2011 Special Interest Seminar

Homeowners Multivariate Trend Analysis – An Example

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Objective and Outline

- Objective
  - To illustrate how multivariate trend analysis can lead to questions of interest in ratemaking and product management

- Outline
  - Overview of a commonly used homeowners loss trend procedure
  - Multivariate loss trend analysis example s

Commonly Used Loss Trend Procedure

- Homeowners loss trend analysis usually includes consideration of:
  - economic data
    - construction cost and consumer price indices
  - Insurance data
    - state or regional pure premium trend
  - changes in the company's environment
    - product design, underwriting guidelines, claims practices

Pure Premium Trend Estimates

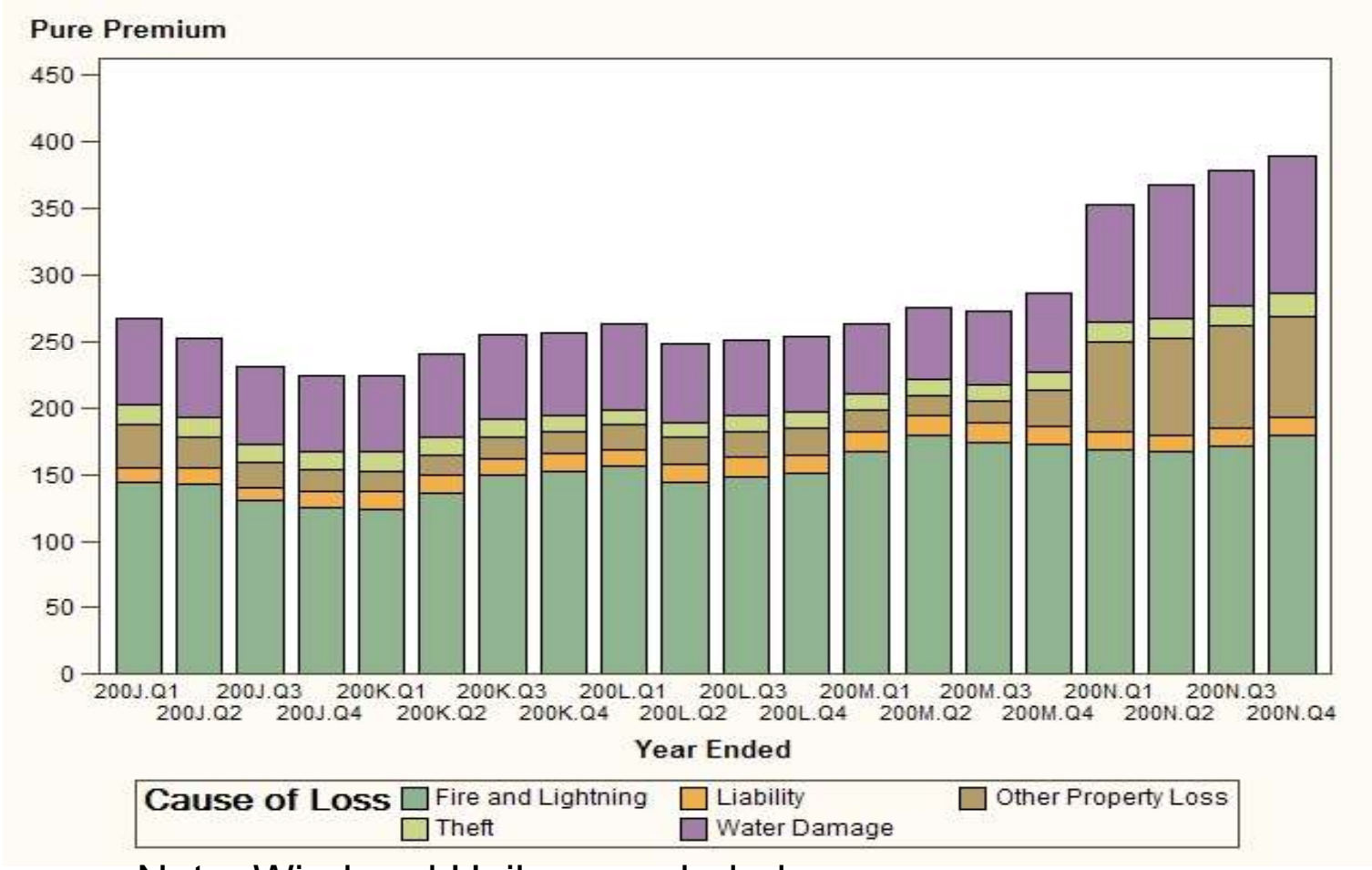
- Based on univariate regression
  - $Ln(P) = a + b \times t + \varepsilon$ $\qquad\qquad$ $P = e^{a} (e^{b})^{t} e^{\varepsilon}$
- pure premium
  - excludes wind and hail
  - based on 4-quarter rolling averages
- range of estimates obtained by including different numbers of points

Pure Premium* Trend
Univariate Analysis Example

Year Ended	Pure Premium	Year Ended	Pure Premium		
200J.Q1	267.68	200L.Q3	251.01		
200J.Q2	252.85	200L.Q4	253.87		
200J.Q3	230.07	200M.Q1	263.10	**20 pt**	9.8%
200J.Q4	224.31	200M.Q2	275.09	**16 Pt**	14.1%
200K.Q1	223.97	200M.Q3	272.43	**12 Pt**	19.3%
200K.Q2	240.49	200M.Q4	285.76	**8 Pt**	29.5%
200K.Q3	255.66	200N.Q1	352.16	**6 Pt**	35.5%
200K.Q4	256.50	200N.Q2	366.91	**4 Pt**	14.0%
200L.Q1	262.88	200N.Q3	378.53		
200L.Q2	248.02	200N.Q4	388.63		

* Excluding Wind and Hail

Does the Trend Vary by Peril?



Note: Wind and Hail are excluded.

Univariate Regression On Each Peril Separately

Combined

20 pt	9.8%
16 Pt	14.1%
12 Pt	19.3%
8 Pt	29.5%
6 Pt	35.5%
4 Pt	14.0%

Data	Points	Fire and Lightning	Water	Theft	Other Property*	Liability
Annual Trend	100 pt	6.9%	8.6%	1.7%	28.4%	4.9%
	80 pt	7.9%	13.4%	6.6%	56.3%	0.7%
	48 pt	6.6%	25.6%	16.1%	86.7%	-0.7%
	40 pt	0.6%	60.2%	23.2%	220.2%	-6.9%
	30 pt	1.6%	73.2%	29.5%	242.4%	-4.2%
	20 pt	8.5%	21.8%	25.3%	15.4%	13.4%
R Squared	100 pt	0.73	0.32	0.04	0.38	0.40
	80 pt	0.75	0.40	0.36	0.65	0.02
	48 pt	0.59	0.58	0.90	0.67	0.01
	40 pt	0.02	0.88	0.93	0.86	0.41
	30 pt	0.08	0.84	0.96	0.76	0.10
	20 pt	0.67	0.79	0.90	0.78	0.70

* Excluding Wind and Hail

Multivariate Regression
Using Peril Pure Premium Information

Peril	Pure Premium	Ln Pure Premium	y	W	T	D	L
Fire and Lightning	169.05	5.13019	1.00	0	0	0	0
Water Damage	87.92	4.47643	1.00	1	0	0	0
Theft	14.02	2.64048	1.00	0	1	0	0
Other Direct Phys. Loss*	68.46	4.22625	1.00	0	0	1	0
Liability	12.71	2.54239	1.00	0	0	0	1
Fire and Lightning	167.01	5.11805	1.25	0	0	0	0
Water Damage	99.05	4.59562	1.25	1	0	0	0
Theft	15.70	2.75366	1.25	0	1	0	0
Other Direct Phys. Loss*	72.74	4.28689	1.25	0	0	1	0
Liability	12.40	2.51770	1.25	0	0	0	1

* Excluding Wind and Hail

Multivariate Regression
Using Peril* Pure Premium Information

$$\ln(P) = \alpha_0 + \alpha_1 y + \alpha_2 W + \alpha_3 T + \alpha_4 D + \alpha_5 L + \alpha_6 yW + \alpha_7 yT + \alpha_8 yD + \alpha_9 yL + \varepsilon$$

P = pure premium for a peril at time t

y = time index corresponding to year ended quarter

W = water damage indicator

T = theft indicator

D = other direct physical loss indicator

L = liability indicator

* Perils other than Wind and Hail

Multivariate Stepwise Regression Using Peril Pure Premium Information

$$\ln(P) = \alpha_0 + \alpha_1 y + \alpha_2 W + \alpha_3 T + \alpha_4 D + \alpha_5 L + \alpha_6 yW + \alpha_7 yT + \alpha_8 yD + \alpha_9 yL + \varepsilon$$

- Full model ←→ Regression on each peril
- Stepwise regression
  - Use test statistics to select model
  - Interactions terms that make it correspond to perils more likely to have significantly different trend

Known Stepwise Regression Issues And Trend Indications by Peril

Predictive Modeling

- Goal is to build a model that will perform well in other data sets
- Danger of letting method automatically select model without a priori hypotheses
- A parameter is more likely to be selected if it is above its expected value than if it is below its expected value

Trend Indications

- Goal is to obtain a range of estimates that will inform the actuary's judgment
- Actuary takes into account changes in company policy and external environment
- Parameters identify perils whose trend is potentially different from the overall trend

R Squared Versus Adjusted R Squared

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}$$

$$adjR^2 = 1 - \frac{(n-1)(1-R^2)}{(n-k-1)}$$

$$k = number\ of\ independent\ variables$$

n	100	60	30
R Sq	0.90	0.90	0.90
k	Adj R Sq	Adj R Sq	Adj R Sq
0	0.90	0.90	0.90
1	0.90	0.90	0.90
2	0.90	0.90	0.89
3	0.90	0.89	0.89
4	0.90	0.89	0.88
5	0.89	0.89	0.88
6	0.89	0.89	0.87
7	0.89	0.89	0.87
8	0.89	0.88	0.86
9	0.89	0.88	0.86
10	0.89	0.88	0.85

Multivariate Stepwise Regression Using Peril Pure Premium Information

$$\ln(P) = \alpha_0 + \alpha_1 y + \alpha_2 W + \alpha_3 T + \alpha_4 D + \alpha_5 L + \alpha_6 yW + \alpha_7 yT + \alpha_8 yD + \alpha_9 yL + \varepsilon$$

```
PROC GLMSELECT DATA=modeling_data;
 MODEL Ln_PurePremium = y W T D L y*W y*T y*D y*L
   / SELECTION=STEPWISE CHOOSE=ADJRSQ
   SELECT=ADJRSQ SHOWPVALUES;
 OUTPUT OUT=predicted_data
   PREDICTED=predicted_Ln_PurePremium;
RUN;
```

Multivariate Stepwise Regression Using Peril Pure Premium Information

Stepwise Selection Summary				
Step	Effect Entered	Effect Removed	Number Effects In	Adjusted R-Square
0	Intercept		1	0.0000
1	L		2	0.2166
2	T		3	0.5737
3	D		4	0.8398
4	W		5	0.9126
5	y*D		6	0.9388
6	y		7	0.9431
7	y*T		8	0.9433*
* Optimal Value Of Criterion				

Multivariate Stepwise Regression Using Peril Pure Premium Information

Parameter Estimates					
Parameter	**DF**	**Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	1	5.023392	0.053846	93.29	<.0001
y	1	0.065871	0.021538	3.06	0.0029
W	1	-0.849798	0.076055	-11.17	<.0001
T	1	-2.441222	0.076245	-32.02	<.0001
D	1	-1.857960	0.076245	-24.37	<.0001
L	1	-2.473611	0.076055	-32.52	<.0001
y*T	1	-0.049202	0.043077	-1.14	0.2563
y*D	1	0.184050	0.043077	4.27	<.0001

Multivariate Stepwise Regression Using Peril Pure Premium Information

$$Ln(P) = 5.023+0.066y-0.850W-1.858D-2.441T-2.474L+0.184yD-0.049yT$$

Peril	Variables	Ln(P)	Trend
Fire and Lightning	W=T=D=L=0	5.023+0.066y	6.8%
Water	W=1, T=D=L=0	5.023−0.850+0.066y	6.8%
Theft	T=1, W=D=L=0	5.023−2.441+(0.066−0.049)y	1.7%
Other Direct Loss	D=1, W=T=L=0	5.023−1.858+(0.066+0.184)y	28.4%
Liability	L=1, W=T=D=0	5.023−2.474+0.066y	6.8%

Comparison of Regression Results

Data	Points	Fire and Lightning	Water	Theft	Other Property Losses*	Liability	**Combined Univariate**
Univariate	20 pt	6.9%	8.6%	1.7%	28.4%	4.9%	**9.8%**
	16 pt	7.9%	13.4%	6.6%	56.3%	0.7%	**14.1%**
	12 pt	6.6%	25.6%	16.1%	86.7%	-0.7%	**19.3%**
	08 pt	0.6%	60.2%	23.2%	220.2%	-6.9%	**29.5%**
	06 pt	1.6%	73.2%	29.5%	242.4%	-4.2%	**35.5%**
	04 pt	8.5%	21.8%	25.3%	15.4%	13.4%	**14.0%**
Multivariate	20 pt×5	**6.8%**	**6.8%**	**1.7%**	**28.4%**	**6.8%**	**9.8%**
	16 pt×5	**7.2%**	**13.4%**	**7.2%**	**56.3%**	**0.7%**	**14.1%**
	12 pt×5	**0.0%**	**25.6%**	**16.1%**	**86.7%**	**0.0%**	**19.3%**
	08 pt×5	**0.0%**	**60.2%**	**23.2%**	**220.2%**	**0.0%**	**29.5%**
	06 pt×5	**0.0%**	**73.2%**	**29.5%**	**242.4%**	**0.0%**	**35.5%**
	04 pt×5	**12.4%**	**21.8%**	**25.3%**	**12.4%**	**12.4%**	**14.0%**

* Excluding Wind and Hail

Conclusion

- Multivariate tend analysis can lead to questions of interest in:
  - Ratemaking
    - How much weight should be given to
      - perils that have a significant marginal trend?
      - perils that are grouped in combined trend?
    - Has the company taken actions regarding any perils?
  - Product management
    - Is it necessary to update underwriting guidelines?
    - Is a change in product design necessary?

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