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RESERVING IN THE 21ST CENTURY: CREDIBILITY AND EXTERNAL DATA

Dave Clark – Munich Reinsurance America

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Objectives of Credibility

Chief goal is more accurate estimate of future loss payments.

We want to make use of all available information in reserving:

- Exposure Bases (Cape Cod, aka Stanard-Bühlmann)
- Paid and Incurred (Quarg & Mack)
- Multiple Triangles (hierarchical or credibility)

Credibility allows “borrowing strength” (John W. Tukey) from other reserve segments. We can improve the reserve estimate by taking advantage of the correlation of one segment with others.

Objectives of Credibility

Many techniques have developed in a variety of scientific and financial fields for making use of expanded data sets and inter-relationships.

- Hierarchical models
- Mixed models
- Random Effects
- Clustered data
- GLMM (Generalized Linear Mixed Models)
- Empirical Bayes

Credibility can be thought of as a linear approximation to any of these.

Multi-dimensional Credibility

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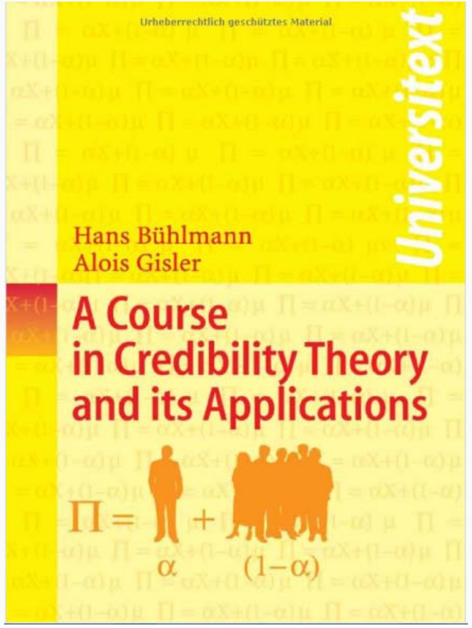
Credibility is typically applied as a weighted average of two estimators.

- Average of a single class of business with a “grand average” in ratemaking
- Average of experience and exposure rating in reinsurance

Multi-dimensional credibility expands this idea to a weighted average of two vectors.

- Development or payment patterns
- Sequence of Loss Ratios



Multi-dimensional Credibility

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The familiar Bühlmann-Straub credibility formula:

$$\hat{\mu}_{CredWtd} = \left(\frac{N}{N+k}\right) \cdot \bar{x}_i + \left(1 - \frac{N}{N+k}\right) \cdot \bar{\bar{x}}_{Overall}$$

$$\text{where:} \quad k = \frac{\sigma^2_{process}}{\sigma^2_{VHM}} = \frac{\text{Expected Process Variance}}{\text{Variance of Hypothetical Means}}$$

This can be rearranged as a weighted average using the inverses of the variances as weights:

$$\left(\frac{N}{\sigma^2_{process}} + \frac{1}{\sigma^2_{VHM}}\right) \cdot \hat{\mu}_{CredWtd} = \left(\frac{N}{\sigma^2_{process}}\right) \cdot \bar{x}_i + \left(\frac{1}{\sigma^2_{VHM}}\right) \cdot \bar{\bar{x}}_{Overall}$$

The multi-dimensional Bühlmann-Straub formula is a generalized form, that replaces all of the elements with matrices:

$$\left(\boldsymbol{\Sigma}_i^{-1} + \boldsymbol{\Sigma}_{Overall}^{-1}\right) \cdot \hat{\boldsymbol{\mu}}_{CredWtd} = \boldsymbol{\Sigma}_i^{-1} \cdot \bar{\boldsymbol{x}}_i + \boldsymbol{\Sigma}_{Overall}^{-1} \cdot \bar{\bar{\boldsymbol{x}}}_{Overall}$$

Multi-dimensional Credibility

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How does this work in practice?

Incremental Loss Ratios by Development Period

Dvlpmnt Period	Credibility Weighted			Company Specifc			Industry Benchmark
1	15.38%			15.17%			16.92%
2	21.24%			21.83%			18.76%
3	12.81%			12.43%			14.06%
4	7.21%			6.98%			9.63%
5	4.58%	=	**W***	3.98%	+	(**I-W**)*	5.82%
6	2.15%			1.53%			2.83%
7	1.47%			1.59%			1.61%
8	0.53%			0.83%			0.74%
9	0.39%			0.49%			0.37%
10	0.26%			0.22%			0.19%

Multi-dimensional Credibility

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For a paid loss triangle, we can calculate incremental loss ratios by development age and historical period, along with standard errors. This is a GLM.

Company-Specific Incremental Paid Loss Ratio

AY	1	2	3	4	5	6	7	8	9	10
1988	14.67%	23.30%	14.24%	6.81%	4.31%	0.53%	1.02%	0.77%	0.11%	0.22%
1989	21.59%	17.33%	15.94%	7.48%	2.82%	1.82%	1.86%	1.42%	0.86%	
1990	13.27%	28.77%	11.63%	5.09%	3.20%	1.48%	2.43%	0.32%		
1991	10.63%	31.79%	9.35%	3.33%	3.95%	2.28%	1.09%			
1992	7.78%	37.10%	12.16%	6.83%	4.82%	1.47%				
1993	14.86%	15.95%	9.55%	13.75%	4.66%					
1994	14.09%	15.51%	15.17%	6.28%						
1995	15.99%	14.63%	11.63%							
1996	18.81%	14.58%								
1997	18.14%									
Average	15.17%	21.83%	12.43%	6.98%	3.98%	1.53%	1.59%	0.83%	0.49%	0.22%
Std Error	0.88%	1.00%	0.94%	0.84%	0.71%	0.54%	0.46%	0.37%	0.32%	0.32%

Multi-dimensional Credibility

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This calculation is repeated for a collection of companies or reserve segments.

The average across all companies becomes the benchmark pattern.

This data also lets us estimate the “spread” of the differences between companies and correlations.

Incremental Loss Ratio by Development Period

Company	1	2	3	4	5	6	7	8	9	10
Co. A	22.87%	19.43%	13.26%	11.34%	2.18%	2.62%	0.67%	0.46%	0.07%	0.02%
Co. B	15.17%	21.83%	12.43%	6.98%	3.98%	1.53%	1.59%	0.83%	0.49%	0.22%
Co. C	18.18%	17.60%	14.63%	11.24%	4.78%	2.00%	1.03%	0.25%	0.09%	0.00%
Co. D	14.91%	25.86%	26.70%	14.99%	4.87%	1.78%	0.75%	0.64%	0.26%	0.00%
Co. E	21.67%	15.64%	13.38%	11.97%	9.32%	4.21%	1.65%	0.81%	0.02%	0.00%
***	***	***	***	***	***	***	***	***	***	***
Co. ZZ	23.84%	17.70%	16.06%	14.13%	5.06%	2.08%	0.89%	0.01%	0.02%	0.00%
Average	16.92%	18.76%	14.06%	9.63%	5.82%	2.83%	1.61%	0.74%	0.37%	0.19%

Multi-dimensional Credibility

The matrix of “total” covariance between incremental loss ratios is calculated from the table of results by company.

We can see that if a company has a higher (or lower) loss ratio in one period, then the next period is generally also higher (or lower).

Matrix of Correlation Coefficients between Loss Ratios

	1	2	3	4	5	6	7	8	9	10
1	1	0.4487267	0.0554446	-0.158143	-0.425792	-0.291124	-0.475995	-0.380101	-0.282516	0.0126847
2	0.4487267	1	0.4457716	-0.127479	-0.432639	-0.365582	-0.348797	-0.395271	-0.275459	0.223431
3	0.0554446	0.4457716	1	0.5866533	0.2429745	0.2442885	-0.04625	0.1486222	-0.221251	0.1340322
4	-0.158143	-0.127479	0.5866533	1	0.6274591	0.5875995	0.2666507	0.5709873	-0.074085	-0.103892
5	-0.425792	-0.432639	0.2429745	0.6274591	1	0.7442758	0.5929477	0.6427857	0.2634285	-0.047521
6	-0.291124	-0.365582	0.2442885	0.5875995	0.7442758	1	0.5881968	0.5562803	0.2764444	-0.072901
7	-0.475995	-0.348797	-0.04625	0.2666507	0.5929477	0.5881968	1	0.5182501	0.6097676	0.1435364
8	-0.380101	-0.395271	0.1486222	0.5709873	0.6427857	0.5562803	0.5182501	1	0.2164061	0.0132059
9	-0.282516	-0.275459	-0.221251	-0.074085	0.2634285	0.2764444	0.6097676	0.2164061	1	0.0632936
10	0.0126847	0.223431	0.1340322	-0.103892	-0.047521	-0.072901	0.1435364	0.0132059	0.0632936	1

Multi-dimensional Credibility

Same table with "heat map" conditional formatting:

Matrix of Correlation Coefficients between Loss Ratios

	1	2	3	4	5	6	7	8	9	10
1	1	0.4487267	0.0554446	-0.158143	-0.425792	-0.291124	-0.475995	-0.380101	-0.282516	0.0126847
2	0.4487267	1	0.4457716	-0.127479	-0.432639	-0.365582	-0.348797	-0.395271	-0.275459	0.223431
3	0.0554446	0.4457716	1	0.5866533	0.2429745	0.2442885	-0.04625	0.1486222	-0.221251	0.1340322
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5	-0.425792	-0.432639	0.2429745	0.6274591	1	0.7442758	0.5929477	0.6427857	0.2634285	-0.047521
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10	0.0126847	0.223431	0.1340322	-0.103892	-0.047521	-0.072901	0.1435364	0.0132059	0.0632936	1

Multi-dimensional Credibility

From this exercise, we see that we can:

- Use a Cape Cod GLM for each individual company to get
  - Company-specific pattern of incremental loss ratios
  - “Process variance” of the company estimated based on volume
- Use the collection of company results to get
  - The “variance of hypothetical means” as a matrix describing how much the individual companies can be different from the overall average
  - Covariances showing how correlated the development ages are

Everything we need for credibility calculation!

Expanding the Model

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Once the basic structure is in place, this model can be expanded further.

- Use fitted curves to smooth the pattern and/or extrapolate a tail factor
- Incorporate case reserves as an additional predictor
- Incorporate market cycle to allow different levels of rate adequacy for different accident years

But what if we have no exposure base or rate level factor?

Latent Variables

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A “Latent Variable” is one that is not directly observable, but can be approximated by a combination of other variables that can be measured, called “indicators.”

Non-Insurance examples:

- Intelligence (does an IQ test measure this?)
- Scholastic Aptitude
- Job Satisfaction
- Happiness
- Credit-worthiness
- Consumer Confidence

Latent Variables

For Insurance and Reserving:

- “Exposure Base” - We really mean something like “propensity for loss” or some value that is proportional to expected loss.
  - Payroll, sales, miles driven, property value, etc, are indicators

- “Market Cycle” – Is a “hard” or “soft” market directly measurable?
  - Market surveys, rate monitors, are indicators

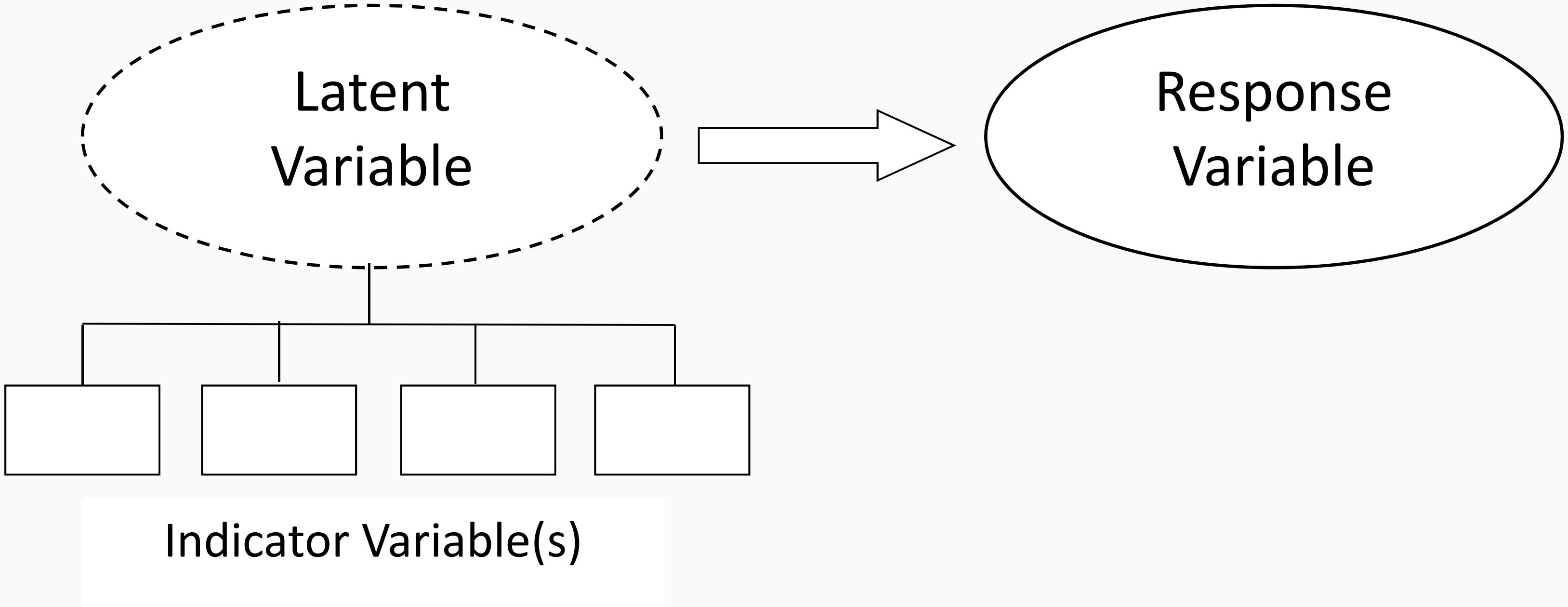
Key idea: We may not have true exposures for a reserve segment; we may not even have onlevel factors – but we do have indicators of these things.

Latent Variables

Here is the magic:

We do not need to have an exact historical exposure base or onlevel factors.

We only need some indicators that are correlated with the onlevel factors.



Latent Variables

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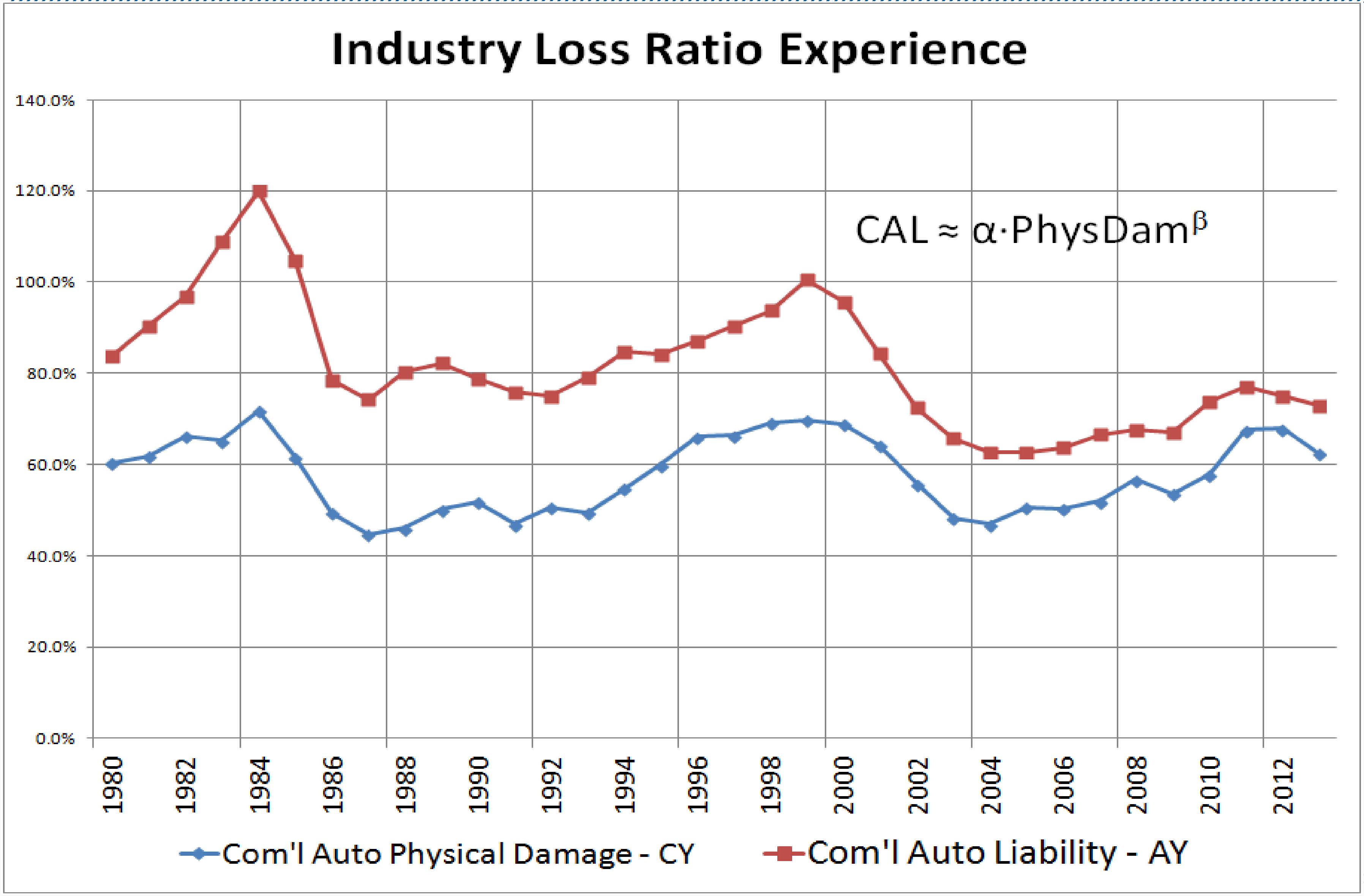

Latent Variables and Credibility

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Coefficients on the indicators can also be part of the credibility:

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7	1.47%			1.59%			1.61%
8	0.53%			0.83%			0.74%
9	0.39%			0.49%			0.37%
10	0.26%			0.22%			0.19%
Beta	0.88			0.80			1.00

The Challenge is not Technical…

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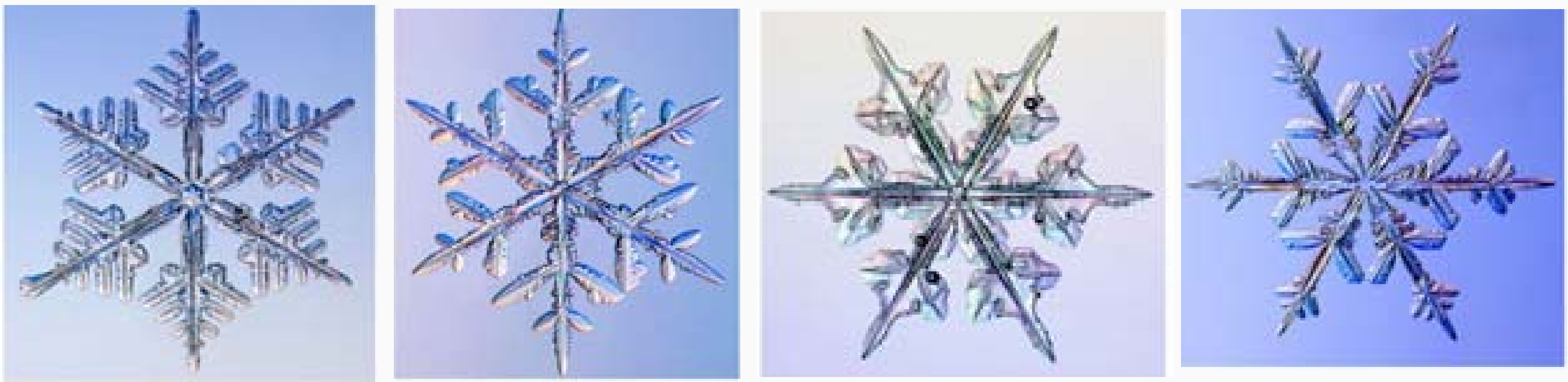
Every Snowflake is different…



And every reserve segment is different…

- Differences exist based on reserving practices, policy structures, etc
- Not all businesses are equally sensitive to the market cycle

But different does not mean completely independent.

Political challenge is to get people to agree that there is some value in external information for a given book of business.

The Challenge is not Technical…

Every Snowflake is different…




But when heat is applied…




Conclusions

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- Credibility theory provides a convenient means for making use of all available information in reserving
- Multi-dimensional credibility is an expansion of the familiar Bühlmann-Straub formula that lets us apply credibility to vectors rather than to single numbers
- A basic application of multi-dimensional credibility is shown using the results of a Cape Cod reserving method for a collection of triangles for Commercial Auto Liability

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THANK YOU FOR YOUR ATTENTION

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