# Stochastic Cape Cod
## An Old Friend in a New Suit

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# Review of Generalized Cape Cod (GCC)

- A method to estimate *a priori* loss ratio
- Similar to traditional Cape Cod (CC) but more flexible with the addition of $Decay \in [0,1]$
- Bridges between Development Factor Method (DFM) and traditional Cape Cod (CC) with a decay factor
- $GCC = \begin{cases} DFM \quad , if\ Decay = 0 \\ Something\ in\ between \\ CC \quad , if\ Decay = 1 \end{cases}$

# Review of GCC (Cont.)

- Mathematically:

$$E[LR_i] = \frac{\sum_j D_j \times F^{|i-j|}}{\sum_j (EXP_j/CDF_j) \times F^{|i-j|}}$$

- where
  - $E[LR_i] = expected\ loss\ ratio\ for\ origin\ period\ i$
  - $F = decay\ factor\ (0 \leq F \leq 1)$
  - $D_j = trended\ latest\ losses\ for\ origin\ period\ i$
  - $CDF_j = cumulative\ development\ factor\ for\ origin\ period\ i$
  - $EXP_j = exposure\ for\ origin\ period\ j$
- $E_j/CDF_j$ is sometimes called the “used-up exposure”

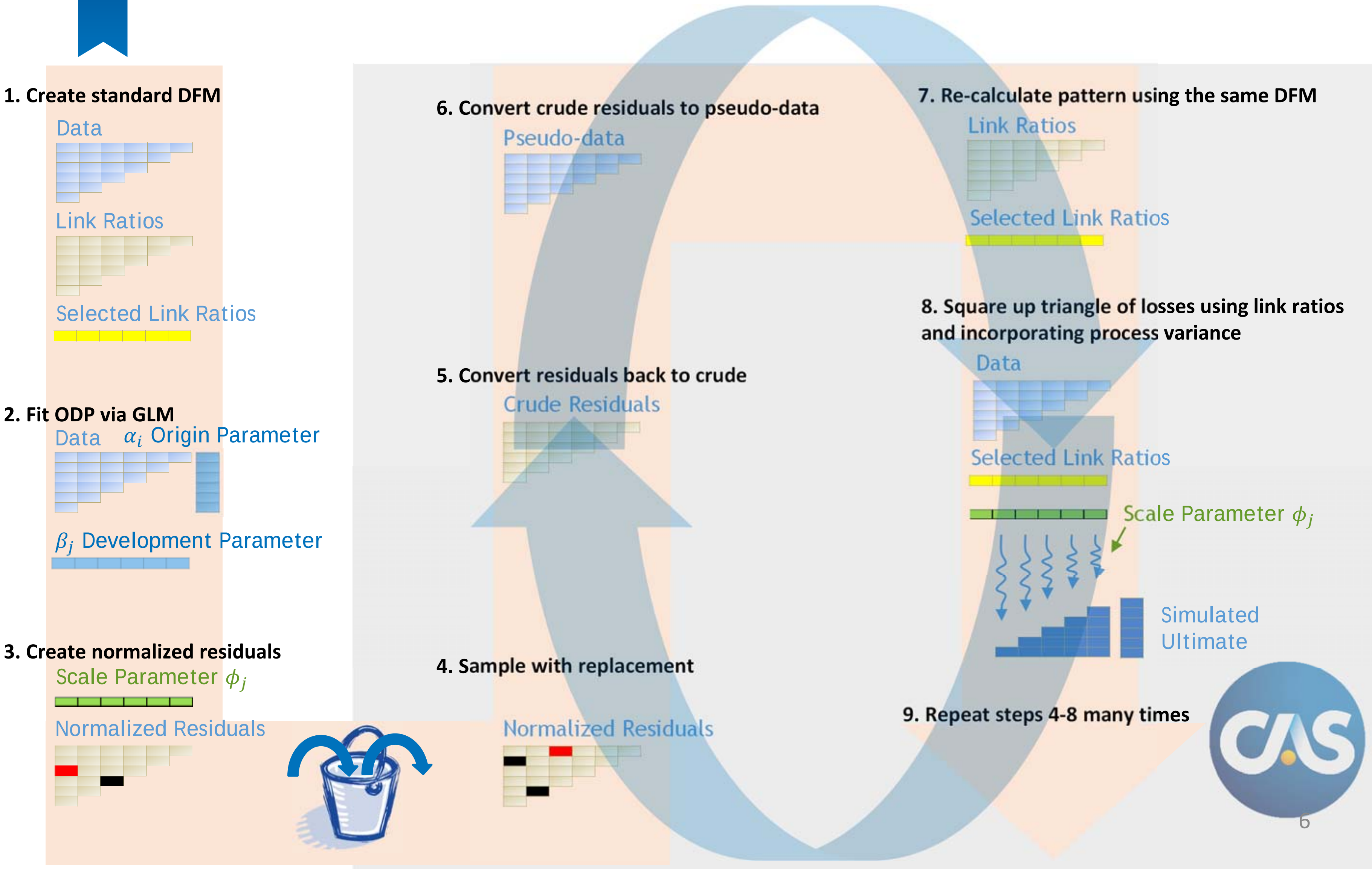
# Review of DFM Bootstrap (Over-Dispersed Poisson)

- $C_{ij} \sim ODP(\mu_{ij}, \phi_j)$
- where
  - $C_{ij}$ is the incremental claim amount in origin period $i$ and development period $j$
  - $E[C_{ij}] = \mu_{ij}$
  - $Var(C_{ij}) = \phi_j \times \mu_{ij}$
- $log(\mu_{ij}) = \alpha_i + \beta_j + c$
  - Log link function
  - The RHS is the linear predictor, estimated via GLM
  - Reproduces the DFM results

# Review of DFM Bootstrap (Cont.)

- $\phi_j$
  - The scale parameter
  - Estimated during the fitting process but assumed to be known (i.e., not variable) for practicality
  - Used to normalize residuals
    - $r_{ij} = \frac{C_{ij} - \mu_{ij}}{\sqrt{\phi_j \times \mu_{ij}}}$
  - Then used to convert sampled normalized residuals to crude for pseudo-data
    - $C^*_{ij} = r^*_{ij} \times \sqrt{\phi_j \times \mu_{ij}} + \mu_{ij}$
  - Can be used to add process variance

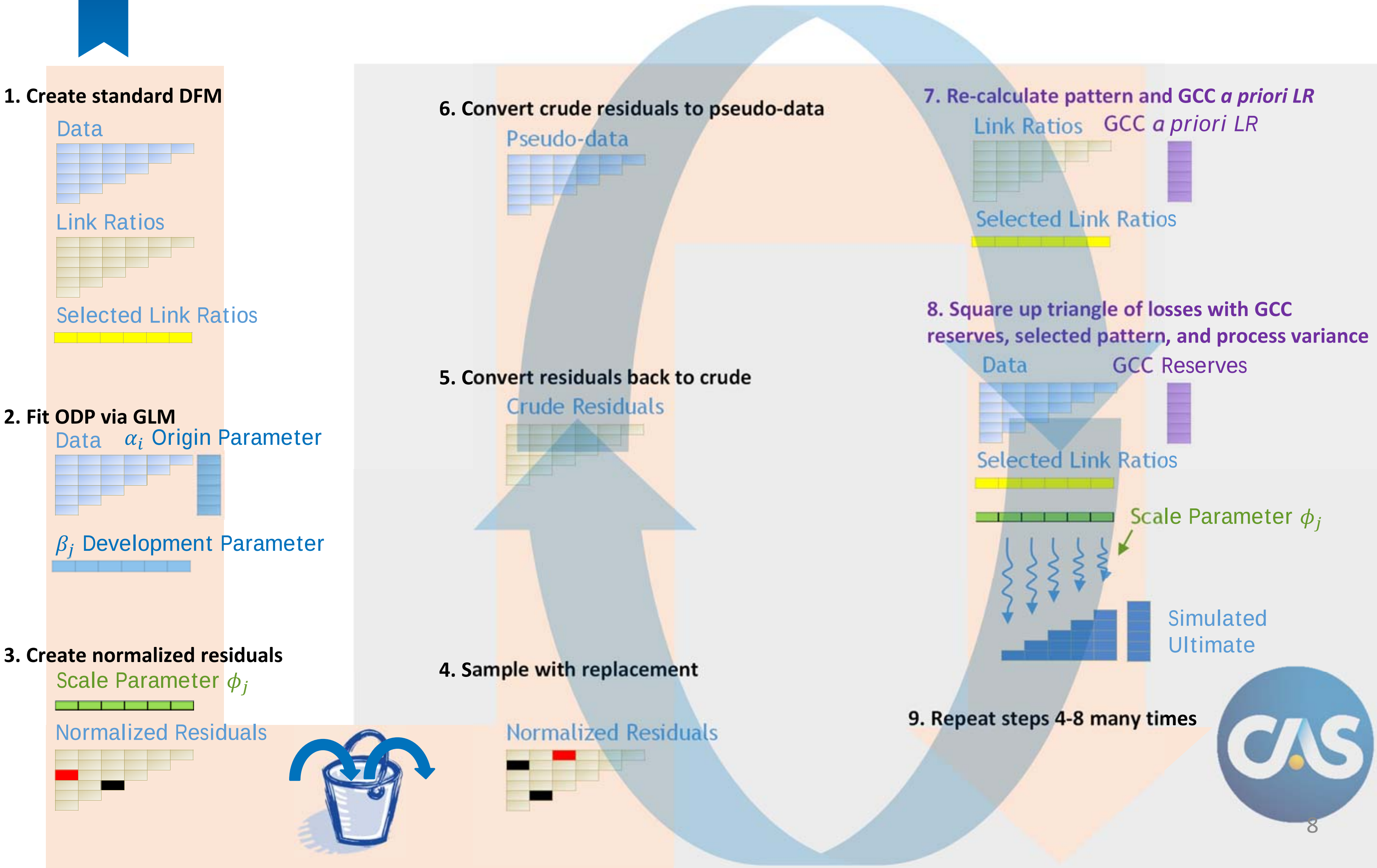
# Map of DFM Bootstrap (ODP)



# Birthing the GCC Bootstrap

- Knowing:
  - Traditional Bootstrap is based on DFM
  - DFM is a special case of GCC
- Replace DFM with GCC in the DFM Bootstrap Map gives the GCC Bootstrap method

# Map of GCC Bootstrap (ODP)

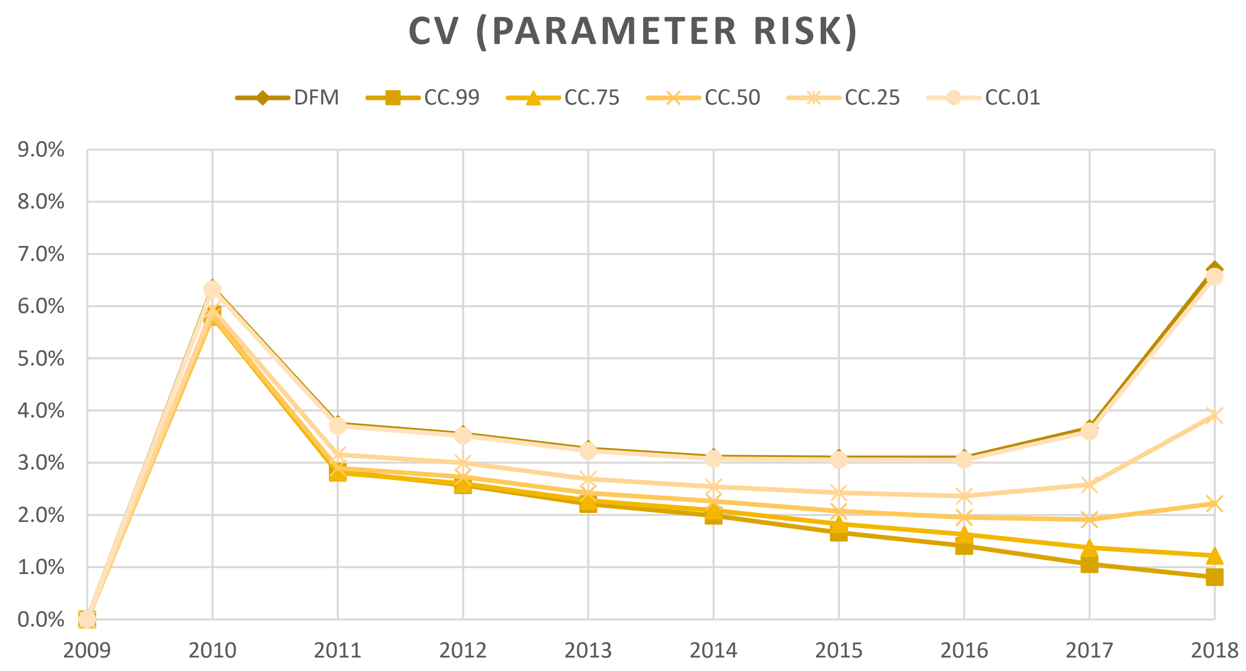


# Implementing in Excel (Ingredients and Recipe)

- Use proprietary or open-source software to obtain DFM Bootstrap results by simulation, including:
  1. Exposure measure (same for all simulations)
  2. Simulated latest diagonal of loss (Step 6 in Map)
  3. Simulated selected link ratios (Step 7)
- Excel macro loops through each DFM Bootstrap simulation:
  - For a given decay factor, calculate the GCC *a priori* LR
  - Calculate B-F reserves using GCC *a priori* LR
  - Project incremental cash flows
  - Add process variance using Scale Parameter (Step 8)

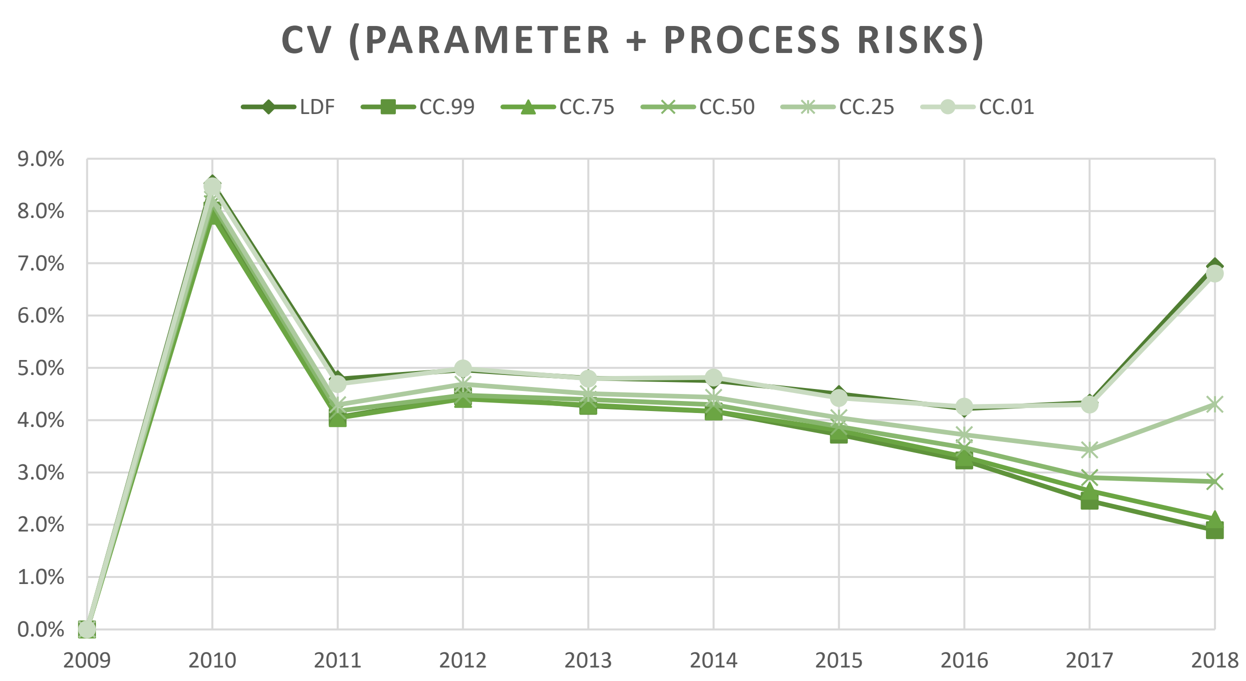
# Implementing in Excel (Sample Results)

- Note that “CC.01” (GCC Bootstrap with decay factor = 0.01) almost reproduces the DFM Bootstrap results
- As decay factor increases (CC.01 => CC.99), CV decreases, because effectively more data is being weighted together



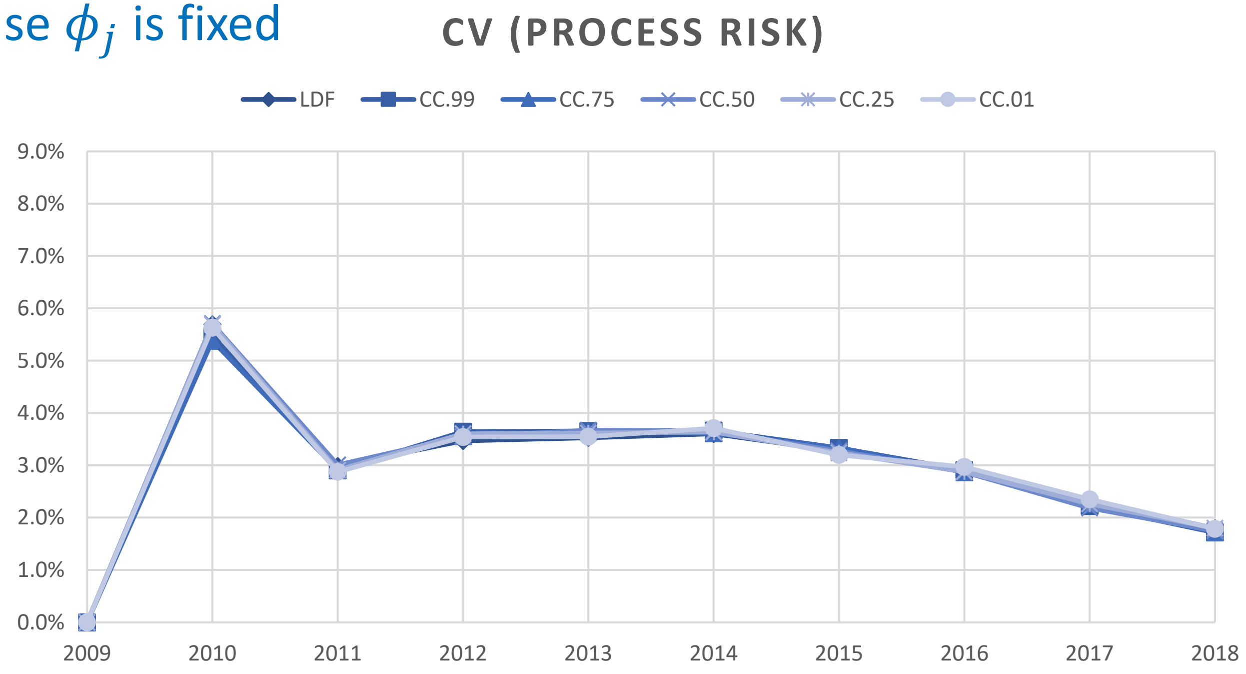
# Implementing in Excel (Sample Results Cont.)

- CVs are higher across the board with the addition of process risk, as expected



# Implementing in Excel (Sample Results Cont.)

- $Process\ Var = Total\ Var - Parameter\ Var$
- $Process\ Risk\ CV = \frac{\sqrt{Process\ Var}}{Mean}$
- Indentical Process Risk CV for all decay factors, which is expected because $\phi_j$ is fixed



# Implementing in R

- LIVE demo

# Compared to DFM Bootstrap

## Pros

- Can reproduce DFM Bootstrap with $F = 0$
- Additional flexibility through decay factor $F \in [0, 1]$
- If an origin period has no latest loss, DFM Bootstrap CV = N/A, but GCC Bootstrap can advise a CV based on the other origin periods and decay factor
- Can provide stability to lines with sparse/volatile data, where DFM Bootstrap yields unrealistically high CVs.

## Cons

- Not readily available in existing software solutions, requiring additional upfront work to set up the algorithm
- Inappropriate decay factor selection can understate variability

# Related Research

- Spencer Gluck's seminal paper *Balancing Development and Trend in Loss Reserve Analysis* offers insights to deriving GCC reserve estimates' variance analytically
- Glenn Meyer's recent CAS Monograph 8 explores stochastic Cape Cod in the Bayesian MCMC framework

# Future Research

- Extend the GCC Bootstrap to the Mack flavor
- Test the model's "reputation" (as Meyers defines it in CAS Monograph 8) based on CAS Loss Reserve Database
- Incorporate a way of suggesting an optimal decay factor as part of the algorithm
  - Gluck's appendix presents a path

# References

- Generalized Cape Cod:
  - https://www.casact.org/pubs/proceed/proceed97/97482.pdf
  - https://www.casact.org/pubs/forum/98fforum/struhuss.pdf
- ODP Bootstrap:
  - https://www.casact.org/pubs/monographs/papers/04-shapland.pdf
- Related Research:
  - https://www.casact.org/pubs/monographs/papers/08-Meyers.pdf
  - https://www.actuaries.org.uk/system/files/documents/pdf/sm0201.pdf

- Q&A

# Thank you!

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