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USING CREDIBILITY TO MITIGATE THE "WINNER'S CURSE"

September 2013

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Agenda

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1. The “Winner’s Curse” – an elephant in the room
2. Basic Credibility Concepts
3. Multivariate Credibility
4. The XOL Reinsurance Problem
5. The Recursive Form for XOL Pricing

The Winner's Curse – An Elephant

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The Winner's Curse:

- Assume pricing is done via an auction (multiple bids)
- Assume there is a random element in the pricing process for each of the bidders

The portfolio written is not a random sample of risks.

Instead, the portfolio is the set of winning bids.

The Winner's Curse – An Elephant Simulation Example (Excel)

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		Example of Winner's Curse			
		Company A	Company B	Company C	"Winner"
"Correct" Premium":		1,000,000	1,000,000	1,000,000	1,000,000
C.V. of Pricing:		0.100	0.100	0.100	
# Quoted:		1,000	1,000	1,000	
Average Premium:		999,425	998,476	999,421	999,108
# Bound:		335	345	320	1,000
Average Premium:		**924,096**	**917,561**	**916,579**	**919,436**
Impact of Curse:		-7.5%	-8.1%	-8.3%	-8.0%
Scenario:	1	980,191	966,664	999,985	966,664
	2	806,302	899,607	1,002,422	806,302
	3	996,435	1,056,072	930,714	930,714
	4	1,083,107	973,006	776,311	776,311
	5	1,070,169	1,271,882	1,017,768	1,017,768
	6	1,140,818	975,723	881,194	881,194
	7	1,068,290	944,245	883,500	883,500
	8	943,281	958,088	1,017,825	943,281
	9	995,775	846,034	954,583	846,034
	10	1,043,841	1,228,726	1,144,771	1,043,841
	11	1,171,744	913,867	1,014,210	913,867

The Winner's Curse – An Elephant Simulation Example (Excel)

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		Example of Winner's Curse			
		Company A	Company B	Company C	"Winner"
"Correct" Premium":		1,000,000	1,000,000	1,000,000	1,000,000
C.V. of Pricing:		0.050	0.100	0.100	
# Quoted:		1,000	1,000	1,000	
Average Premium:		1,000,115	1,000,025	1,001,051	1,000,397
# Bound:		278	354	368	1,000
Average Premium:		**968,793**	**913,176**	**914,234**	**929,027**
Impact of Curse:		-3.1%	-8.7%	-8.7%	-7.1%
Scenario:	1	1,030,850	932,531	979,909	932,531
	2	1,066,441	1,092,306	982,305	982,305
	3	981,758	966,095	1,028,692	966,095
	4	934,596	991,287	981,753	934,596
	5	1,097,762	916,302	885,274	885,274
	6	1,008,027	998,314	860,096	860,096
	7	966,520	998,013	1,009,339	966,520
	8	970,755	974,602	1,066,335	970,755
	9	965,096	897,087	1,170,621	897,087
	10	961,391	942,408	1,102,551	942,408
	11	1,068,667	879,760	1,237,398	879,760

The Winner's Curse – An Elephant

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The Winner's Curse leads to downward bias in pricing.

The downward bias is due to variance in the pricing estimation process. The greater the variance, the more the downward bias.

We can mitigate this downward bias by using minimum variance estimators.

Credibility can help!

Basic Credibility Concepts - Preliminaries

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Criteria for an estimator of future losses:

- **Unbiased** = the expected value of the estimator is equal to the "true" expected loss

$$E(\hat{\mu}) = \mu$$

- **Minimum Variance** = on average the value produced by this estimator will be closer to the true expected loss than other estimates
- **Robust** = the estimator behaves well even if model assumptions are not exactly met; stable results even given outliers

Basic Credibility Concepts – Venter's *Credibility for Dummies*

Credibility theory that focuses on the goal of minimum variance is also known as "least squares" or "greatest accuracy" credibility.

The goal is simple to state: **We want to make use of all the available and relevant information, giving the proper weight to each piece of information.**

"Credibility theory is all about weighted averages."

-Gary Venter

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A credibility-weighted (cw) average of two estimators is given as a linear weighted average:

$$\widehat{\mu_{cw}} = w \cdot \widehat{\mu_1} + (1-w) \cdot \widehat{\mu_2}$$

The two estimators are unbiased and independent:

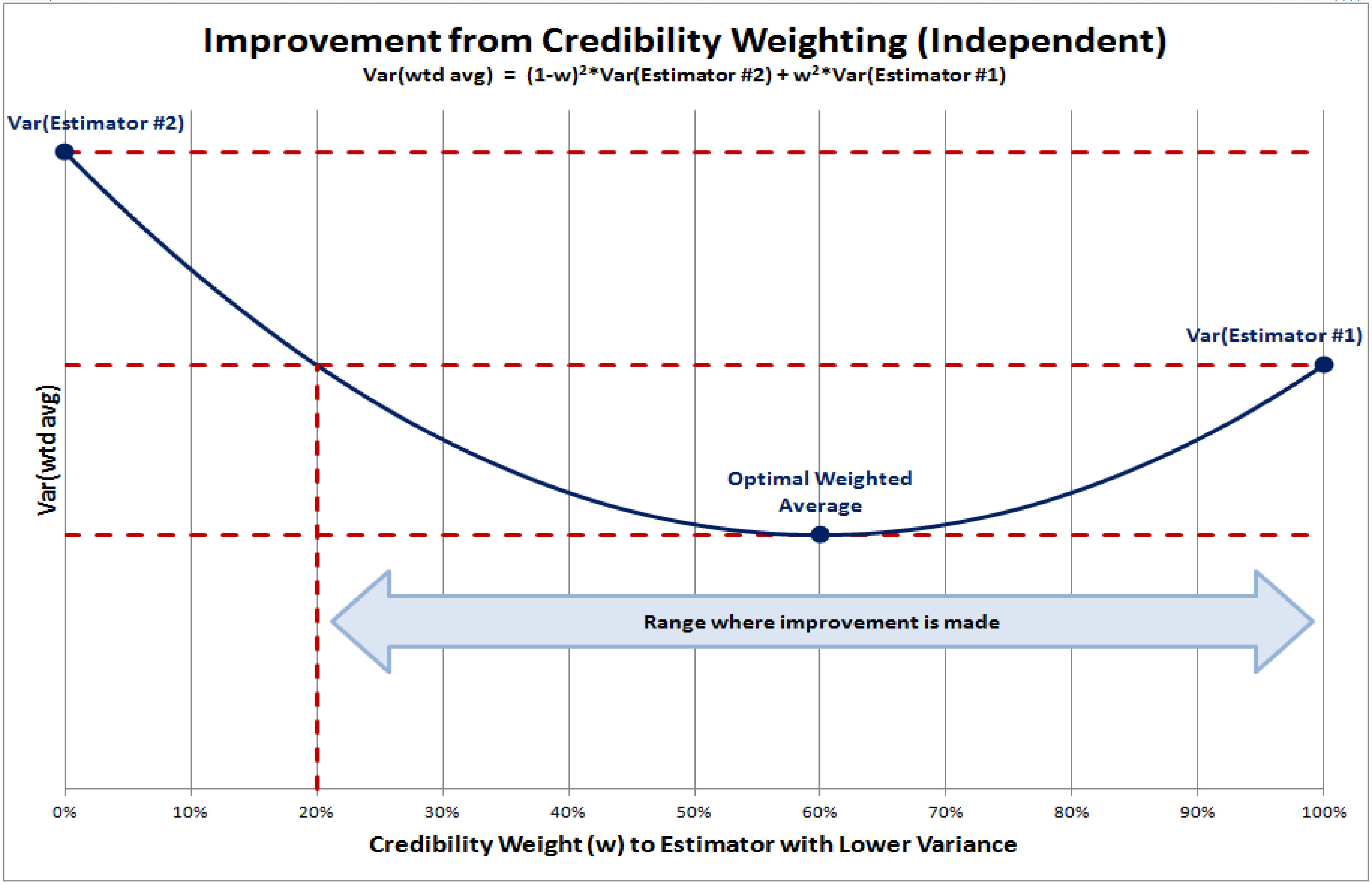
$$E(\widehat{\mu_1}) = E(\widehat{\mu_2}) = \mu$$

$$Cov(\widehat{\mu_1}, \widehat{\mu_2}) = 0$$

The variance of the credibility-weighted average is written as:

$$Var(\widehat{\mu_{cw}}) = w^2 \cdot Var(\widehat{\mu_1}) + (1-w)^2 \cdot Var(\widehat{\mu_2})$$

Basic Credibility Concepts



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We can find the "best" credibility weight as the w that minimizes the variance of the credibility-weighted average.

$$\frac{\partial\, Var(\widehat{\mu_{cw}})}{\partial\, w_1} = 0$$

The result is that the "best" weight is inversely proportional to the variance of the estimator.

$$\widehat{w_1} = \frac{Var(\widehat{\mu_2})}{Var(\widehat{\mu_1}) + Var(\widehat{\mu_2})} = \frac{Var(\widehat{\mu_1})^{-1}}{Var(\widehat{\mu_1})^{-1} + Var(\widehat{\mu_2})^{-1}}$$

A credibility-weighted (cw) average of multiple estimators:

$$\widehat{\mu_{cw}} = \sum_{i=1}^{n} w_i \cdot \widehat{\mu_i} \qquad \sum_{i=1}^{n} w_i = 1$$

If all estimators are assumed to be unbiased and independent:

$$Var(\widehat{\mu_{cw}}) = \sum_{i=1}^{n} w_i^2 \cdot Var(\widehat{\mu_i})$$

Multivariate Case

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Assuming independence among the various estimators, the "best" weights are again inversely proportional to the individual variances.

$$\widehat{w_i} = \frac{Var(\widehat{\mu_i})^{-1}}{\sum_{j=1}^{n} Var\left(\widehat{\mu_j}\right)^{-1}}$$

Substituting these weights back into the variance equation produces the following:

$$Var(\widehat{\mu_{cw}} \mid \widehat{w_i}) = \frac{1}{\frac{1}{Var(\widehat{\mu_1})} + \frac{1}{Var(\widehat{\mu_2})} + \cdots + \frac{1}{Var(\widehat{\mu_n})}}$$

Multivariate Case

Where there is correlation between the estimators, we define a covariance matrix containing the covariance between every pair of estimators.

For the three variable case, we have:

$$\boldsymbol{\Sigma} = \begin{bmatrix} Var(\widehat{\mu_1}) & Cov(\widehat{\mu_1}, \widehat{\mu_2}) & Cov(\widehat{\mu_1}, \widehat{\mu_3}) \\ Cov(\widehat{\mu_2}, \widehat{\mu_1}) & Var(\widehat{\mu_2}) & Cov(\widehat{\mu_2}, \widehat{\mu_3}) \\ Cov(\widehat{\mu_3}, \widehat{\mu_1}) & Cov(\widehat{\mu_3}, \widehat{\mu_2}) & Var(\widehat{\mu_3}) \end{bmatrix}$$

Multivariate Case

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The weights to be applied to the estimators are represented as a vector of numbers.

$$\overrightarrow{\boldsymbol{W}} = \langle w_1, w_2, \cdots, w_n \rangle^T$$

The "best" value for the weights, constrained so that they sum to unity, is found by matrix operations.

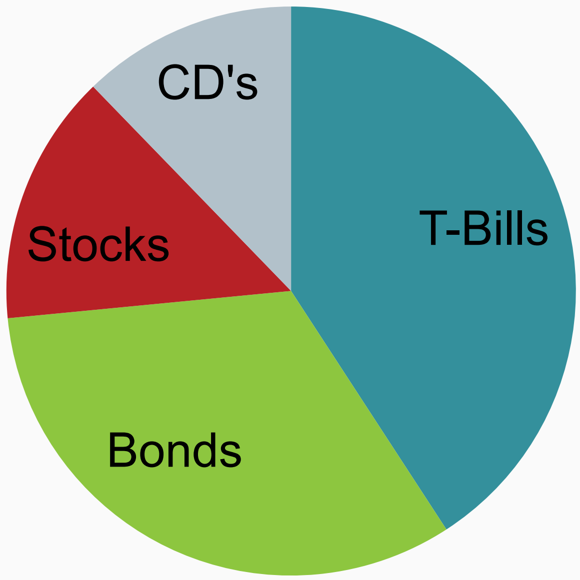
$$\overrightarrow{\boldsymbol{W}} = \frac{\boldsymbol{\Sigma}^{-1} \cdot \mathbf{1}_{\mathrm{n}}}{\mathbf{1}_n^T \cdot \boldsymbol{\Sigma}^{-1} \cdot \mathbf{1}_{\mathrm{n}}}$$

This is calculated by taking the inverse of the covariance matrix and then dividing each column total by the overall total.

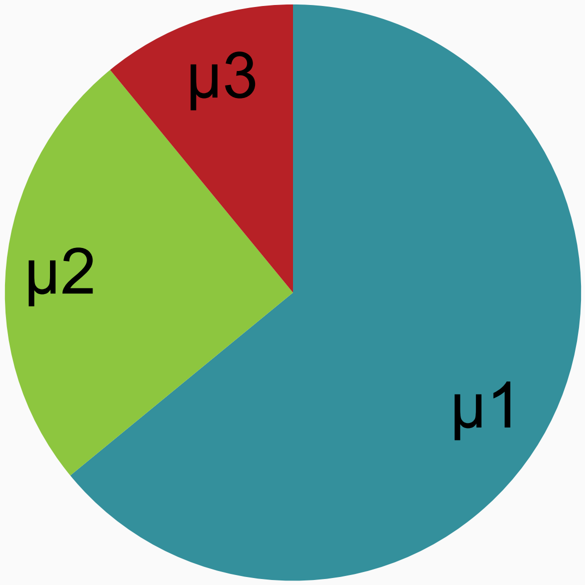
Interesting Tangent:

The math is equivalent to minimum variance portfolio optimization.

Portfolio Asset Allocation



Credibility Weights



The XOL Reinsurance Problem

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The experience rate is an estimator of the future loss. $\hat{\mu}_{exper}$

With variance based on:

- Number of years and losses in the historical period
- Attachment Point and Limit of layer being priced
- Changing operations of the client company

The exposure rate is an estimator of the future loss. $\hat{\mu}_{expos}$

With variance based on:

- Volume of loss experience in the industry
- Relevance of industry experience to a specific client

Credibility – Market Heterogeneity (variance in exposure rate)

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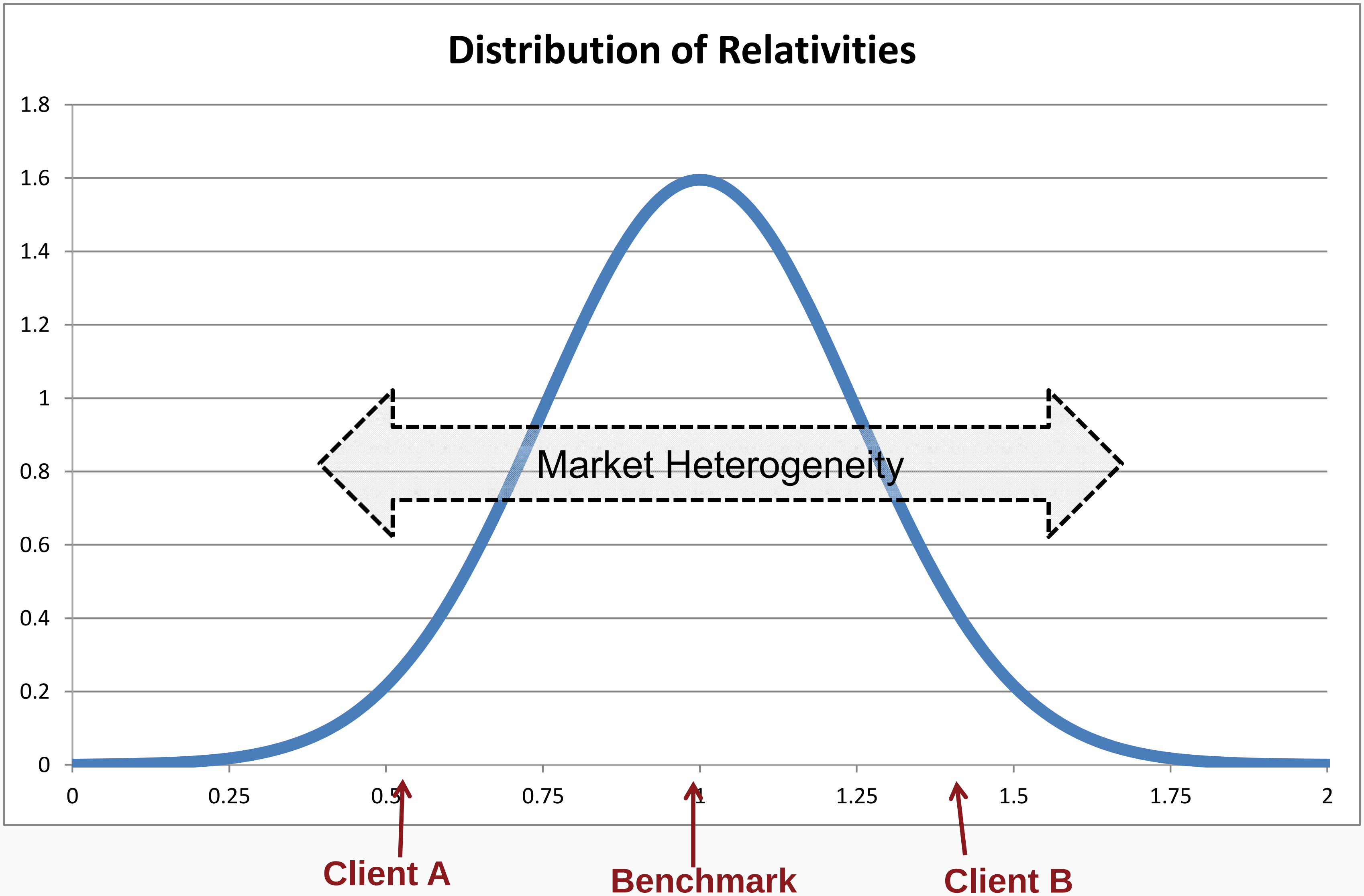

Traditional Credibility Weighted Average

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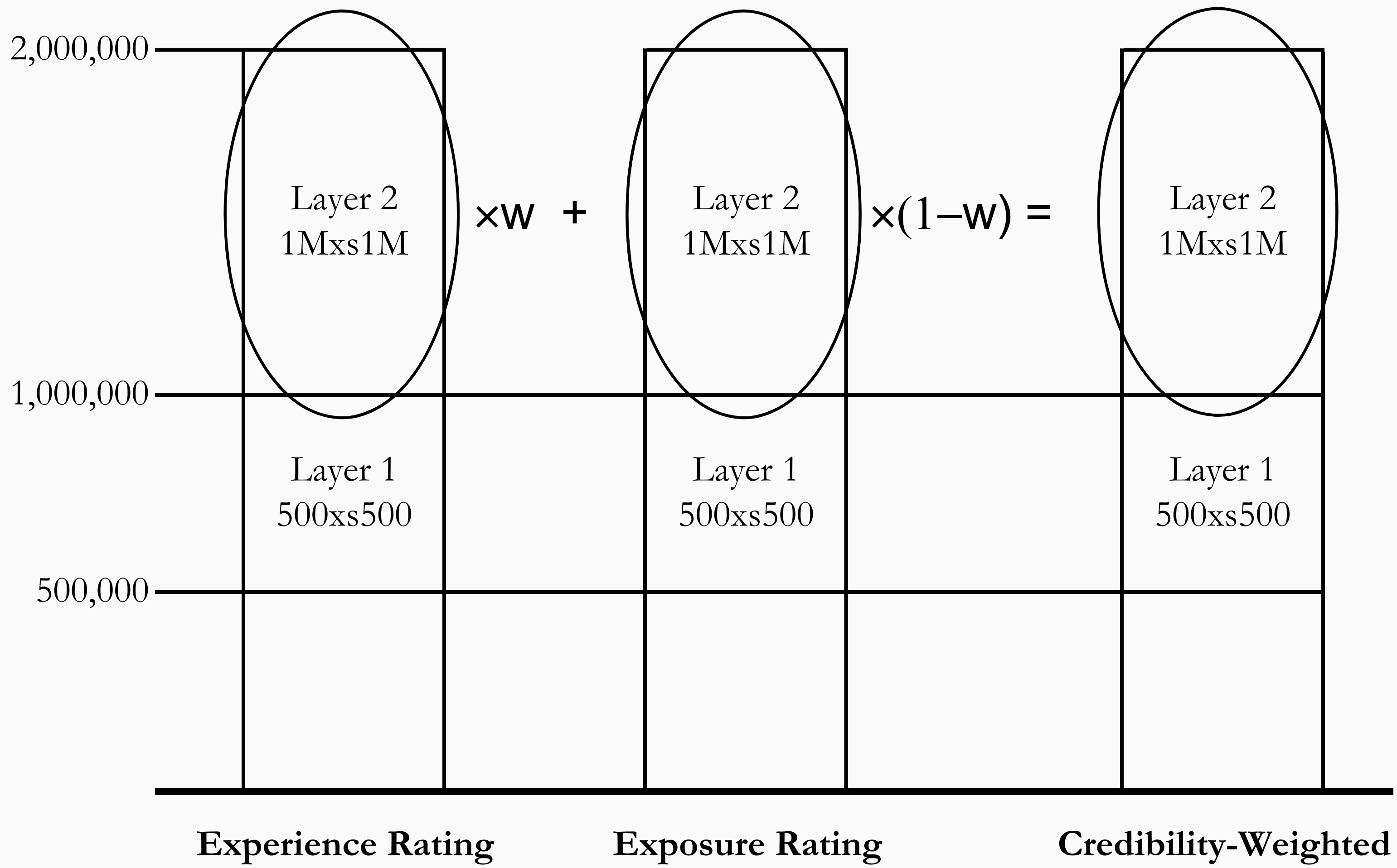
Example of Standard Credibility Procedure



The XOL Reinsurance Problem

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Our goal is to produce an unbiased, minimum variance estimator of the expected loss in the prospective period.

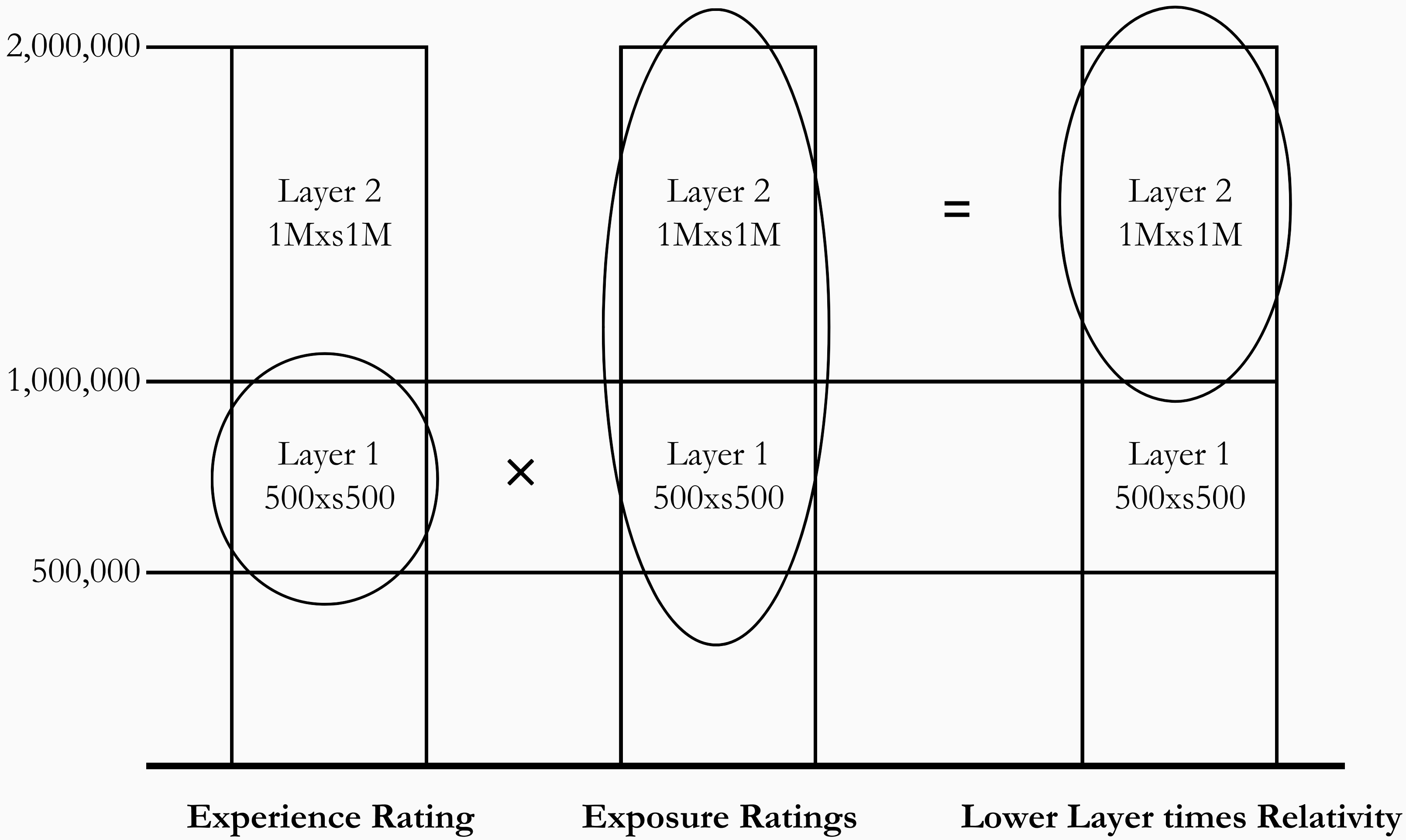
The traditional credibility weighting can bring us part of the way, but it does not make use of all the available information. Namely, the experience in lower layers is ignored.

An additional estimate can be produced using exposure-rating relativities applied to a lower layer (e.g., 500,000 xs 500,000).

$$\hat{\mu}_{rel} = \hat{\mu}_{exper\_500x500} \cdot \left\{ \frac{\hat{\mu}_{expos\_1Mx1M}}{\hat{\mu}_{expos\_500x500}} \right\}$$

Estimating Higher Layer based on Exposure-Rating Relativities Applied to Lower Layer

Using Exposure-Rating Relativities



Calculating Variances - For Numerical Example

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For a numerical example in the paper, we estimate variances for the three methods:

- **Experience Rate** – based on loss volume in historical period (ignores uncertainty for changing exposures, etc)
- **Exposure Rate** – based on uncertainty in Pareto distribution used for size-of-loss and on uncertainty in overall frequency
- **Relativity Method** – based on Pareto distribution in size-of-loss curve and on variance of experience rate for lower layer

Note: The covariances between the methods are set by the structure of the model and do not have to be separately estimated.

Calculating Variances - Covariance Matrix

	Exposure	Experience	Relativity
Covariance	1.573E+11	0	3.790E+10
Matrix:	0	1.716E+11	7.322E+10
	3.790E+10	7.322E+10	8.788E+10
Inverse:	7.580E-12	2.165E-12	-5.073E-12
	2.165E-12	9.663E-12	-8.986E-12
	-5.073E-12	-8.986E-12	2.105E-11
Row Total:	4.672E-12	2.843E-12	6.996E-12
Weights:	32.2%	19.6%	48.2%

Total Variance: 6.891E+10

Recursive Credibility Form

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The result is a three-factor credibility formula.

$$\hat{\mu}_{cw} = w_1 \cdot \hat{\mu}_{expos\ \_1Mx1M}$$
$$+w_2 \cdot \hat{\mu}_{exper\ \_1Mx1M}$$
$$+w_3 \cdot \hat{\mu}_{rel}$$

We can rearrange this expression into a recursive form:

$$\hat{\mu}_{cw\,500x500} = \left(\frac{w_1}{w_1 + w_3}\right) \cdot \hat{\mu}_{expo\ s_{500x500}} + \left(\frac{w_3}{w_1 + w_3}\right) \cdot \hat{\mu}_{expe\ r_{500x500}}$$

$$\hat{\mu}_{cw\,1Mx\,1M} = (w_1 + w_3) \cdot \hat{\mu}_{cw\,500x500} \cdot \left\{\frac{\mu_{expo\ s_{1Mx\,1M}}}{\mu_{expo\ s_{500x500}}}\right\} + w_2 \cdot \hat{\mu}_{exper\ 1Mx\,1M}$$

Recursive Credibility Form
Numerical Example

Alternative Recursive Form

	Experience Rating		Exposure Rating			Credibility-Weighted	
	Loss Cost	Cred%	Loss Cost	Relativity	Cred%	Loss Cost	Cred%
500 xs 500	5,000,000	60.0%	4,000,000	1.000	40.0%	**4,600,000**	100.0%
1M xs 1M	4,000,000		3,000,000	0.750			

	Experience Rating		Complement of Credibility			Credibility-Weighted	
	Loss Cost	Cred%	Loss Cost	Relativity	Cred%	Loss Cost	Cred%
500 xs 500	5,000,000		**4,600,000**	1.000			
1M xs 1M	4,000,000	19.6%	3,450,000	0.750	80.4%	3,557,800	100.0%

Numbers for illustration only

Conclusions

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- It is desirable to have a price based on a minimum variance estimator, so as to mitigate the “Winner’s Curse”
- Minimum variance credibility is a good framework for combining all sources of information
- For towers of excess layers, the minimum variance credibility formula is equivalent to a recursive application of exposure-rating relativities

References

"The Reinsurer's Conundrum" by Russ Wenitsky

Best's Review – July 2009

"Credibility Theory for Dummies" by Gary Venter

Casualty Actuarial Society Forum 2003: Winter

http://www.casact.org/pubs/forum/03wforum/03wf621.pdf

"Credibility for a Tower of Excess Layers" by David R. Clark

Variance Casualty Actuarial Society 2011: , Volume 05, Issue 01

http://www.variancejournal.org/issues/05-01/32.pdf

THANK YOU VERY MUCH FOR YOUR ATTENTION.

Dave Clark

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