TOWERS PERRIN

Internal Stochastic Risk Models

CANE Meeting -- Sturbridge

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Why build an internal stochastic EC/RBC model?

1. The calibration of the standard factor approach (used by NAIC, Solvency II, AMB, S&P) may be set conservatively
   - Rating agencies and regulators will ultimately give credit in their ratings for internal capital models
   - ➔ *Insurers without internal stochastic models will be handicapped by higher capital requirements*
2. Improves perception of company with the rating agencies regulators, and possibly analysts
3. Insurers need internal models to compete effectively
   - Internal models can reflect the actual risks more accurately
   - Internal models are an integral part of advanced risk management; can be a source of advantage

S&P has established criteria for reviewing internal company EC models

- Multiple risk measures used
- Encompassing all major risks; both gross and net
- Explicit calculation of diversification benefit – with conservative tail correlation
- Robustness
- Validation testing and methodology
- ECM used for strategic risk management

S&P has indicated that a strong ERM rating requires an internal model

"Companies that use standard [RBC] formulas without modifications will be likely to make poor decisions… If companies use these standard formulas without modification, S&P will view this as a weak [ERM] practice."

"Some companies have risk positions that are so complex that simple linear formulas are not adequate to estimate risk capital accurately."

Solvency II requirements for internal models will be demanding

Use Test

- Widely used, important role in risk management, decision-making and capital allocation within company
- Frequency of calculation consistent with frequency of use
- Responsibility of management

Statistical Quality

- Current, credible, realistic, justified assumptions
- Complete and appropriate data
- Consistent ranking of risks for use test and decision-making
- Adequate measurement of diversification benefits
- Reasonable management actions, with regard to time-to-implement

Solvency II requirements for internal models will be demanding

Calibration Standards

- VaR favoured as risk measure
- Flexibility but must be at least equivalent to 99.5% VaR over 1 year

P&L Attribution

- Analysis of profit and loss by source for each major Business Unit
- Link risk categories and sources of profit and loss

Validation Standards

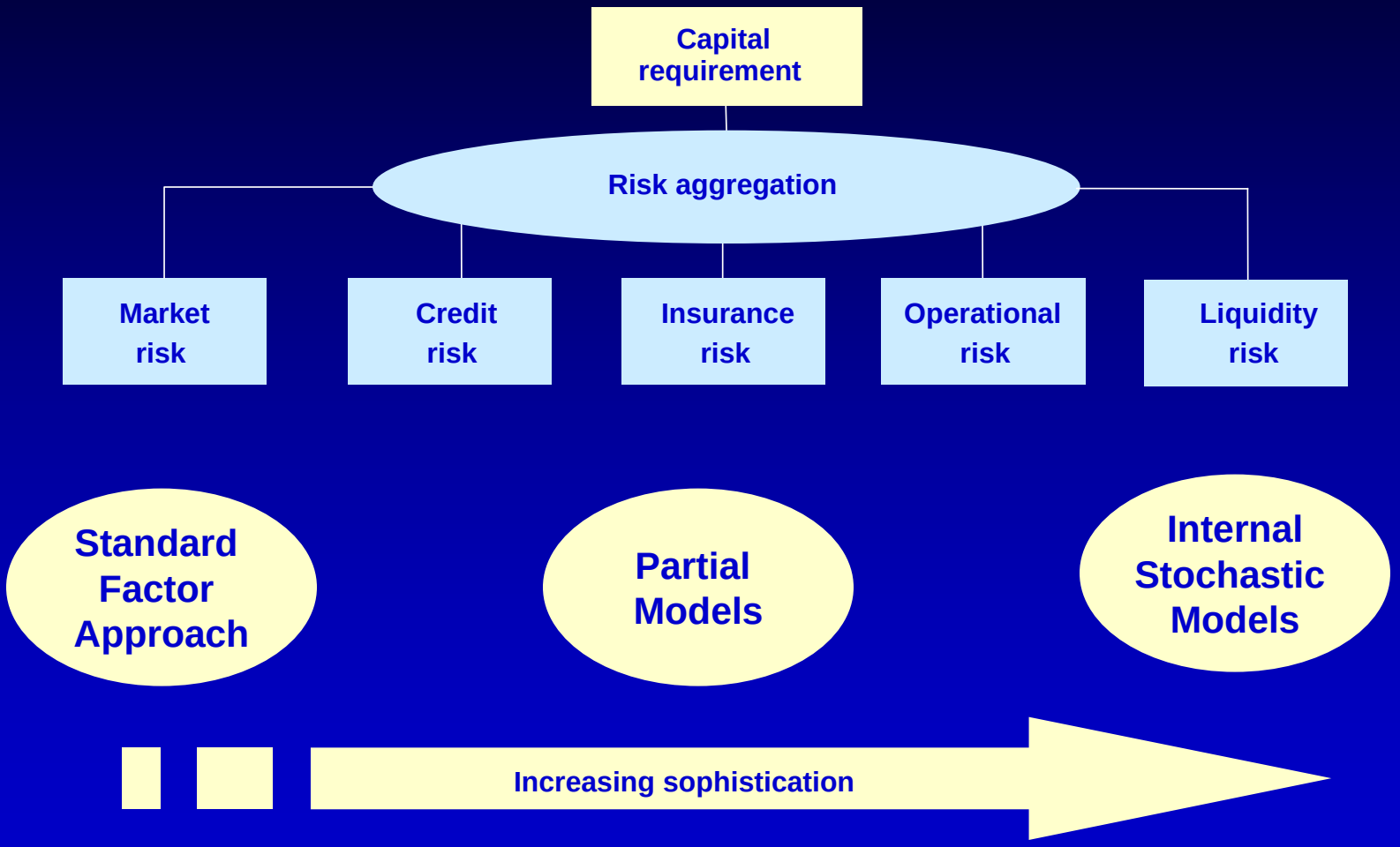
- Regular validation cycle, including performance of internal model, appropriateness, testing against experience
- Effective statistical processes to demonstrate appropriateness
- Analysis of actual versus expected

Being clear with terminology — what is an internal model under Solvency II?

Internal model = economic capital
+ risk management processes

An 'internal' model needs to be demonstrably embedded and should be consistent with the firm's approach to enterprise risk management

Approaches to EC present a spectrum of systems requirements and sophistication



Stochastic models come in two loosely defined categories

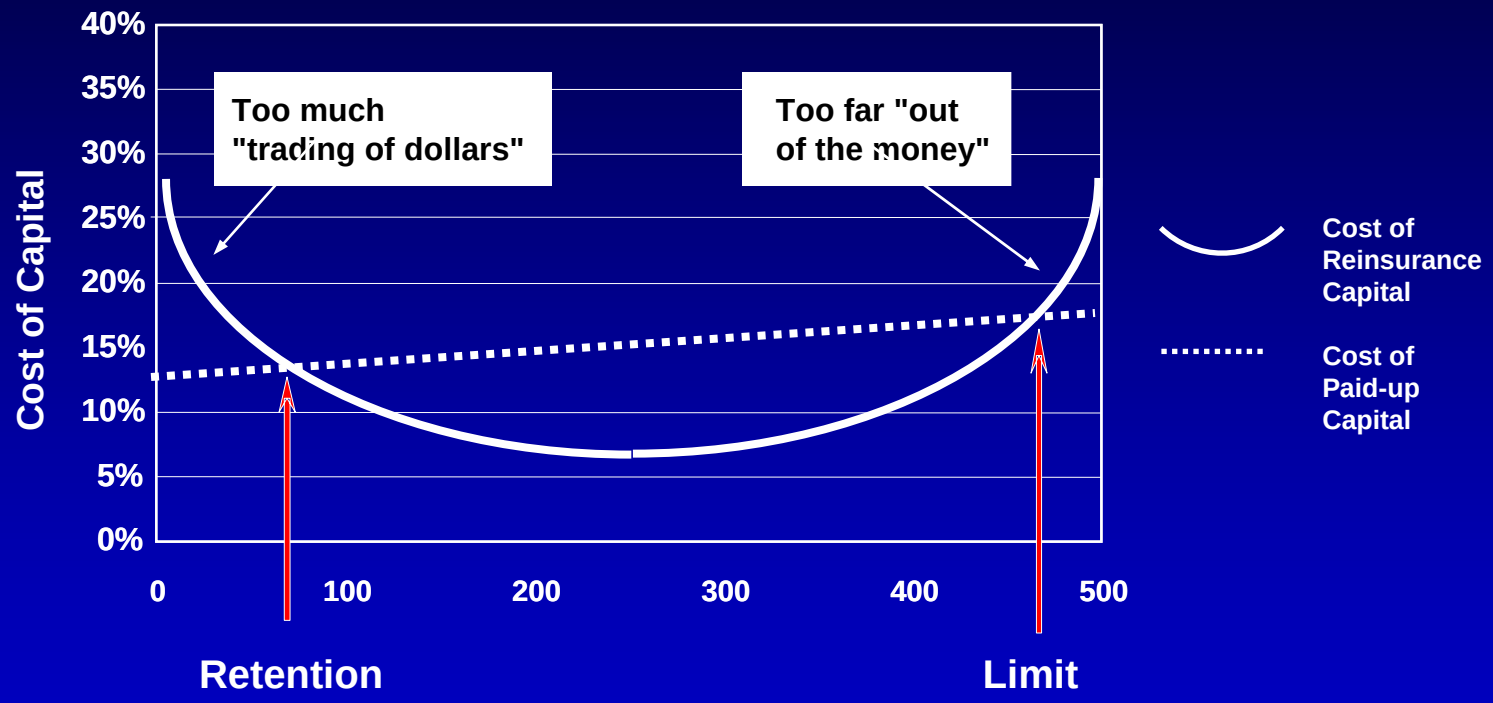
- Statistical models
  - Described entirely by a set of random variables
  - Each variable has an associated distribution and parameters
  - Correlation is specified via copulas
  - *Example: tornado loss model*
- Structural models
  - Described by system of equations that specify deterministic interactions, and random elements
  - Volatility can vary over time and be state-specific
  - Correlations are emergent properties
  - *Example: hurricane loss model*

Statistical models seek to measure prediction error

Category	Authors / Approaches
Analytic	■ Mack: Chain Ladder Estimation Error ■ Murphy: Regression Estimation Error ■ Wright: Poisson/Gamma Collective Risk Model ■ Scollnick: Bayesian Approach ■ Van Kampen: Loss Ratio Distribution ■ Wacek Loss Ratio Path
Simulation	■ England & Verrall: Bootstrap Simulation ■ Hodes, Feldblum & Blumsohn: WC Model ■ Kelly: Practical Approach

Statistical approach can be used to optimize property reinsurance retentions

ILLUSTRATIVE



$$\text{Cost of Reinsurance Capital} = \frac{\text{Change in Contract Risk Margin}}{\text{Change in Paid-Up Risk Capital}}$$

Risk has structure, due to underlying systemic drivers

- Inter-temporal
  - Reversion to normative conditions
  - Momentum induces cyclical behavior
- Inter-variable
  - Risk premia across asset class returns
  - Purchase power parity across currencies
  - Inflation impact on loss costs

To manage the risks of an insurer, we need a multi-period economic model that robustly captures the structure of the key elements of systemic risk

Economic scenarios can be used to introduce structure to the model

"Risk Drivers"

Economic Scenario Generator

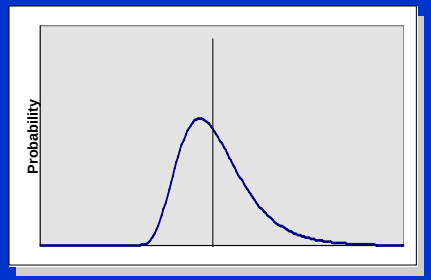
- Inflation
- Interest Rates
- Credit Spreads
- Currency Exchange
- GDP

Asset Behavior Model

Product Behavior Model

Projected Financials

Risk Profile = Distribution of Future Financial Results



- Required Economic Capital
- Tangible Economic Value

"Risk Strategies"

Company Strategy

- Asset Mix
- Product Mix
- Capital Structure
- Reins/Hedging

Optimization

Our Global CAP:Link economic scenario generator is a system of stochastic equations

- Stochastic equations generate time series for each variable:

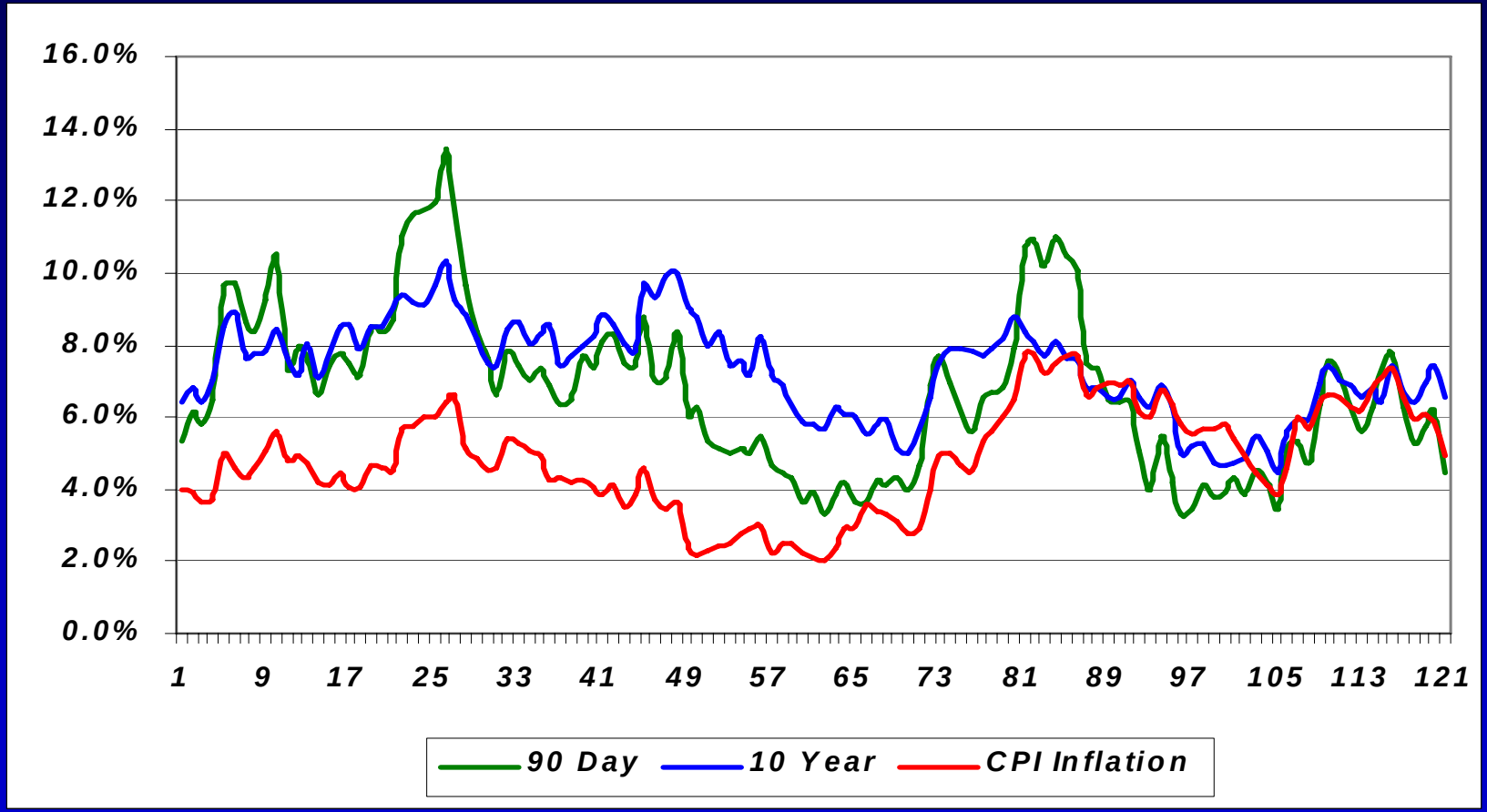
$$dr_t = f_1(r_u - r_t)dt + f_2(r_t, p_t, \ldots)dt + f_3(r_t)dZ_1$$

Mean Reversion — **Variable Links** — **Random Element**

- Models the change in a variable, as a function of a deterministic system and a stochastic overlay
- The equation creates a direct link between
  - the variable through time
  - other variables in the system
  - the random nature of the variable

Stochastic equations produce a plausible set of scenarios for all systemic risk variables

Global CAP:Link Scenario of Interest and Inflation Rates for Ten Years



Case study: what is the asset mix that minimizes the risk to an excess WC insurer?

- A matched set of Treasury bonds?
- What are the drivers of risk?
  - Medical inflation drives ultimate claim costs
  - Inflation and interest rates are linked
  - Equity returns are linked to inflation
- *Minimum risk position includes equities, as a natural hedge against inflation*

Major failures of stochastic risk models

- Oct 1987 — Black Monday Stock Market Crash
- Sep 1998 — Long Term Capital Management Fails
- Oct 2001 — Enron Fails
- Sep 2006 — Hurricane Katrina Destroys LA, MS, AL
- Aug 2007 — Subprime Credit Crisis Begins

"Theoretically…such a loss… unlikely to occur even once over the entire lifetime of the universe"

"No company has a better handle on its enterprise risk"

"Funds…hit by moves that…models suggested were 25 standard deviations away from normal."

Source: Steve Mildenhall

The failure of the banks' sub-prime models is instructive

Company	Total Equity Aug-07 ($ billions)	Total Assets Aug-07 ($ billions)	Subprime Markdown ($ billions)	Reported VaR Metric ($ billions)	Subprime Loss Relative to Reported VaR
Merrill Lynch	42	1,076	8.4	0.05	162 x
UBS	41	2,042	3.4	0.14	24 x
Citigroup	128	2,221	3.5	0.11	33 x
DeutschBank	47	2,523	3.1	0.10	31 x
Morgan Stanley	35	1,185	2.4	0.09	27 x
Goldman Sachs	39	1,046	1.7	0.10	17 x
Lehman Brothers	21	606	0.7	0.04	17 x
Bear Stearns	13	397	0.7	0.03	24 x
Bank of America	136	1,579	1.5	0.04	35 x

- VaR metrics typically based on daily trading volatility, assuming no "change in state"
- To be effective models must capture "unknown unknowns"

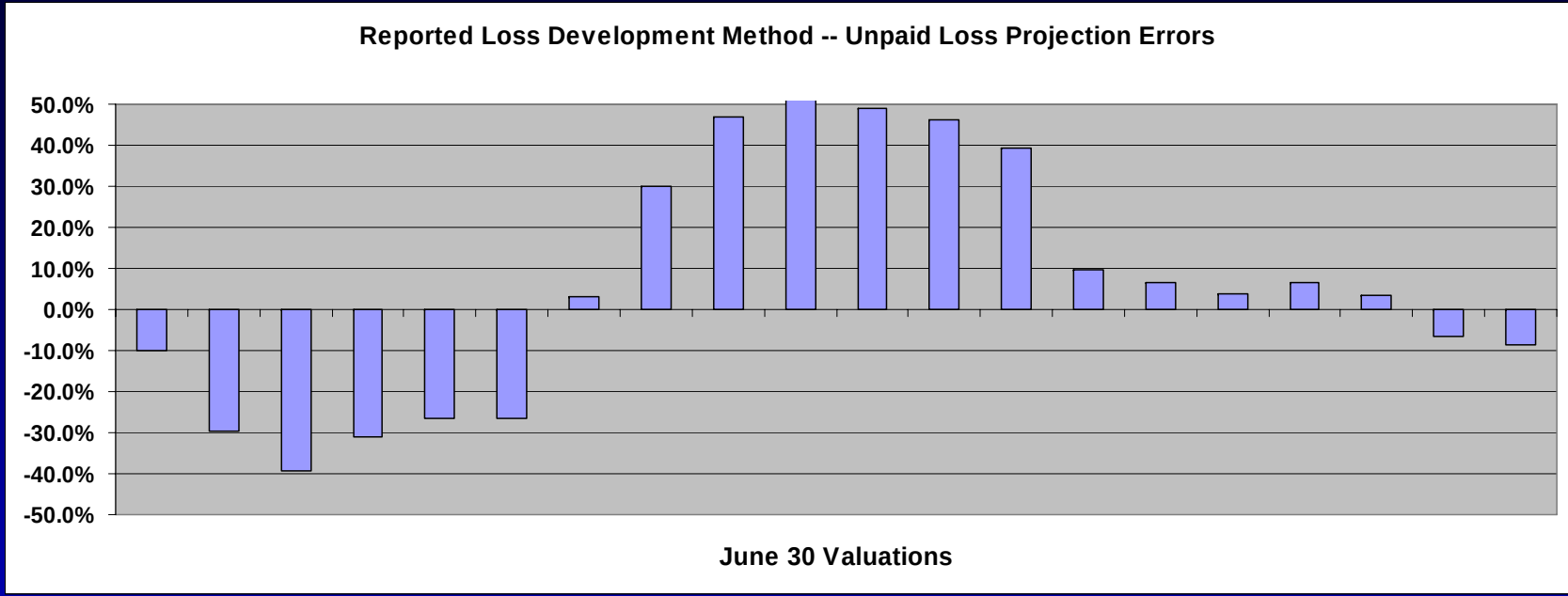
Source: Steve Mildenhall

The same issue, closer to home...

- GIRO: Test results indicate that Mack method for measuring reserve risk may understate true risk
  - Assumes that loss development is a *stationary* stochastic process
    - But greatest risk is when development "stretches out" due to economic or social inflation
  - May confuse MSE with MSEP
    - Need to test with "out-of-sample" data

Models need empirical validation !!

Hindsight testing is an analysis of historical claim liability estimation errors



- Requires a lot of history
- May need to separate management decisions from actuarial indications
- Provides concrete, non-parametric, empirical evidence that can be used to validate/invalidate models

Empirical hindsight data indicates that Mack understates reserve risk

- Sample of 20 lines of business, “more difficult” casualty lines
- Experience over a 15-20 year period
- Mack includes parameter risk and tail factor volatility