	445,521	445,521	445,521	445,521	445,521	445,521	445,521	445,521	-	
19	86,555	86,555	86,555	86,555	86,555	86,555	86,555	86,555	86,555	
20	-	-	-	-	-	-	-	-	-	
Total	**18,680,856**	**13,454,320**	**9,274,925**	**6,143,258**	**4,015,986**	**2,454,107**	**1,276,363**	**532,076**	**86,555**	

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Mack Model

- Cash Flow Runoff Uncertainty

Runoff of Mack Model — Cash Flow Standard Deviations by Valuation Period										
k	0	1	2	3	4	5	6	7	8	
11	665,562	-	-	-	-	-	-	-	-	
12	609,716	609,716	-	-	-	-	-	-	-	
13	558,467	558,467	558,467	-	-	-	-	-	-	
14	445,167	445,167	445,167	445,167	-	-	-	-	-	
15	353,389	353,389	353,389	353,389	353,389	-	-	-	-	
16	248,729	248,729	248,729	248,729	248,729	248,729	-	-	-	
17	142,151	142,151	142,151	142,151	142,151	142,151	142,151	-	-	
18	118,457	118,457	118,457	118,457	118,457	118,457	118,457	118,457	-	
19	70,421	70,421	70,421	70,421	70,421	70,421	70,421	70,421	70,421	
20	-	-	-	-	-	-	-	-	-	
CVA	*2,106,547*	*1,446,529*	*1,030,589*	*697,312*	*463,054*	*292,154*	*173,670*	*81,200*		
Total	**2,447,095**	**1,788,912**	**1,340,940**	**954,131**	**663,602**	**431,762**	**263,362**	**159,952**	**70,421**	

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Merz-Wüthrich Model

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Merz-Wüthrich Model

- Unpaid Claim Uncertainty
  - Variance of Development Periods
    - Same as for Mack
  - Variance of Accident Year
    - Decompose process and parameter variance
    - Include both process and parameter variance for first diagonal
    - Include only *partial* parameter variance for remaining diagonals
  - Variance of Total of All Accident Years
    - Similar to Mack, excluding process variance for future diagonals

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Merz-Wüthrich Model

- Unpaid Claim Uncertainty
  - Variance of Development Periods

$$\hat{\sigma}_d^2 = \frac{1}{N-d-1} \times \sum_{j=1}^{N-d} c(j,d) \times \left\{ \frac{c(j,d+1)}{c(j,d)} - \hat{F}(d) \right\}^2 \; ; \; 1 \le d \le n-2$$

  - Variance of Accident Year

$$Var\left[R^1(w)\right] = \hat{c}(w,n)^2 \times \frac{\hat{\sigma}_{N+1-w}^2}{\hat{F}(N+1-w)^2} \times \left\{ \frac{1}{c(w,N+1-w)} + \frac{1}{\sum_{j=1}^{w-1} c(j,N+1-w)} \right\}$$

$$+\hat{c}(w,n)^2 \times \sum_{d=n+2-w}^{n-1} \frac{\hat{\sigma}_d^2}{\hat{F}(d)^2} \times \left\{ \alpha_d^1 \times \frac{1}{\sum_{j=1}^{N-d} c(j,d)} \right\}$$

Using weight function:

$$\alpha_d^1 = \frac{c(N+1-d,d)}{\sum_{j=1}^{N+1-d} c(j,d)} \; ; for \; d = 1,2,\ldots,N$$

  - Variance of Total of All Accident Years

$$Var\left[R^1(tot)\right] = \sum_{w=2}^{N} \left(Var\left[R^1(w)\right] + 2\hat{c}(w,n) \times \left(\sum_{i=w+1}^{N} c(i,n) \right) \times \left\{ \frac{\hat{\sigma}_{N+1-w}^2}{\hat{F}(N+1-w)^2} \times \frac{1}{\sum_{j=1}^{w-1} c(j,N+1-w)} + \sum_{d=n+2-w}^{n-1} \left(\frac{\hat{\sigma}_d^2}{\hat{F}(d)^2} \times \alpha_d^1 \times \frac{1}{\sum_{j=1}^{N-d} c(j,d)} \right) \right\} \right)$$

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Merz-Wüthrich Model

- Unpaid Claim Uncertainty (1-Year Time Horizon)

Merz-Wüthrich Model			
Year (w)	Unpaid Claims	Standard Deviation	CoV
1	-	-	0.0%
2	94,634	75,535	79.8%
3	469,511	105,309	22.4%
4	709,638	79,846	11.3%
5	984,889	235,115	23.9%
6	1,419,459	318,427	22.4%
7	2,177,641	361,089	16.6%
8	3,920,301	629,681	16.1%
9	4,278,972	588,662	13.8%
10	4,625,811	1,029,925	22.3%
CVA		*1,025,050*	
Total	18,680,856	1,778,968	9.5%

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Merz-Wüthrich Model

- Unpaid Claim Runoff Uncertainty
  - Runoff of Mean Unpaid
    - Same as Mack
  - Runoff of Accident Year Uncertainty
    - Roll forward to remove *t* diagonals
    - Include both process and *partial* parameter variance for first diagonal (for *t*>1)
    - Include only *partial* parameter variance for remaining diagonals (less than for *t*=1)
  - Runoff of Total of All Accident Year Uncertainty
    - Roll forward to remove *t* diagonals
    - For all runoff years, the totals reconcile to Mack

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Merz-Wüthrich Model

- Unpaid Claim Runoff Uncertainty (Time Window)

Runoff of Merz-Wüthrich Model Standard Deviations by Time Window										
Year	1	2	3	4	5	6	7	8	9	TOTAL
1	-	-	-	-	-	-	-	-	-	-
2	75,535	-	-	-	-	-	-	-	-	75,535
3	105,309	60,996	-	-	-	-	-	-	-	121,699
4	79,846	91,093	56,232	-	-	-	-	-	-	133,549
5	235,115	60,577	82,068	51,474	-	-	-	-	-	261,406
6	318,427	233,859	57,825	82,433	51,999	-	-	-	-	411,010
7	361,089	328,989	243,412	59,162	85,998	54,343	-	-	-	558,317
8	629,681	391,249	359,352	266,320	64,443	94,166	59,533	-	-	875,328
9	588,662	554,574	344,763	318,493	236,576	56,543	83,645	52,965	-	971,258
10	1,029,925	538,726	511,118	317,142	293,978	218,914	51,661	77,317	49,055	1,363,155
CVA	*1,025,050*	*676,444*	*449,236*	*288,887*	*164,691*	*92,828*	*57,595*	*24,085*	-	*1,353,961*
Total	1,778,968	1,177,727	885,178	607,736	428,681	267,503	128,557	96,764	49,055	2,447,095

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Merz-Wüthrich Model

- Comparison with Mack Runoff

Valuation Period	Unpaid Claims	Mack Model Standard Deviation	Mack Model CoV	Merz-Wüthrich Model Standard Deviation	Merz-Wüthrich Model CoV	Ratio
0	18,680,856	2,447,095	13.1%	1,778,968	9.5%	72.7%
1	13,454,320	1,788,912	13.3%	1,177,727	8.8%	65.8%
2	9,274,925	1,340,940	14.5%	885,178	9.5%	66.0%
3	6,143,258	954,131	15.5%	607,736	9.9%	63.7%
4	4,015,986	663,602	16.5%	428,681	10.7%	64.6%
5	2,454,107	431,762	17.6%	267,503	10.9%	62.0%
6	1,276,363	263,362	20.6%	128,557	10.1%	48.8%
7	532,076	159,952	30.1%	96,764	18.2%	60.5%
8	86,555	70,421	81.4%	49,055	56.7%	69.7%

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Merz-Wüthrich Model

- Cash Flow Uncertainty
  - Cash Flow Mean Unpaid
    - Sum the mean along the diagonal
  - Cash Flow Uncertainty
    - Calculate standard deviation along the diagonal
    - For k=1, first diagonal includes process and parameter variance
    - For k>1, first diagonal includes process and *partial* parameter variance
  - Cash Flow Total Uncertainty
    - Calculate standard deviation for all diagonals
    - More covariance since need to include development period correlation

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Merz-Wüthrich Model

- Cash Flow Uncertainty (1-Year Time Horizon)

k	Merz-Wüthrich Model Cash Flow Uncertainty Unpaid Claims	Standard Deviation	CoV
11	5,226,536	665,562	12.7%
12	4,179,394	111,733	2.7%
13	3,131,668	108,154	3.5%
14	2,127,272	95,702	4.5%
15	1,561,879	83,976	5.4%
16	1,177,744	76,031	6.5%
17	744,287	67,017	9.0%
18	445,521	55,652	12.5%
19	86,555	40,213	46.5%
CVA		*1,632,904*	
Total	**18,680,856**	**1,778,968**	**9.5%**

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Merz-Wüthrich Model

- Cash Flow Runoff Uncertainty (Time Window)

Runoff of Merz-Wüthrich Model
Cash Flow Standard Deviations by Time Window

k	1	2	3	4	5	6	7	8	9	TOTAL
11	665,562	-	-	-	-	-	-	-	-	665,562
12	111,733	599,391	-	-	-	-	-	-	-	609,716
13	108,154	86,156	541,078	-	-	-	-	-	-	558,467
14	95,702	76,066	64,691	423,136	-	-	-	-	-	445,167
15	83,976	62,836	53,438	45,235	330,124	-	-	-	-	353,389
16	76,031	51,412	40,766	35,443	29,206	222,871	-	-	-	248,729
17	67,017	38,525	27,466	21,825	18,919	16,823	111,174	-	-	142,151
18	55,652	31,819	22,446	17,711	15,257	13,786	11,586	92,455	-	118,457
19	40,213	20,602	13,965	10,286	8,620	7,577	6,974	5,476	49,055	70,421
20	-	-	-	-	-	-	-	-	-	-
CVA	*1,632,904*	*1,002,522*	*693,268*	*431,394*	*270,680*	*146,137*	*63,121*	*28,026*	-	*2,106,547*
Total	1,778,968	1,177,727	885,178	607,736	428,681	267,503	128,557	96,764	49,055	2,447,095

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Alternative Model

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Alternative Model

- Unpaid Claim Runoff Uncertainty
  - Runoff of Mean Unpaid
    - Same as Mack
  - Runoff of Accident Year Uncertainty
    - Roll forward to remove *t* diagonals
    - Include both process and parameter variance for first diagonal (for *t*>1)
    - Include only *partial* parameter variance for remaining diagonals (same as *t*=1)
  - Runoff of Total of All Accident Year Uncertainty
    - Roll forward to remove *t* diagonals
    - For each runoff year, the first diagonal reconciles to Mack

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Alternative Model

- Unpaid Claim Runoff Uncertainty (Time Horizon)

Runoff of Alternative Model
1-Year Time Horizon Standard Deviations by Valuation Period

Year	0	1	2	3	4	5	6	7	8
1	-	-	-	-	-	-	-	-	-
2	75,535	-	-	-	-	-	-	-	-
3	105,309	74,931	-	-	-	-	-	-	-
4	79,846	100,806	74,041	-	-	-	-	-	-
5	235,115	68,535	93,353	69,186	-	-	-	-	-
6	318,427	240,563	67,590	95,673	71,982	-	-	-	-
7	361,089	336,607	255,033	70,558	102,361	78,029	-	-	-
8	629,681	400,731	374,947	284,965	79,593	116,320	90,307	-	-
9	588,662	562,933	356,774	334,233	253,564	69,171	101,939	77,826	-
10	1,029,925	544,418	521,865	329,305	308,794	234,466	62,194	92,663	70,421
CVA	*1,025,050*	*787,105*	*592,464*	*434,573*	*299,857*	*212,772*	*154,021*	*79,424*	-
Total	1,778,968	1,258,989	987,439	713,534	521,112	353,057	214,796	144,746	70,421

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Alternative Model

- Comparison with Mack Runoff

Valuation Period	Unpaid Claims	Mack Model Standard Deviation	Mack Model CoV	Alternative Model Standard Deviation	Alternative Model CoV	Ratio
0	18,680,856	2,447,095	13.1%	1,778,968	9.5%	72.7%
1	13,454,320	1,788,912	13.3%	1,258,989	9.4%	70.4%
2	9,274,925	1,340,940	14.5%	987,439	10.6%	73.6%
3	6,143,258	954,131	15.5%	713,534	11.6%	74.8%
4	4,015,986	663,602	16.5%	521,112	13.0%	78.5%
5	2,454,107	431,762	17.6%	353,057	14.4%	81.8%
6	1,276,363	263,362	20.6%	214,796	16.8%	81.6%
7	532,076	159,952	30.1%	144,746	27.2%	90.5%
8	86,555	70,421	81.4%	70,421	81.4%	100.0%

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Alternative Model

- Cash Flow Runoff Uncertainty (Time Horizon)

Runoff of Alternative Model
1-Year Time Horizon Cash Flow Standard Deviations by Valuation Period

k	0	1	2	3	4	5	6	7	8
11	665,562	-	-	-	-	-	-	-	-
12	111,733	609,716	-	-	-	-	-	-	-
13	108,154	109,882	558,467	-	-	-	-	-	-
14	99,702	98,675	90,084	445,167	-	-	-	-	-
15	83,976	85,291	77,554	70,476	353,389	-	-	-	-
16	76,031	76,123	65,148	60,230	54,280	248,729	-	-	-
17	67,017	66,535	53,523	45,973	43,151	41,759	142,151	-	-
18	55,652	55,325	44,294	37,931	35,483	34,692	32,406	118,457	-
19	40,213	40,009	31,628	25,970	23,895	22,903	22,997	19,221	70,421
20	-	-	-	-	-	-	-	-	-
CVA	*1,632,904*	*1,110,741*	*813,592*	*553,671*	*378,763*	*246,460*	*157,423*	*81,200*	
Total	1,778,968	1,284,315	999,009	719,407	524,388	353,079	215,796	144,896	70,421

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Conclusions

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Model Comparisons

	Mack	Merz-Wüthrich	Alternative
• Current Calendar Year:			
	– First diagonal: both process and parameter variance	– First diagonal: both process and parameter variance	– First diagonal: both process and parameter variance
	– Other diagonals: both process and parameter variance	– Other diagonals: only *partial* parameter variance	– Other diagonals: only *partial* parameter variance
• Future Runoff Years:			
	– First diagonal: both process and parameter variance	– First diagonal: both process and *partial* parameter variance	– First diagonal: both process and parameter variance
	– Other diagonals: both process and parameter variance	– Other diagonals: only *partial* parameter variance	– Other diagonals: only *partial* parameter variance
• Reconciliation:			
	– Not Applicable	– All runoff years reconcile to Mack	– Not Applicable

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Conclusions

• Calculation of Risk Margin (using Merz-Wüthrich)

Cost of Capital for Merz-Wüthrich Model using a VaR Capital Profile						
Valuation Period	Unpaid Claims	Standard Deviation	99.5th Percentile	99.5% VaR	6.0% CoC	Discounted CoC
0	18,680,856	1,778,968	23,753,426	5,072,570	304,354	301,328
1	13,454,320	1,177,727	16,785,734	3,331,414	199,885	193,982
2	9,274,925	885,178	11,799,479	2,524,553	151,473	144,092
3	6,143,258	607,736	7,882,818	1,739,561	104,374	97,323
4	4,015,986	428,681	5,252,966	1,236,980	74,219	67,836
5	2,454,107	267,503	3,227,797	773,690	46,421	41,590
6	1,276,363	128,557	1,645,023	368,659	22,120	19,425
7	532,076	96,764	833,102	301,026	18,062	15,548
8	86,555	49,055	293,233	206,679	12,401	10,464
Total						891,587
Percent of Unpaid Claims:						*4.8%*

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Conclusions

- Calculation of Risk Margin (using Alternative Model)

Cost of Capital for Alternative Model using a VaR Capital Profile						
Valuation Period	Unpaid Claims	Standard Deviation	99.5th Percentile	99.5% VaR	6.0% CoC	Discounted CoC
0	18,680,856	1,778,968	23,753,426	5,072,570	304,354	301,328
1	13,454,320	1,258,989	17,038,055	3,583,735	215,024	208,674
2	9,274,925	987,439	12,123,409	2,848,484	170,909	162,580
3	6,143,258	713,534	8,222,165	2,078,907	124,734	116,308
4	4,015,986	521,112	5,555,442	1,539,456	92,367	84,424
5	2,454,107	353,057	3,512,025	1,057,918	63,475	56,868
6	1,276,363	214,796	1,935,777	659,413	39,565	34,745
7	532,076	144,746	1,021,830	489,754	29,385	25,295
8	86,555	70,421	421,013	334,458	20,067	16,933
Total						1,007,157
Percent of Unpaid Claims:						*5.4%*

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Conclusions

- Calculation of Risk Margin (using BE Runoff)

Cost of Capital using a BE Runoff Capital Profile						
Valuation Period	Unpaid Claims	BE Runoff Ratio	99.5th VaR	BE Runoff Ratio VaR	6.0% CoC	Discounted CoC
0	18,680,856	100.0%	5,072,570	5,072,570	304,354	301,328
1	13,454,320	72.0%		3,653,365	219,202	212,729
2	9,274,925	49.6%		2,518,499	151,110	143,746
3	6,143,258	32.9%		1,668,131	100,088	93,327
4	4,015,986	21.5%		1,090,494	65,430	59,803
5	2,454,107	13.1%		666,384	39,983	35,821
6	1,276,363	6.8%		346,582	20,795	18,262
7	532,076	2.8%		144,479	8,669	7,462
8	86,555	0.5%		23,503	1,410	1,190
Total						873,668
Percent of Unpaid Claims:						*4.7%*

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Conclusions

- Runoff of Merz-Wüthrich standard deviation tends to underestimate the Risk Margin (under Solvency II)
- Runoff of the Alternative standard deviation corrects for the underestimation
- Covariance Adjustment
  - Examples in this presentation all show separate CVA
  - Paper includes formulas to allocate CVA to respective years
  - This may aide in comparisons with other models

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Any Final Questions?

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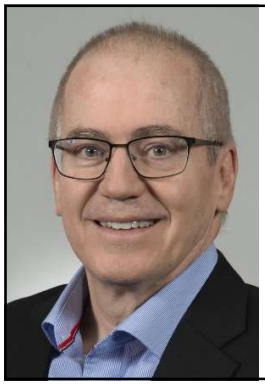

About the speaker

- **Name**
  - Mark R. Shapland, FCAS, FSA, MAAA
  - Past member of CAS Board of Directors
  - Frequent Speaker at CAS and International Events
- **Recent Publications**
  - Cash Flow and Unpaid Claim Runoff Estimates Using Mack and Merz-Wüthrich Models
  - Back-Testing the ODP Bootstrap & Mack Bootstrap Models
  - Using the Hayne MLE Models: A Practitioners Guide
  - The Actuary and Enterprise Risk Management: Integrating Reserve Variability
  - Using the ODP Bootstrap Model: A Practitioner's Guide
  - The Analysis of "All-Prior" Data

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