



The Global Underwriting Cycle

An Application of Dynamic Factor Analysis

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Overview

- Research Question
- Model Requirements
- Dynamic Factor Analysis (DFA)
- Global Underwriting Cycle
- Risk Modeling in DFA
- Discussion
- Appendix



Research Question

Definition of Underwriting Cycle

- Explain and forecast the underwriting cycle in multiple countries within the limitations of internationally available data
 - The underwriting cycle is represented by the trajectory of the nonlife insurance price level, as measured by the (log) ratio of nonlife gross premium to (nominal) GDP
 - It is assumed that the risk in the economy associated with nonlife insurance is proportional to the dollar value of goods and services that is produced in a given time period (e.g., a calendar year)
 - The proportionality assumption may be compromised during times of economic recessions as some risk may be tied to balances (e.g., the capital stock) rather than flows (e.g., GDP)

Research Question

Structural Drivers

- The primary macroeconomic drivers of the underwriting cycle are considered to be the real interest rate and real GDP growth
 - The real interest rate affects expected investment returns and the discounting of future losses
 - For a discussion of the actuarial implications of changes in the rates of inflation and interest, see Feldblum (2001), who points to the ambiguity of the effect of the interest rate on insurance income
 - Real GDP growth serves as a summary statistic for the state of the economy
- The two predictors are lagged by one year to account for the data lag in pricing decisions

Model Requirements

Dependencies, Common Trends, and Predictors

- There are five major requirements on the model
 - Dimension reduction
 - Uncovering common trends
 - Accommodating predictors (structural drivers)
 - Quantifying dependencies across countries
 - Parsimony
- DFA meets these requirements

Model Requirements

Parsimony and Mean Squared Error

- A high number of predictors may subtract from the forecasting performance
 - For simplicity, assume the textbook case of a (centered) univariate normal linear model with (orthogonal) stochastic predictors (that are independent of the error term)
 - When estimated with ordinary least squares, the forecast error variance is proportional to n/T , where n is the number of predictors and T is the length of the time series
 - When n is large, the MSE (mean squared error) of the forecasts may be higher than when using no predictors at all—see Stock and Watson (2006)

Model Requirements

Parsimony and Mean Squared Error

- Consistent with the econometric argument presented by Stock and Watson (2006), research studies in the field of sociology have found that in statistical models of decision-making the number of predictors should be (very) limited—see Gigerenzer and Brighton (2009) for a survey of this line of research
 - Although limiting the number of covariates may introduce a bias in the prediction error, this adverse effect on the MSE may be more than offset by a drop in variance

Dynamic Factor Analysis

Origins

- The origins of DFA (Dynamic Factor Analysis) date back to a 1977 paper by Sargent and Sims on "unobservable index models"—the paper was titled "Business Cycle Modeling Without Pretending to Have Too Much A Priori Economic Theory"
- Although little may be known about the individual decisions that contribute to aggregate data (such as aggregate measures of the underwriting cycle), DFA is capable of identifying the processes that generate these data
- Sargent and Sims (1977) were able to show that a "small number of factors can account for much of the observed variation of major economic aggregates" (Stock and Watson, 2006)

Sargent and Sims were jointly awarded the 2011 Nobel Prize in economics



Dynamic Factor Analysis

Model Properties

- The dynamic, time-varying factors are manifestations of latent processes—these hidden processes (HP) may be thought of as summary statistics of structural influences that are not readily identifiable and, potentially, common to all time series
- In addition to hidden processes, identifiable structural influences (e.g., a set of economic predictors, possibly reduced to their principal components) may contribute to the variation of the dependent variable
- Stock and Watson (2012) show that DFA tends to outperform shrinkage methods when it comes to forecasting
- For a survey on DFA, see Stock and Watson (2011)

Dynamic Factor Analysis

Model Structure

- The model is written in state-space notation—the design follows Holmes, Ward, and Wills (2012); see also Zuur et al. (2003) and Drukker and Gates (2011)
- The transition equation describes the hidden processes:

$$\mathbf{x}_t \sim MVN(\mathbf{B} * \mathbf{x}_{t-1}, \mathbf{Q}), \mathbf{B} \text{ diagonal}, \mathbf{Q} \text{ diagonal}$$

- The hidden processes are defined as being independent of each other
- The measurement equation defines the likelihood

$$\mathbf{y}_t \sim MVN(\mathbf{Z} * \mathbf{x}_t + \mathbf{D} * \mathbf{d}_t, \mathbf{R}), \mathbf{R} \text{ diagonal}$$

- It is assumed that all cross-equation correlation is captured by the hidden processes
- The matrices \mathbf{Z} and \mathbf{D} indicate the loadings on the hidden processes and the regression coefficients of the predictors, respectively

Dynamic Factor Analysis

Model Structure

- Any hidden process may be specified as an AR(1) process, an AR(2) process, or a random walk—the AR processes were modeled following Johnson and Hoeting (2003)
 - The standard DFA implementation defines the hidden processes as (independent) random walks—see Holmes, Ward, and Wills (2012); Drukker and Gates (2011); Zuur, Tuck, and Bailey (2003); Zuur et al.(2003)
- For the purpose of model identification, time series n does not load on hidden process k , where k is greater than n

Dynamic Factor Analysis

Model Structure

- DFA is implemented as a Bayesian model and estimated by means of MCMC (Markov Chain Monte Carlo simulation)
 - MCMC is fast, flexible, and can handle arbitrarily large systems
- The model can handle missing values in the dependent variable—these values are estimated like any other parameter in the model
- All dependent and explanatory variables are standardized (i.e., each series adds up to zero and the sum of squares equals the number of observations)
- Substituting t distributions (with endogenous degrees of freedom) for the normal distributions for the innovations in the hidden processes (or the likelihood) is straightforward*

*A sensitivity analysis with Student's t distribution for the innovations in the hidden processes shows no material difference in the regression coefficients and the loadings

Dynamic Factor Analysis

DIC Criterion for Model Selection

- Model selection (i.e., selection of the number of hidden processes) is based on the DIC
 - The DIC can be viewed as a generalization of Akaike's Information Criterion (AIC)
- The DIC is an approximation of the expected predictive deviance, which is a measure of out-of-sample predictive power
- The DIC can be computed as the sum of the posterior mean deviance and the effective number of parameters
- Differences in the DIC between 5 and 10 are considered substantial

Dynamic Factor Analysis

DIC Criterion for Model Selection

- The deviance equals twice the negative log likelihood
- The effective number of parameters (pD) can be calculated as the difference between the posterior mean deviance and the deviance at the posterior means of the parameters
 - It takes a very high number of draws from the posterior distribution to arrive at a stable measure for pD
- An alternative concept, pV , calculates the effective number of parameters as half the variance of the deviance—see Gelman et al. (2004)
 - pV is invariant to parameterization, robust, and trivial to calculate—for a discussion, see <http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/dicpage.shtml#q9>

Data Set

Data

- Nonlife premium (gross) is available for the 34 member countries of the OECD
 - Some time variation in the data may be due to changes in reporting (improved data quality or coverage)
 - To the degree that countries share such changes, this time variation may contribute to a hidden process
 - In the statistical model, the (log) ratio of nonlife premium to (nominal) GDP is centered—hence, cross-country differences in the coverage ratio do not affect the regression results

Data Set

Data

- Nominal and real GDP data are from the IMF Global Economic Outlook April 2013, 1980–2018 (past 2011: forecasts)
- The rate of interest (as it manifests itself in the bank lending rate) is from the OECD StatExtracts, 1960–2012
 - For some countries, interest rate data is sparse—only 11 countries have a full data set for 1981–2012
 - No interest rate data is available for the OECD member country Turkey
- For details on the insurance and macroeconomic data sources, see the appendix

Interest Rates

Model Description

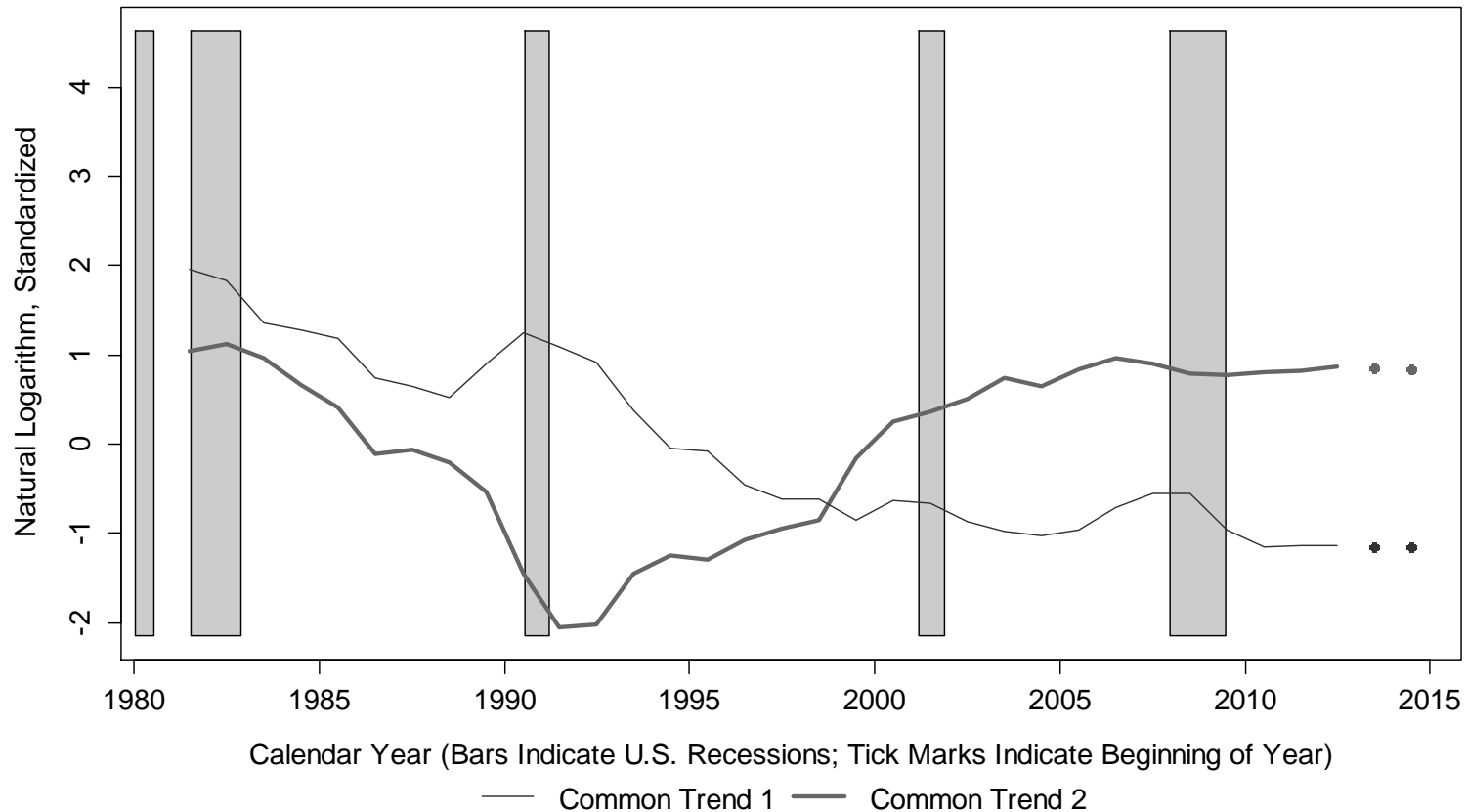
- DFA is performed on interest rates, 34 OECD countries with the exception of Turkey, 1981–2011, two hidden processes
 - DIC: 1,483 (1 hidden process); 1,204 (2); 1,015 (3; not identified*)
 - Forecasts are generated for the time period 2012–2014
 - No covariates are employed
 - All hidden processes are specified as random walks
 - The precisions of the normal likelihood are credibility-adjusted**

* For three hidden processes, the model is not identified. See the appendix on identification.

** See Schmid (2012)

Interest Rates

Hidden Processes

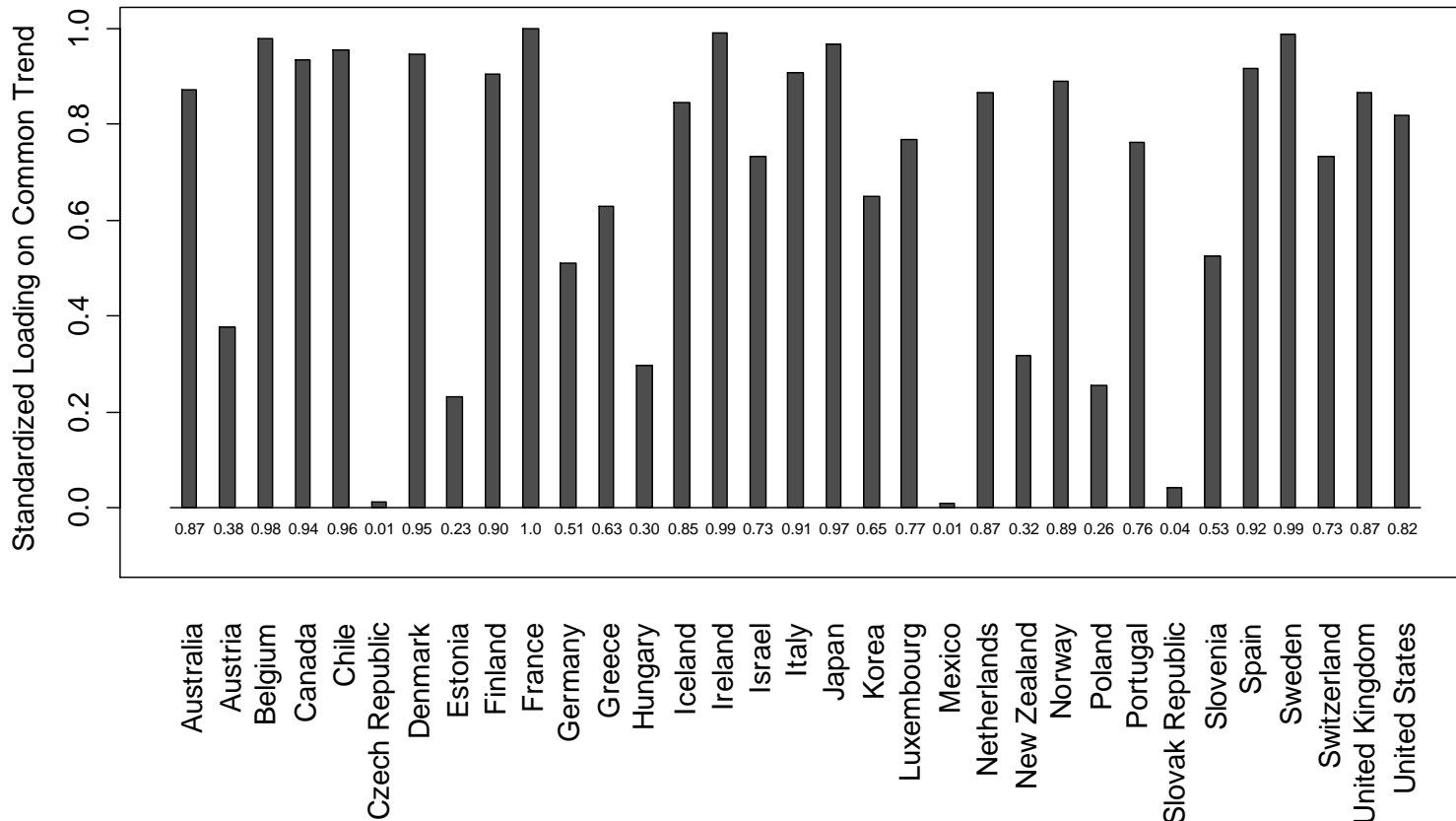


Both hidden processes are standardized for the purpose of illustration



Interest Rates

Loadings, First Hidden Process

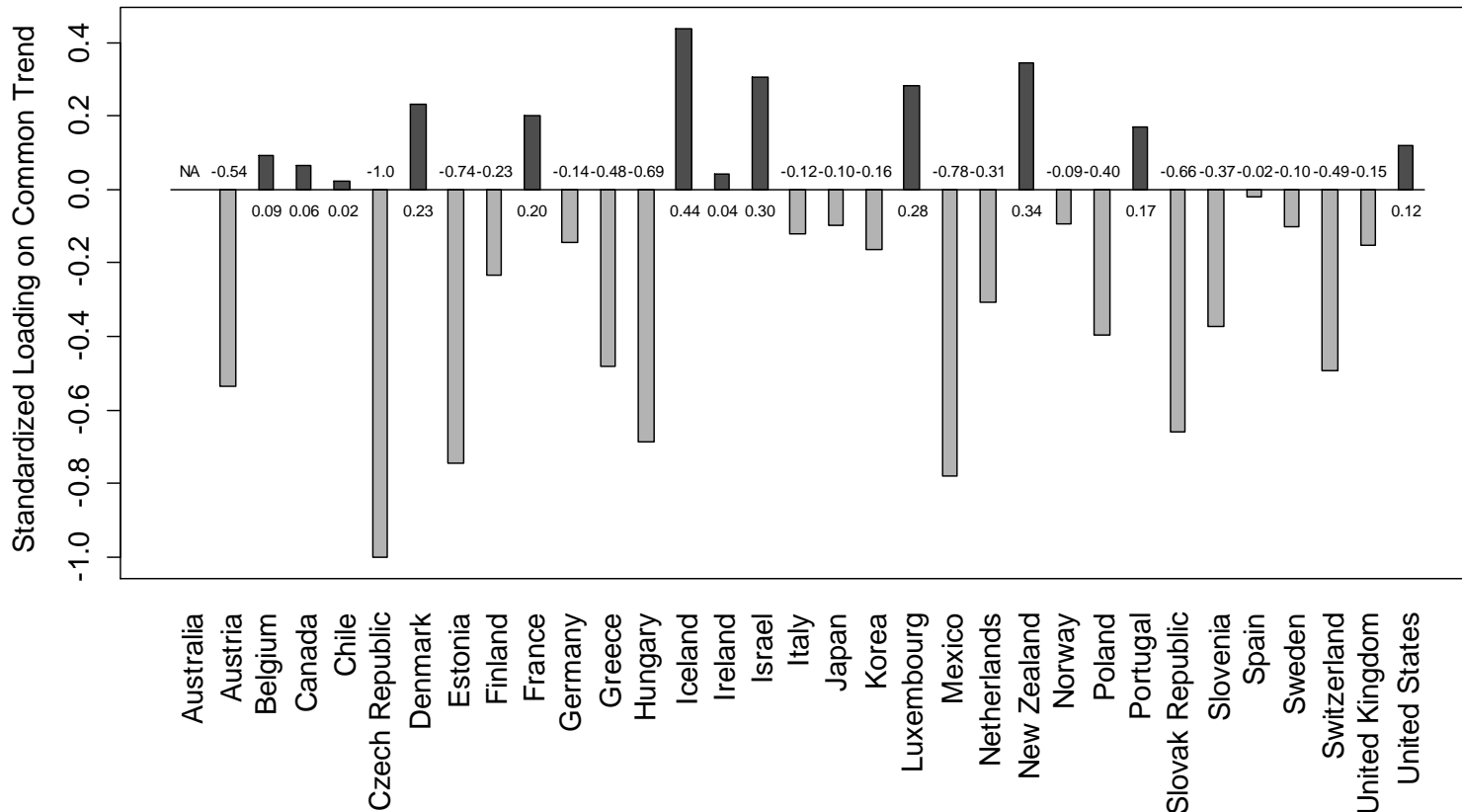


The loadings are based on standardized dependent variables and scaled to a maximum absolute value of unity. Loadings should not be interpreted as causal relations between country (cause) and hidden process (effect); rather, countries with high standardized loadings (in absolute value terms) should be considered as highly susceptible to the economic forces (or the inverse thereof) behind the respective hidden process



Interest Rates

Loadings, Second Hidden Process

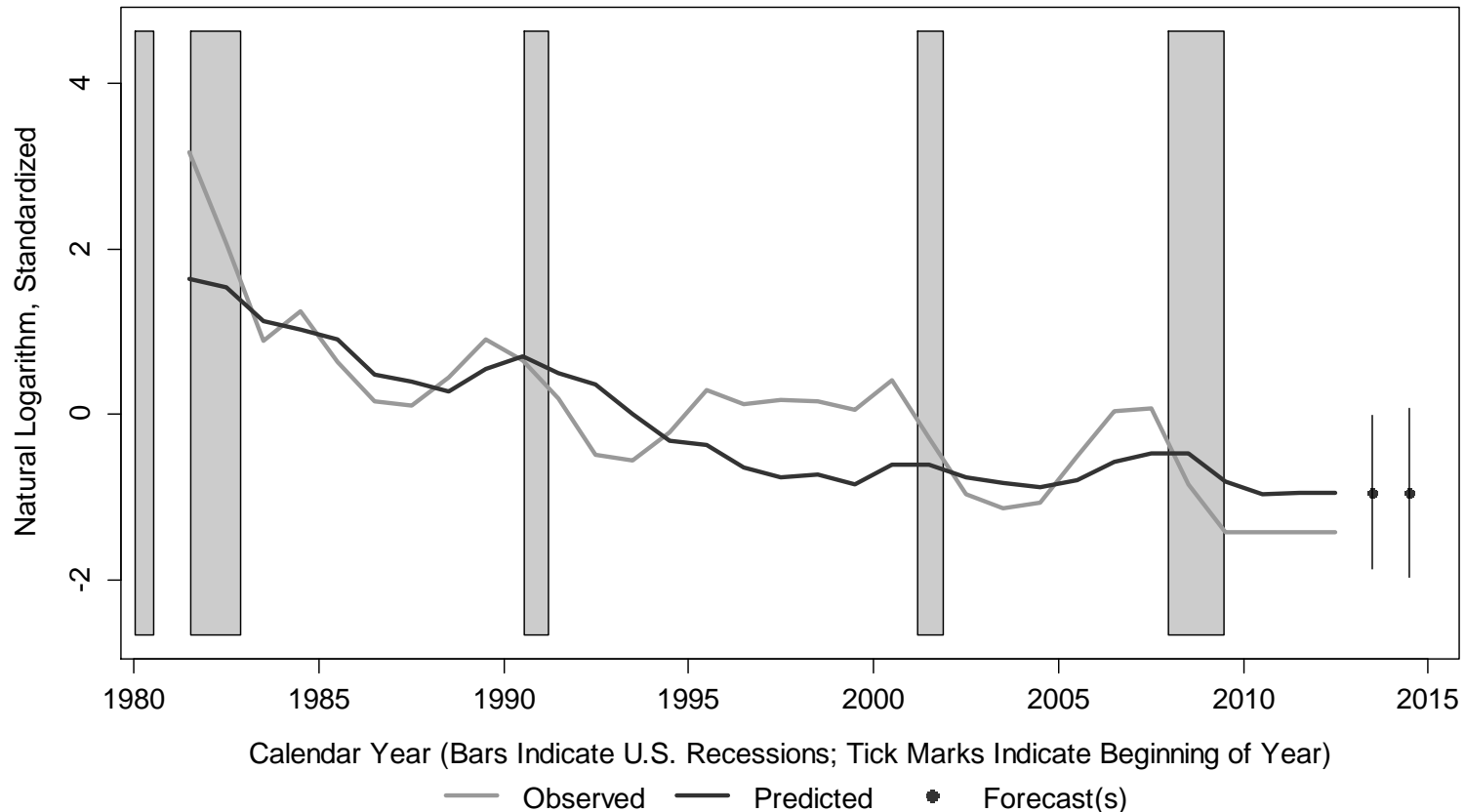


The loadings are based on standardized dependent variables and scaled to a maximum absolute value of unity. Loadings should not be interpreted as causal relations between country (cause) and hidden process (effect); rather, countries with high standardized loadings (in absolute value terms) should be considered as highly susceptible to the economic forces (or the inverse thereof) behind the respective hidden process



Interest Rates

United States

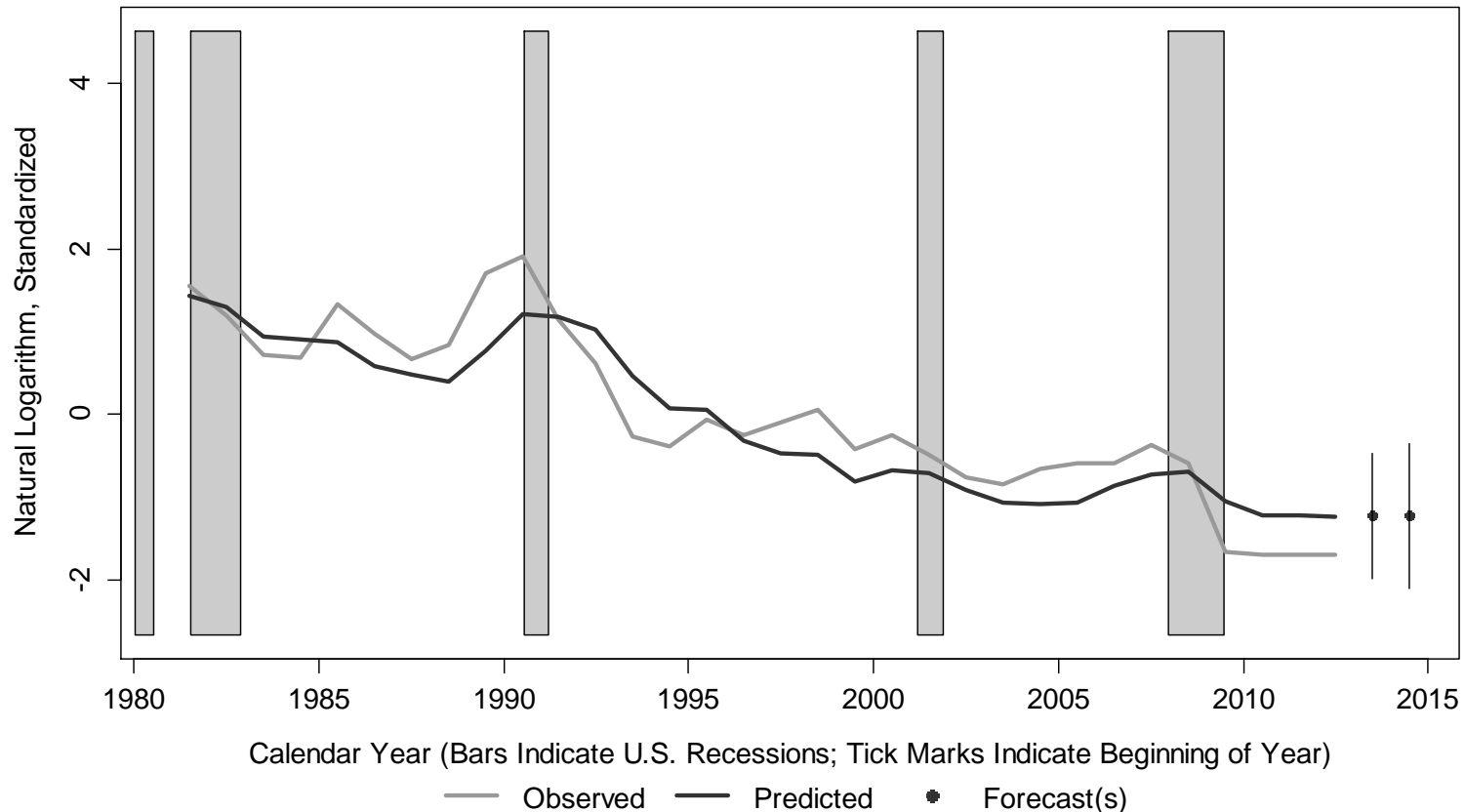


The model does not entirely replicate the sharp drop in interest rates resulting from the Quantitative Easing policy action of the Federal Reserve, which was implemented in response to the 2007-2008 financial crisis. The vertical bars surrounding the forecasts indicate 80 percent credible intervals



Interest Rates

United Kingdom

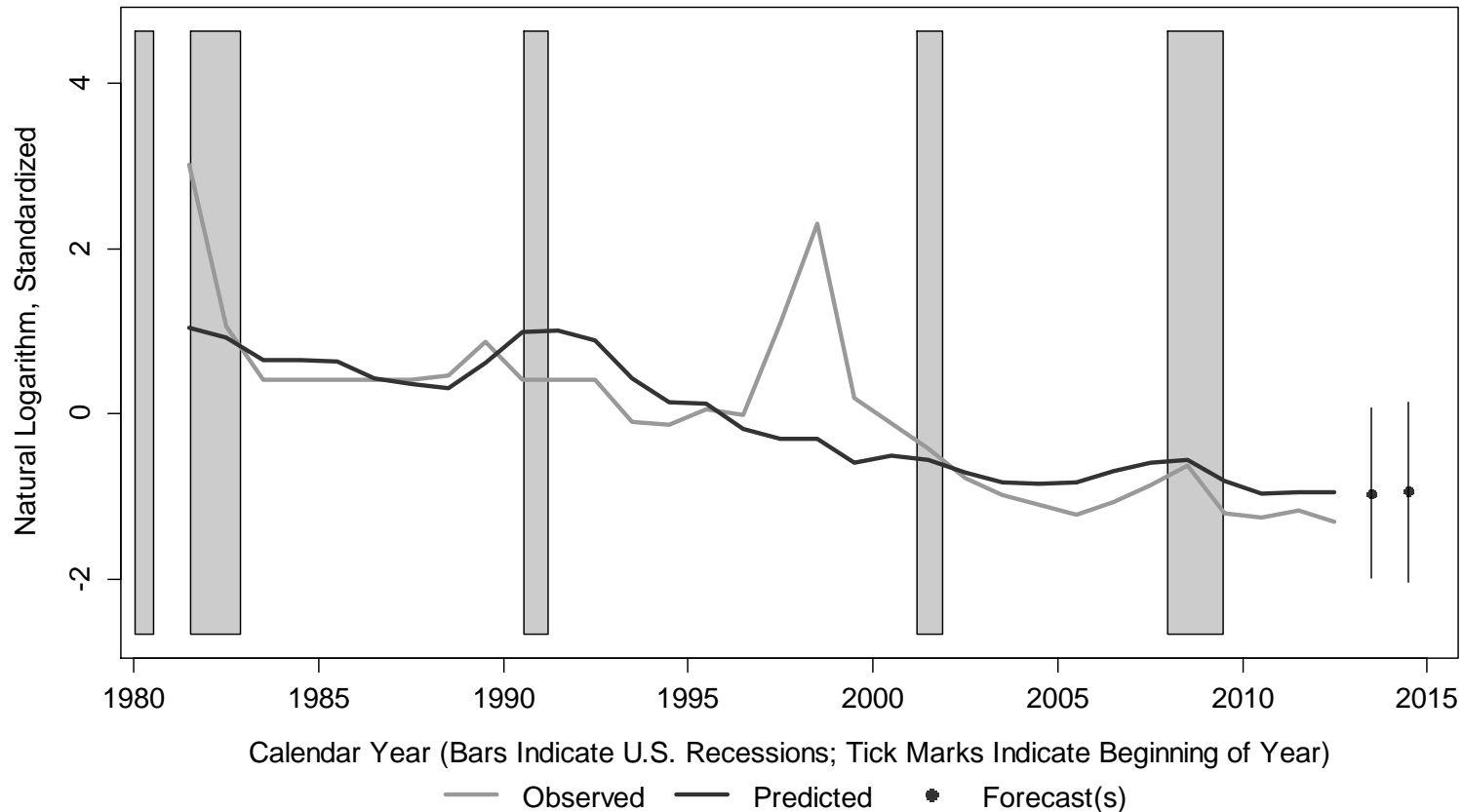


The model does not entirely replicate the sharp drop in interest rates resulting from the Quantitative Easing policy action of the Bank of England, which was implemented in response to the 1987-1988 financial crisis. The vertical bars surrounding the forecasts indicate 80 percent credible intervals



Interest Rates

South Korea

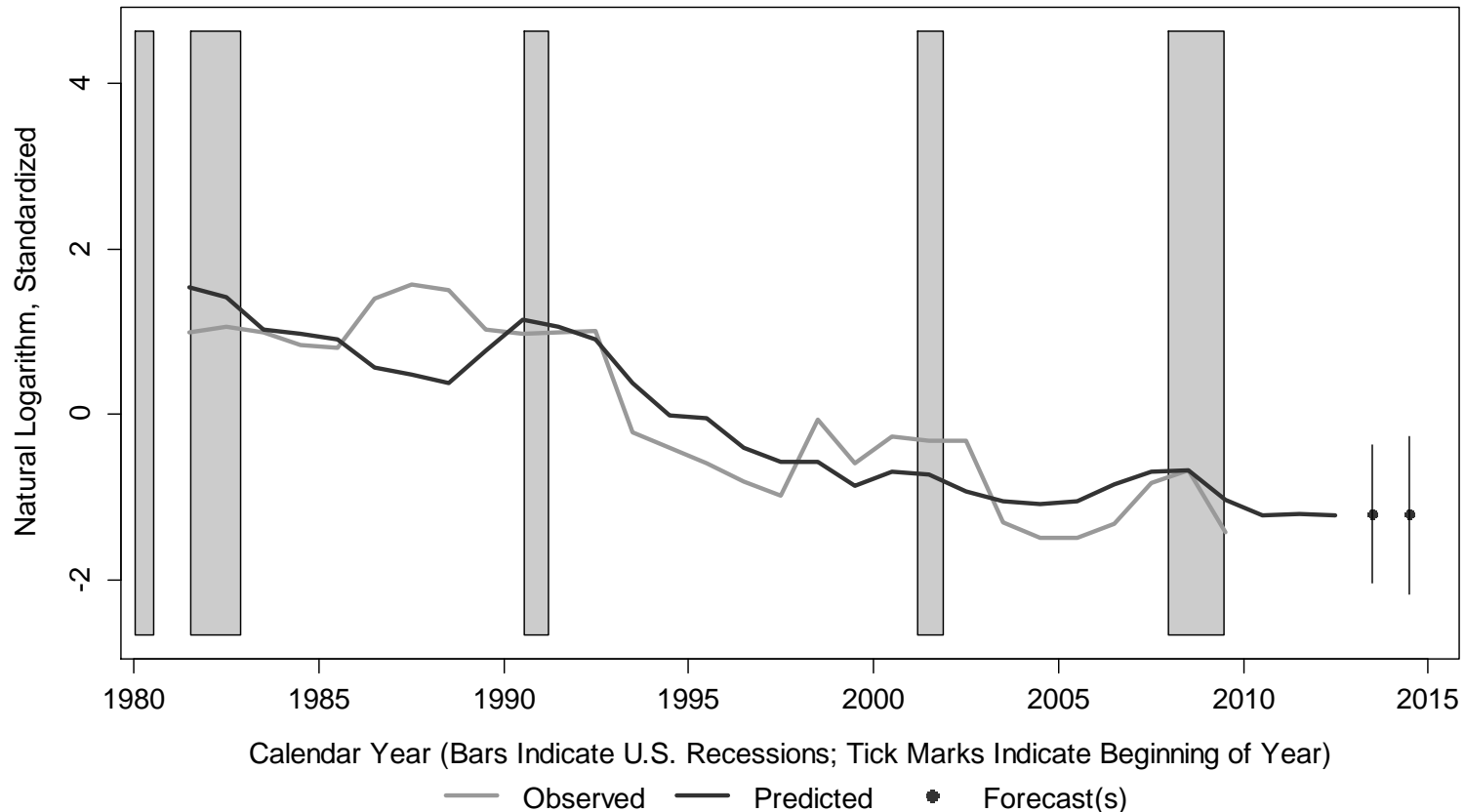


Interest rates in South Korea have not been subject to Quantitative Easing policy actions comparable to what was implemented in the USA, the UK, and the Euro zone following the 2007-08 financial crisis. The vertical bars surrounding the forecasts indicate 80 percent credible intervals



Interest Rates

Norway



Norway is not a member of the Euro zone and has not pursued a policy of Quantitative Easing similar to what was implemented in the United States, the UK, and the Euro zone following the 2007-08 financial crisis. The vertical bars surrounding the forecasts indicate 80 percent credible intervals

Global Underwriting Cycle

Model Description

- DFA is performed on the (log) ratio of nonlife (gross) premium to nominal GDP, 29 OECD countries*, 1983–2011, one hidden process
 - DIC: 2,074 (1 hidden process); 1,951 (2); 2,039 (3)
 - One-year lags of the real interest rate and the rate of real GDP growth are used as covariates
 - The real interest rate is defined as the differences between the predicted values of the nominal interest rate and the rate of change of the GDP deflator
 - Forecasts are generated for the time period 2012–2014
 - All hidden processes are specified as random walks
 - The precisions of the normal likelihood are credibility-adjusted**

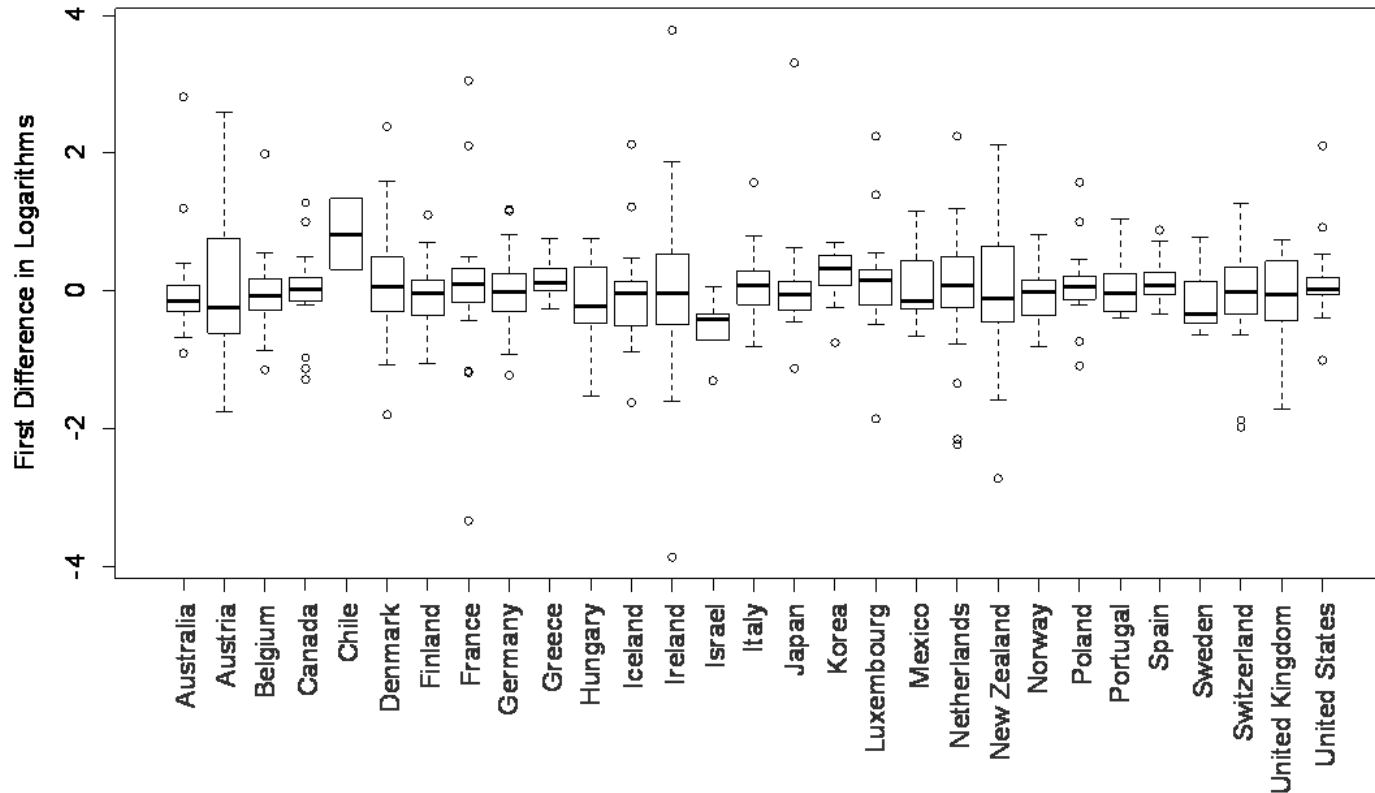
* GDP deflator information is not available for the entire 1982-2013 time window for the Czech Republic, Slovenia, and the Slovak Republic

** See Schmid (2012)



Global Underwriting Cycle

Boxplots of First Difference of Dependent Variable

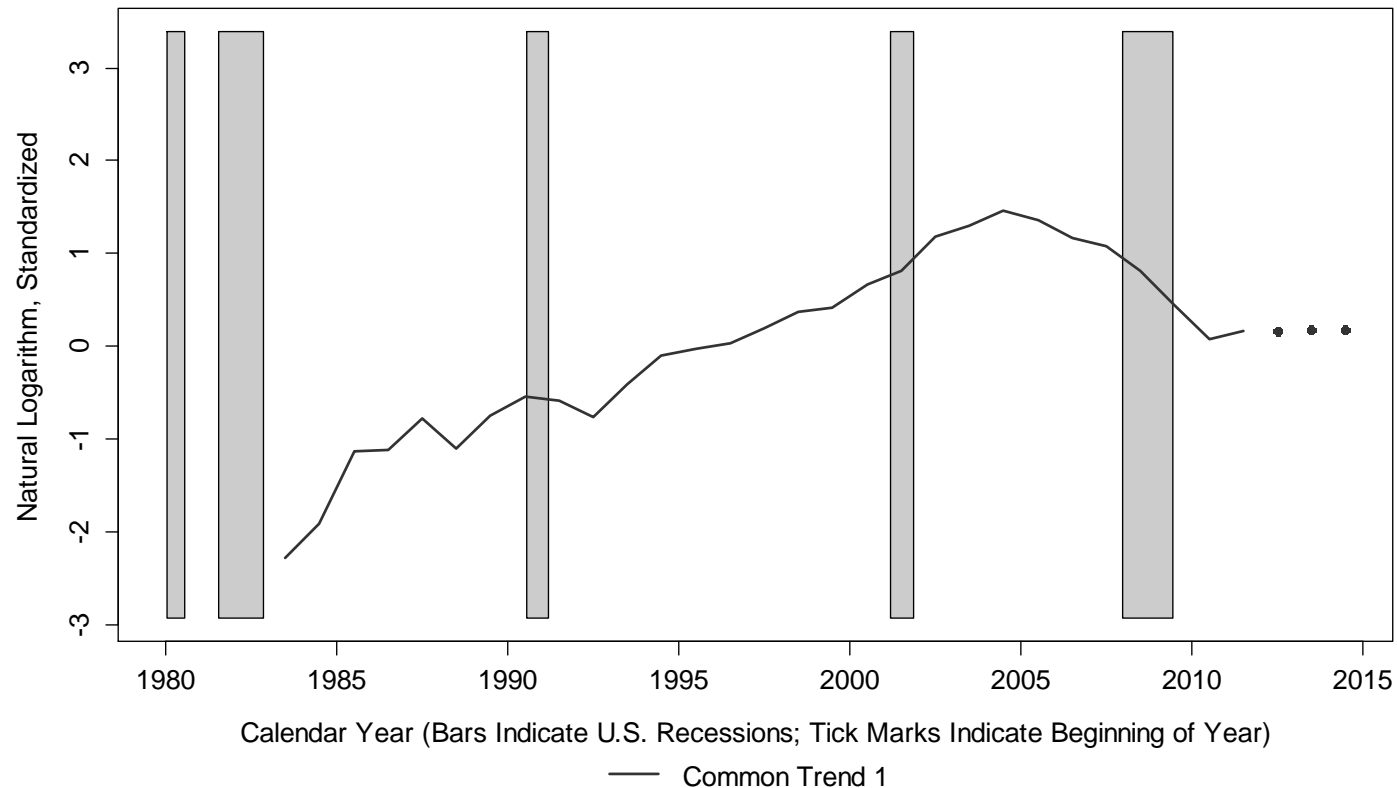


The log ratio of nonlife premium to nominal GDP shows little skew (with the exception of Chile)



Global Underwriting Cycle

Hidden Process

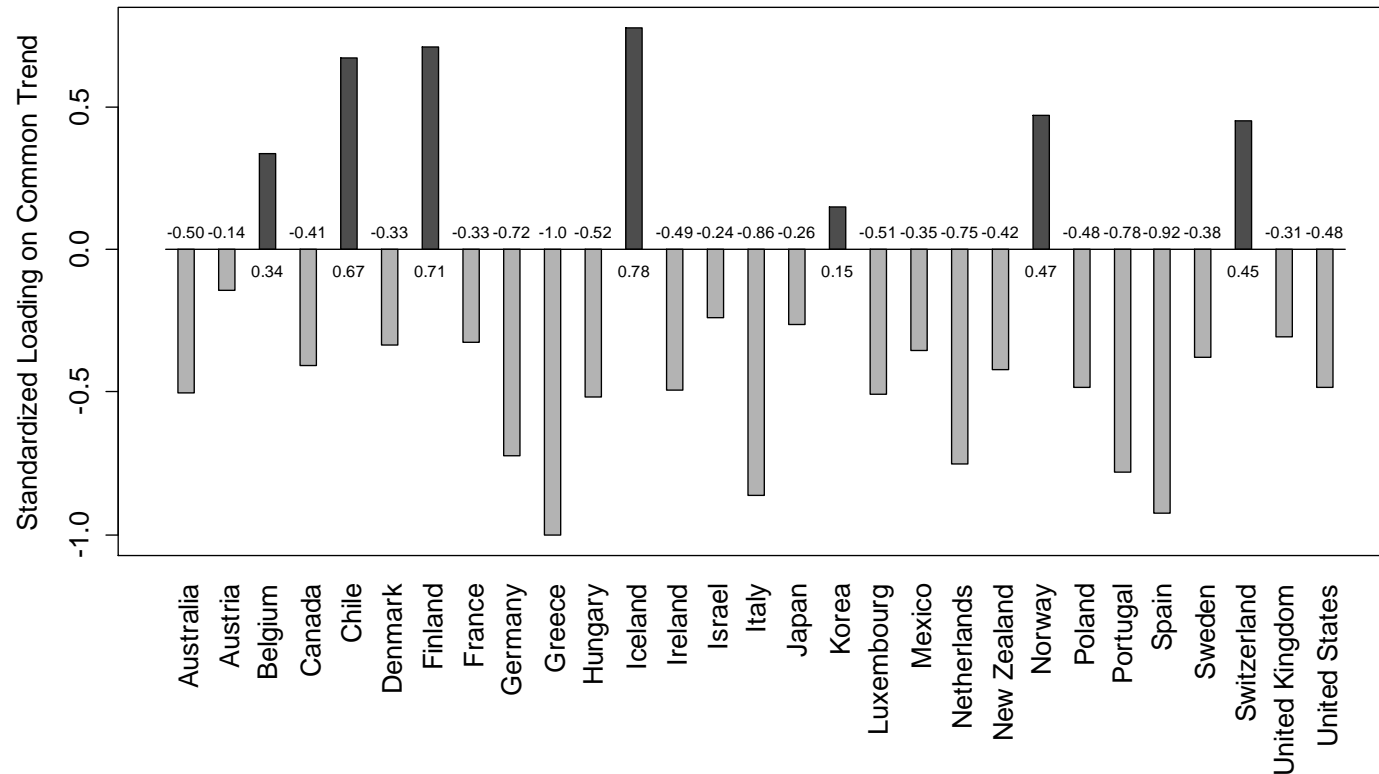


The hidden process is standardized for the purpose of illustration



Global Underwriting Cycle

Loadings, Hidden Process

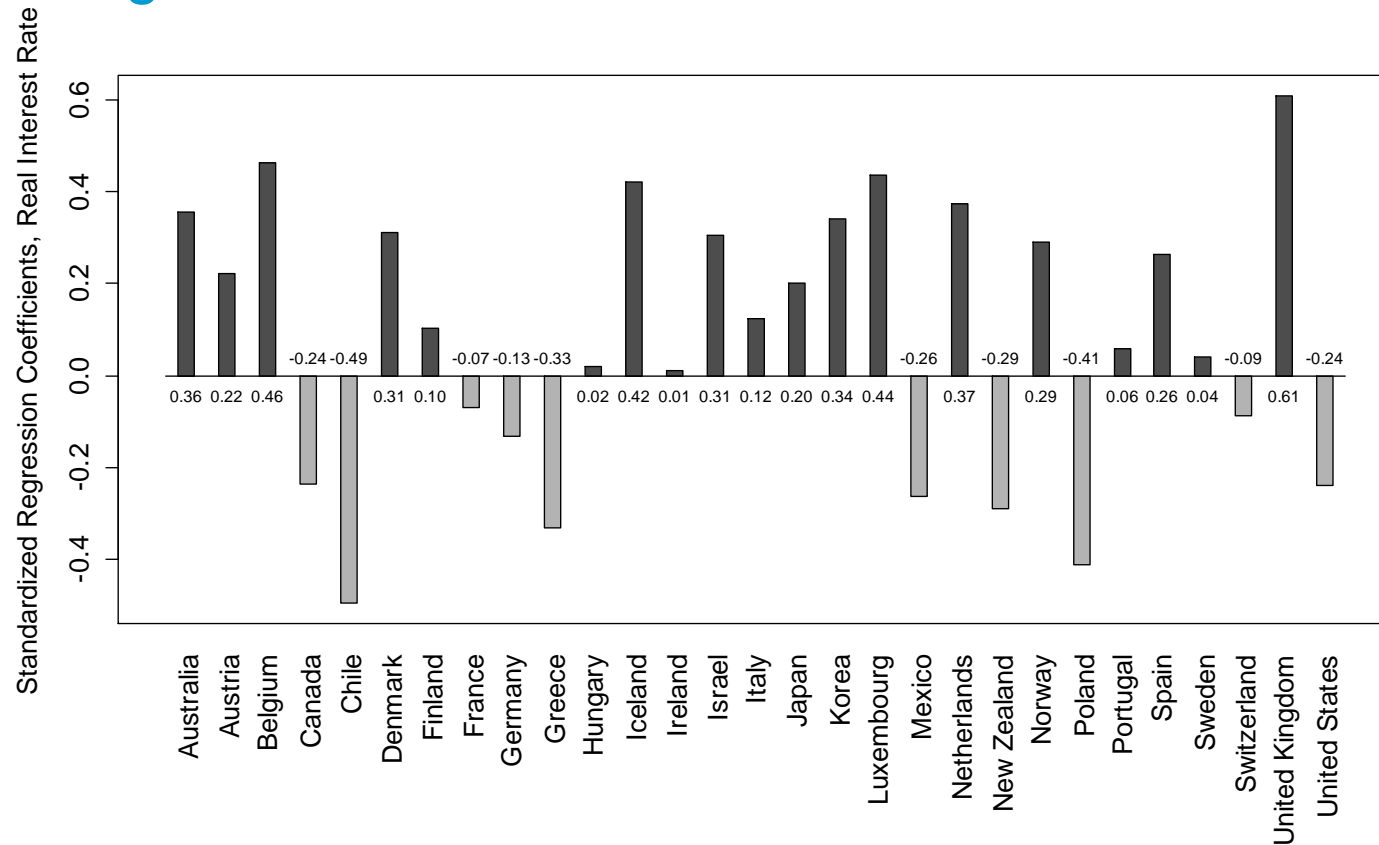


The loadings are based on standardized dependent variables and scaled to a maximum absolute value of unity. Loadings should not be interpreted as causal relations between country (cause) and hidden process (effect); rather, countries with high standardized loadings (in absolute value terms) should be considered as highly susceptible to the economic forces (or the inverse thereof) behind the respective hidden process



Global Underwriting Cycle

Regression Coefficients, Real Interest Rate

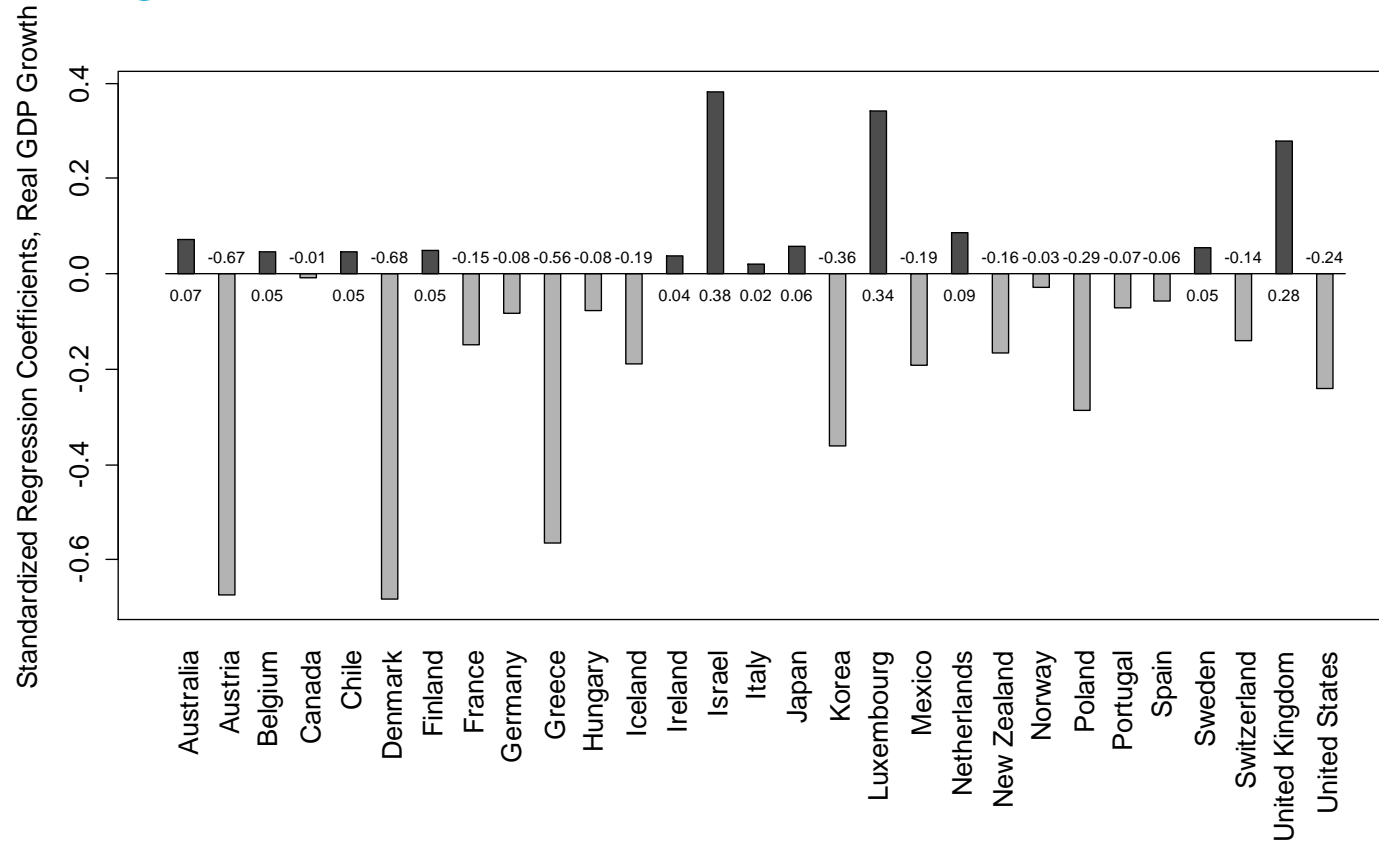


The displayed regression coefficients report the effect in terms of standard deviations on the log ratio of nonlife premium to nominal GDP in response to a one-standard deviation change in the real interest rate



Global Underwriting Cycle

Regression Coefficients, Real GDP Growth

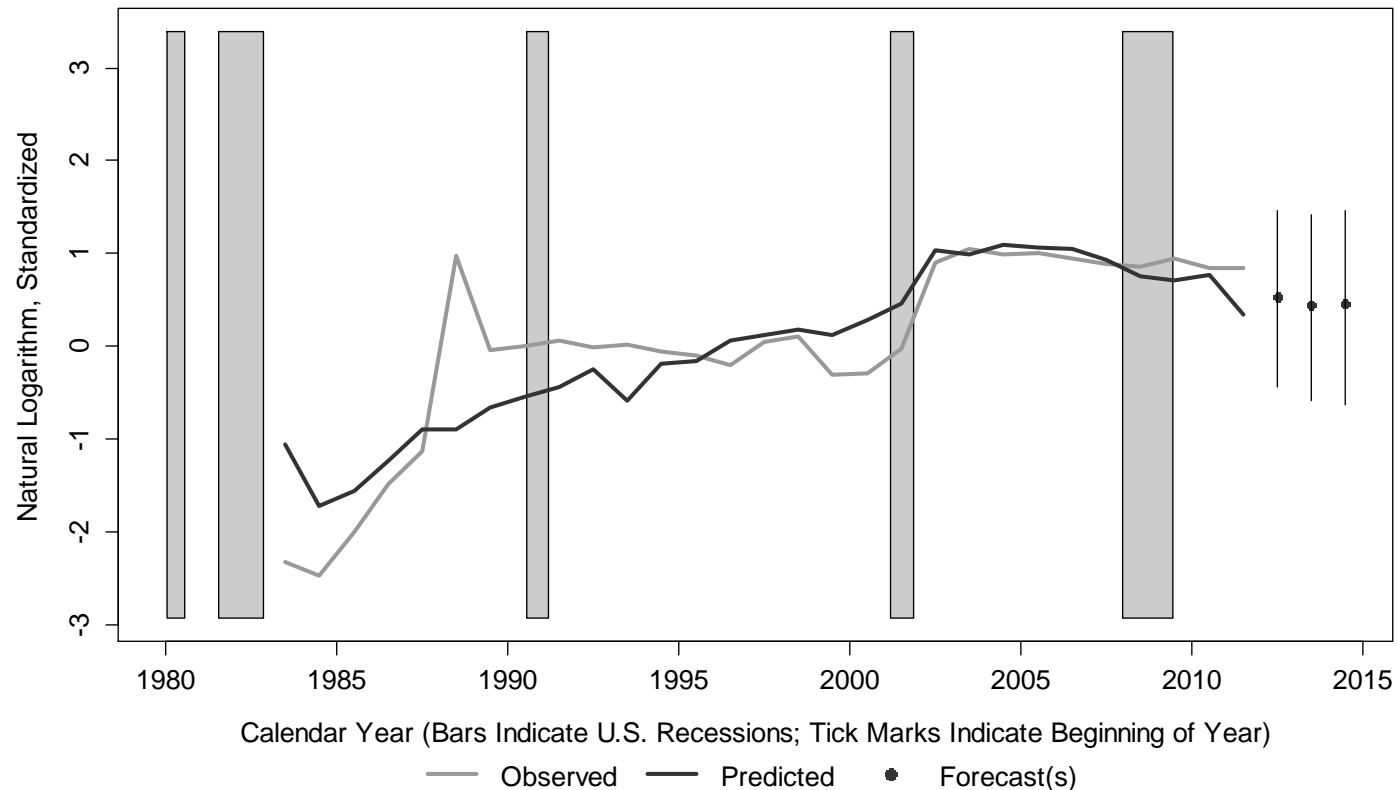


The displayed regression coefficients report the effect in terms of standard deviations on the log ratio of nonlife premium to nominal GDP in response to a one-standard deviation change in the rate of real GDP growth



Global Underwriting Cycle

United States

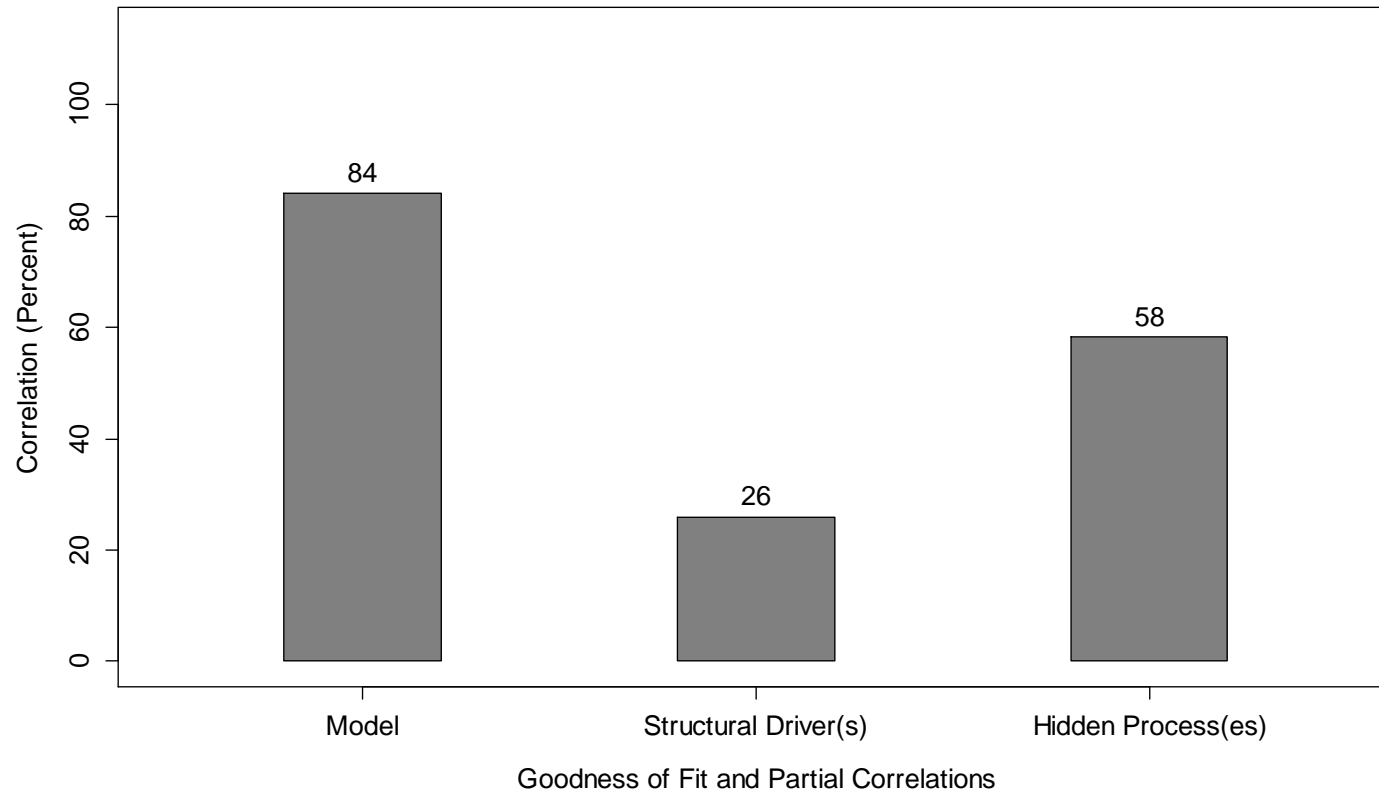


The displayed 80 percent credible forecast intervals do not account for the uncertainty that surrounds the estimates of the interest rate—such uncertainty can be accounted for by means of resampling from the posteriors of the interest rate estimates



Global Underwriting Cycle

United States

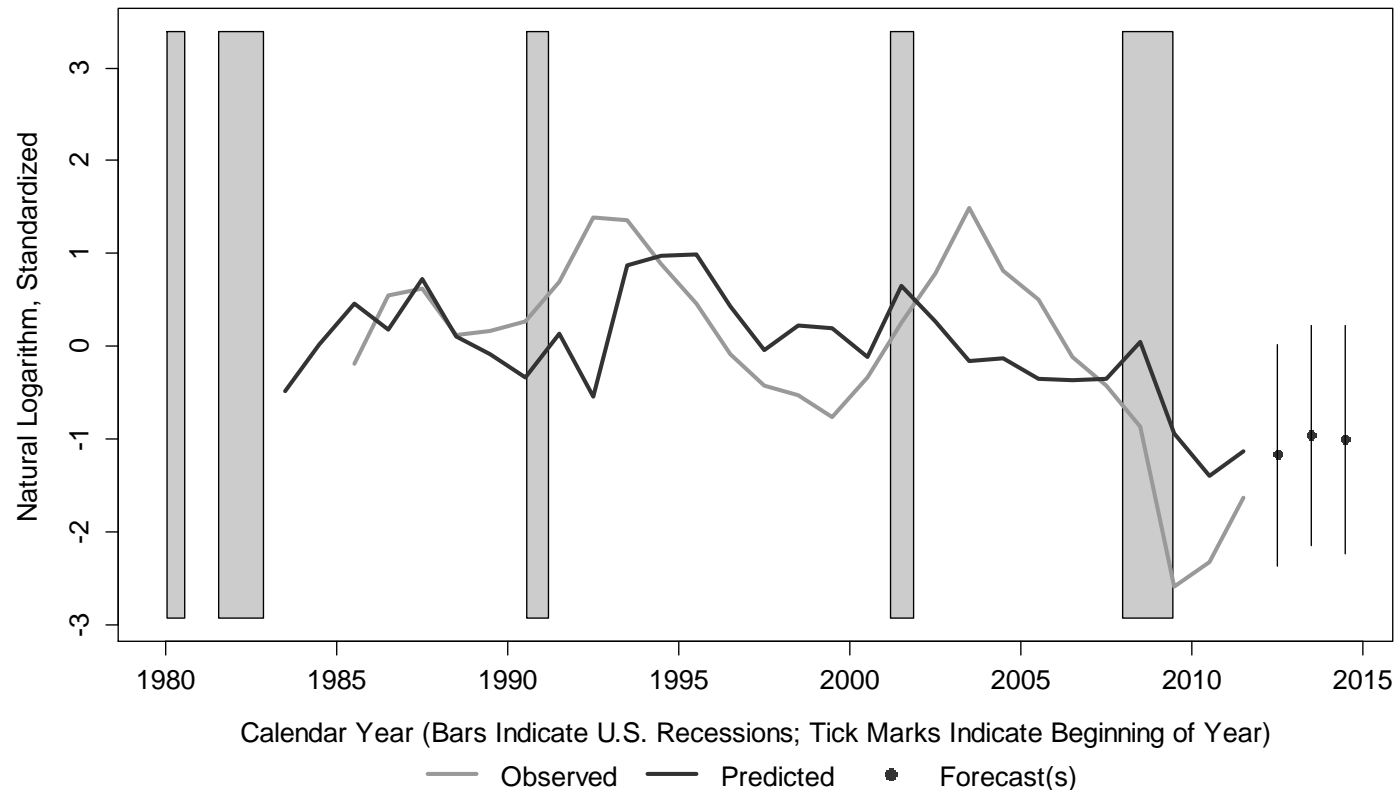


The goodness of fit ("Model") is expressed as the correlation coefficient between observed and predicted values of the dependent variable. This measure corresponds to the square root of the Pseudo-R²



Global Underwriting Cycle

United Kingdom

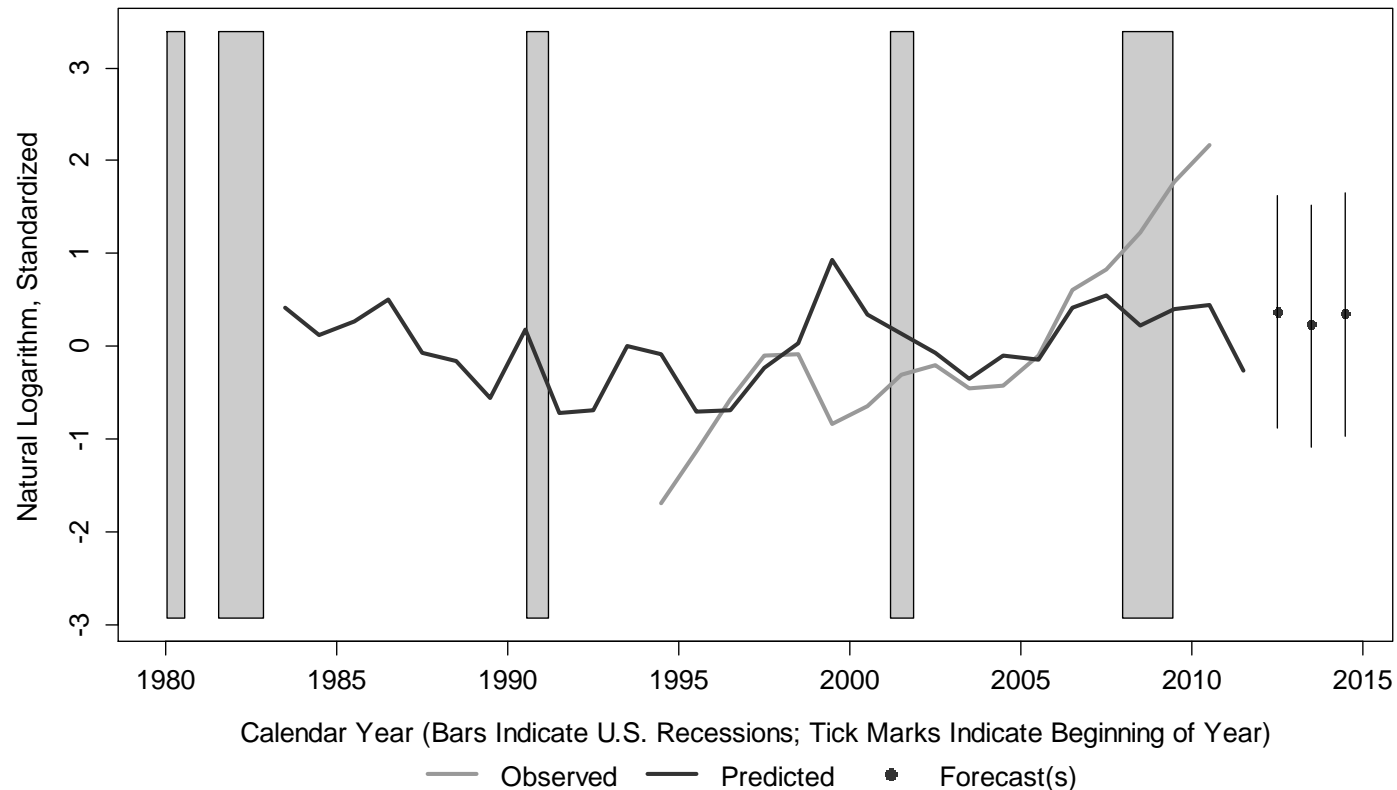


The displayed 80 percent credible forecast intervals do not account for the uncertainty that surrounds the estimates of the interest rate—such uncertainty can be accounted for by means of resampling from the posteriors of the interest rate estimates



Global Underwriting Cycle

South Korea

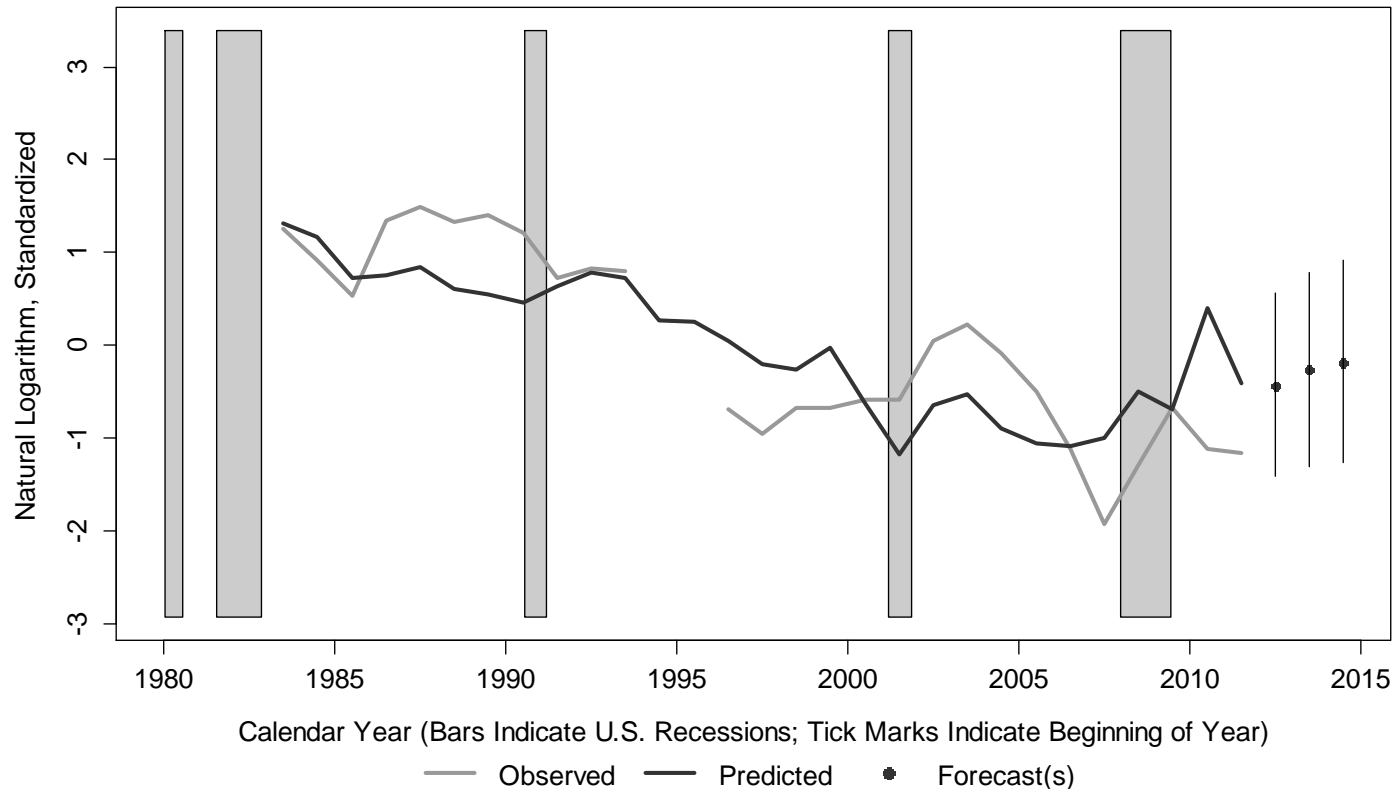


The displayed 80 percent credible forecast intervals do not account for the uncertainty that surrounds the estimates of the interest rate—such uncertainty can be accounted for by means of resampling from the posteriors of the interest rate estimates



Global Underwriting Cycle

Norway



The displayed 80 percent credible forecast intervals do not account for the uncertainty that surrounds the estimates of the interest rate—such uncertainty can be accounted for by means of resampling from the posteriors of the interest rate estimates



Global Underwriting Cycle

Conclusion

- In the United States, the influence of the real interest rate and real GDP growth on the nonlife premium level is equally powerful* and negative
 - By way of contrast, for the United Kingdom, the influences of the two covariates are positive
 - In regard to cross-country differences in the influence of the real interest, one possible explanation is the mentioned ambiguity of the influence of the interest rate on insurance income
 - Another possible explanation is cross-country differences in the financial system and the market structure (e.g., the market share of foreign insurers)

*As calculated by the ratio of the standardized regression coefficients



Global Underwriting Cycle

Conclusion

- There is only one hidden process in the global underwriting cycle
 - The purpose of the hidden process is dimension reduction in a situation where the explanatory variables cannot be itemized or measured, or are large in number
 - Possibly, the hidden process is driven by systematic changes of data coverage
- A necessary (yet not sufficient condition) for the global underwriting cycle being driven by U.S. macroeconomic financial conditions is that the United States does not load on the hidden process—this is not the case

Global Underwriting Cycle

Conclusion

- There are two hidden processes in the global cycle of (nominal) interest rates
 - With the exception of a Eastern European countries (including Austria, the economy of which is tied to those of Eastern European countries) and Mexico, all OECD countries load on the first hidden process to about the same degree
 - It is primarily the Eastern European countries (and Mexico) that load on the second hidden process
- In conclusion, interest rates show a very high degree of dependence across OECD countries, where one cycle governs Eastern Europe (and Mexico) and another one drives the remaining OECD nations

Discussion



Forecasting Performance

Real-Time Data Sets

- An evaluation of the forecasting performance (using a holdout period) requires real-time data sets
 - Real GDP, nominal GDP (and, hence, the GDP deflator) are subject to revisions
 - Assuming that nonlife premium and the (nominal) lending rate are not subject to revisions, vintage data sets can be created using real GDP and nominal GDP series from historical IMF World Economic Outlook data sets

Shrinkage

LASSO for Regression Coefficients

- Andrew Gelman: "I see routine regression analysis all the time that does no regularization and as a result suffers from the usual problem of noisy estimates and dramatic overestimates of the magnitudes of effect."
<http://andrewgelman.com/2013/03/18/tibshirani-announces-new-research-result-a-significance-test-for-the-lasso/>
- LASSO may be performed on the regression coefficients across equations, within groups of regression coefficients (e.g., real interest rate, real GDP growth) across equations, or on groups of regression coefficients (Grouped LASSO) themselves

Appendix



Appendix

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Appendix

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Appendix

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Appendix

Data Sources

- Nonlife gross premium, 1983–1991, stated in U.S. dollars, OECD, <http://stats.oecd.org>
- Nominal GDP (stated in U.S. dollars) and GDP deflator, 1980–2018 (estimates start after 2011), IMF World Economic Outlook Database April 2013, <http://www.imf.org/external/pubs/ft/weo/2013/01/weodata/index.aspx>
- Lending rate, 1960–2012, World Bank, <http://data.worldbank.org/indicator/FR.INR.RINR>

Appendix

Note on Model Identification (1/2)

- As the number of hidden processes increases, the model may become unidentified—in this case, more than one structural form of the model is compatible with the reduced form of the model
- A major source of nonidentification is overparameterization, which results in high DIC values
- Nonidentifiability causes Markov chains to "flip"
 - For some countries, the signs of the loadings (for a given absolute value) switch change during the iteration process—see following slide

Appendix

Note on Model Identification (2/2)

