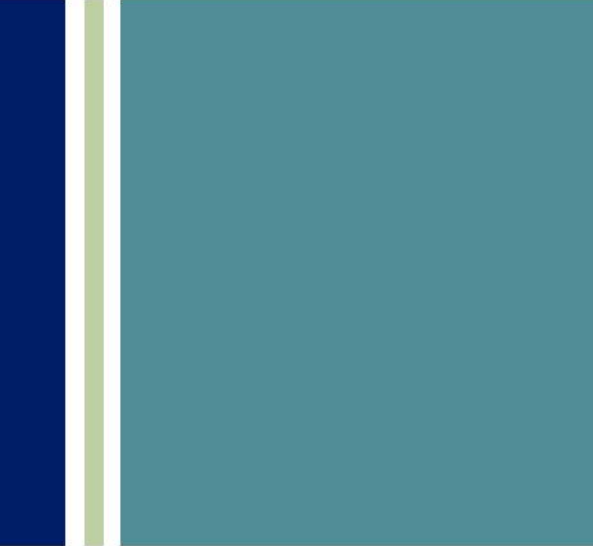watsonwyatt.com

CAS Ratemaking Seminar

Use of GLMs in Rate Filing

Gaétan Veilleux, FCAS
March 17-18, 2008

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Statement of Principals…

"A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer."

- CAS Statement of Principals Regarding Property & Casualty Insurance Ratemaking
- ASOP 9

Agenda

- Defining your actuarial support
  - Method
  - Data
  - Assumptions
  - Model and variable structure
  - Output
  - Refinements
  - Selections and impacts
- Sample exhibits
- Know the regulations

Method used

- If you can't explain it, don't expect others to understand it.
- A generalized linear model (GLM) is …
  - Layman's explanation
  - Statistician's explanation

$$E[Y_i] = \mu_i = g^{-1}(\Sigma X_{ij}\beta_j + \xi_i)$$

$$Var[Y_i] = \phi V(\mu_i)/\omega_i$$

- CAS syllabus - "A Practitioner's Guide to Generalized Linear Models"
- "Generalized Linear Models" by P. McCullagh and J. A. Nelder

Data used

- Actuaries should be accustomed to explaining this
  - How much (e.g. exposure, claim counts)
  - States, products, years
  - Nature of explanatory variables
  - Any data adjustments
  - Data splits (e.g. hold-out samples for model validation)

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Assumptions

- Model assumptions
  - Link function
    - log = multiplicative
    - Identity = additive

  - Distribution assumptions
    - Tweedie for fitting models directly to loss costs
    - Frequency (Poisson)
    - Severity (gamma) models
    - Combining Freq and Sev results

Model structure

Clearly define which variables are in your models and the structure of the model.

Equation format:

$$E[\text{Loss Cost}] = g^{-1} (\alpha + \beta.\text{Symbol} + \gamma.\text{Deductible} + \ldots)$$

Table format:

Variable	Frequency	Severity	Loss Cost
Symbol	Yes	Yes	Yes
Variable 2	Yes	No	Yes
Variable 3	…		

Variable structure

Explain how specific variables are defined in your models.

- Continuous
- Categorical
- Interactions
- Offsets or restrictions

Output

- Explain clearly how to interpret output and any diagnostics used – (and in context of how you described method)
- Graphs work well
- Tables are more traditional
- Validation techniques (e.g. lift curves)

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Commercial refinements

- Constrained parameter estimates for particular variable
- Grouping categorical levels (for credibility)
- Smoothed anomalous effects

Selections and impact

- Again – not new! Just new framework and better tools
- How were selections made – in consideration of:
  - Model results
  - Competitive analysis
  - Dislocation management
  - Other corporate objectives
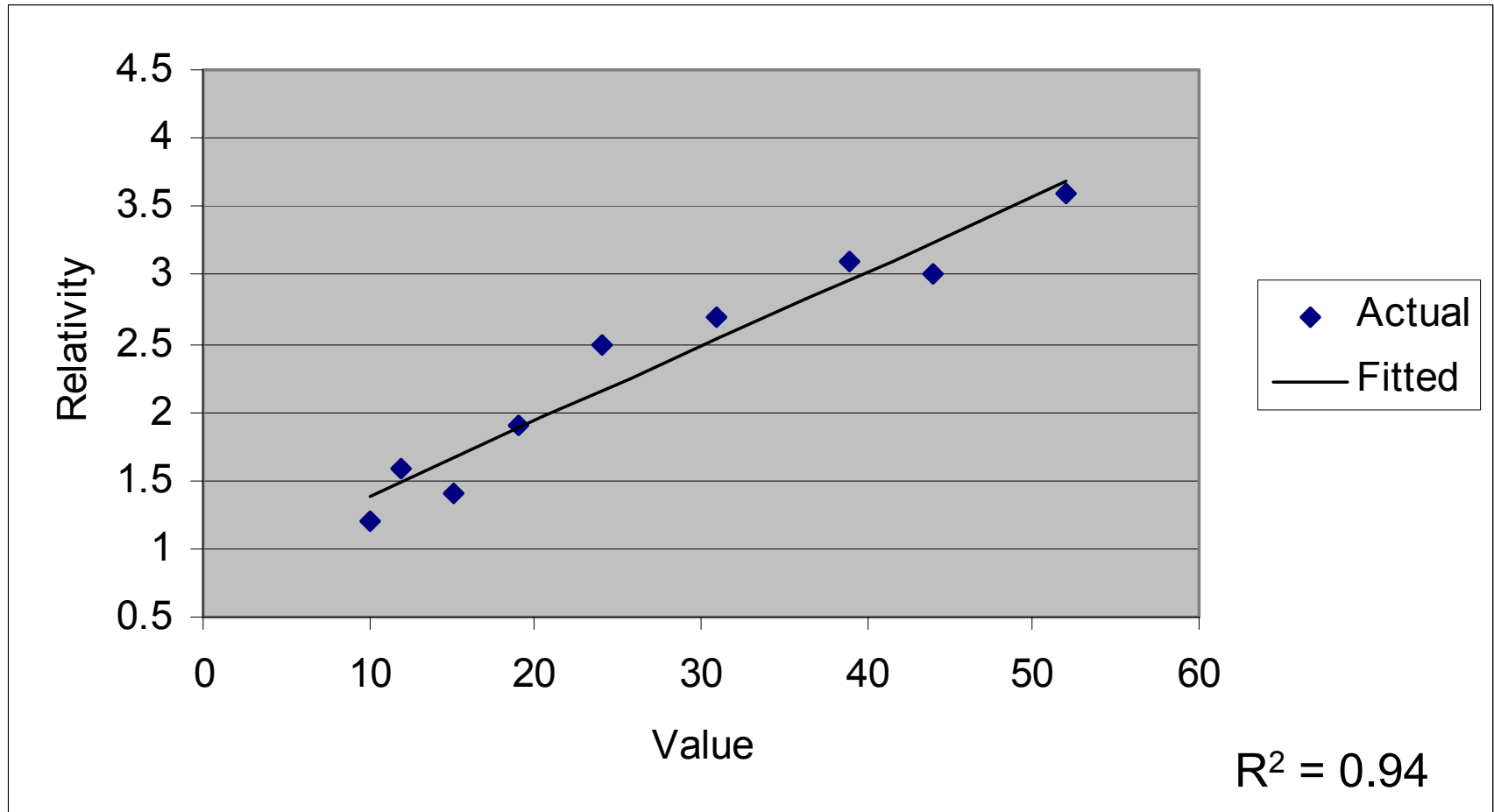
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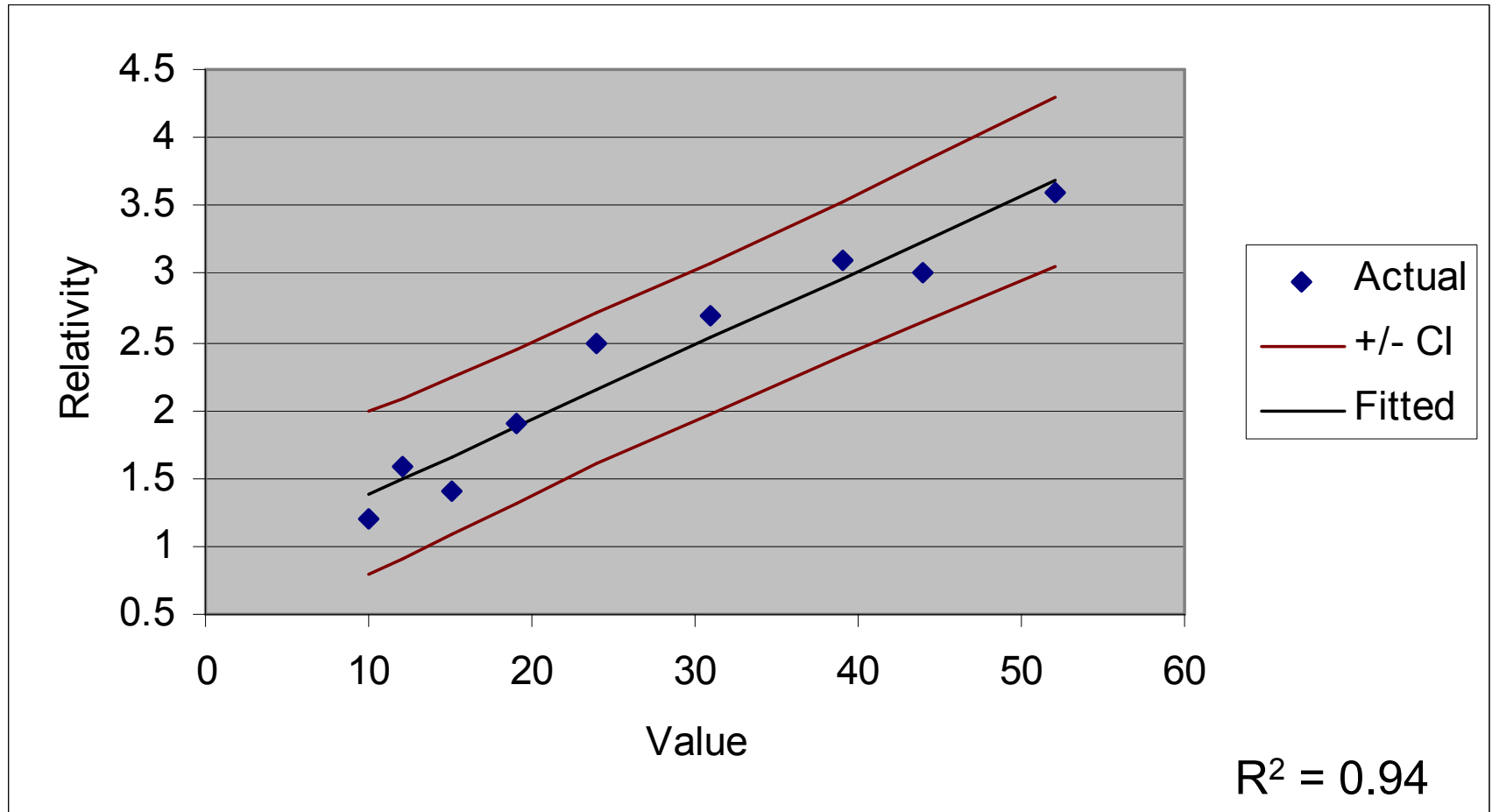- **Sample exhibits**
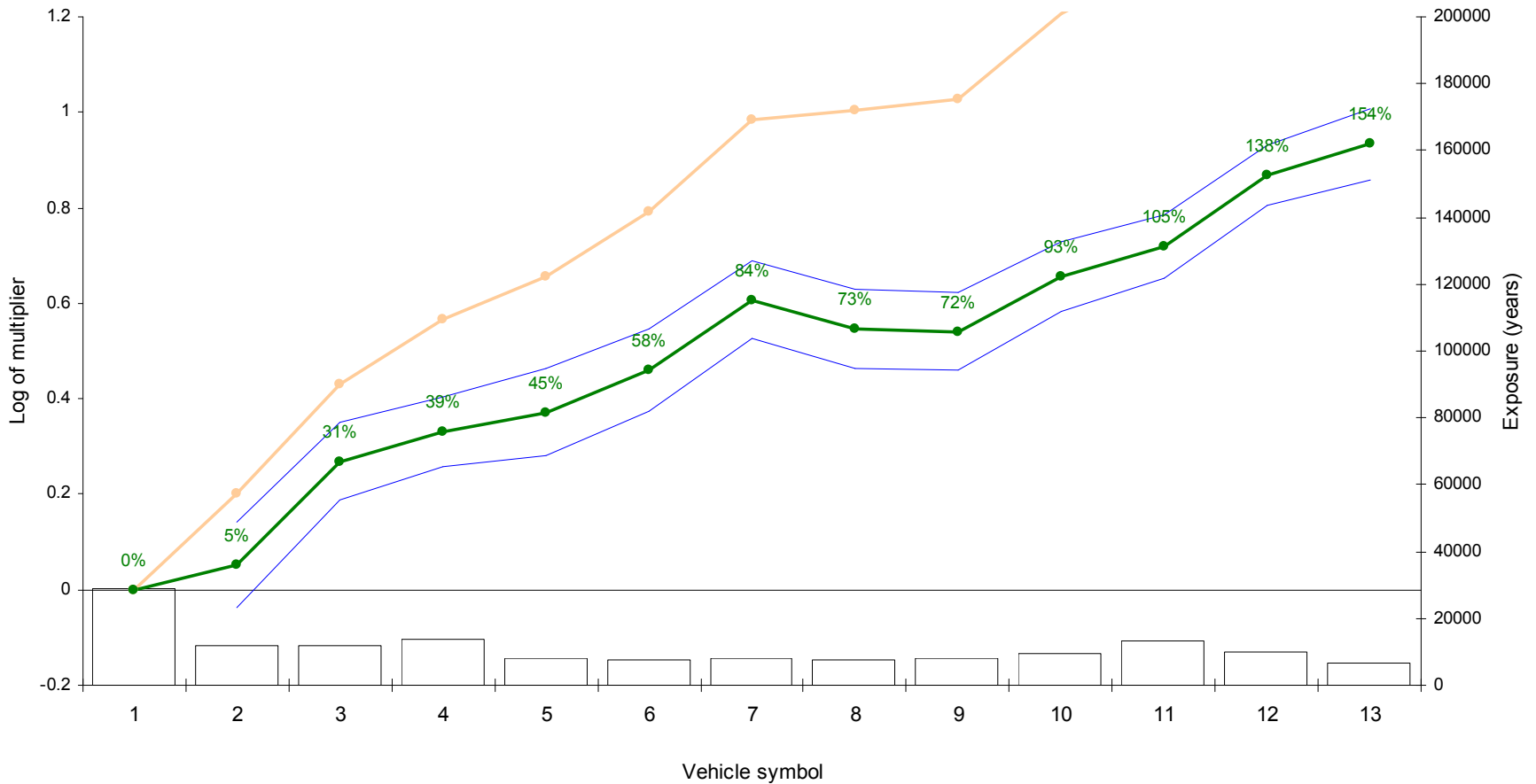- Know the regulations
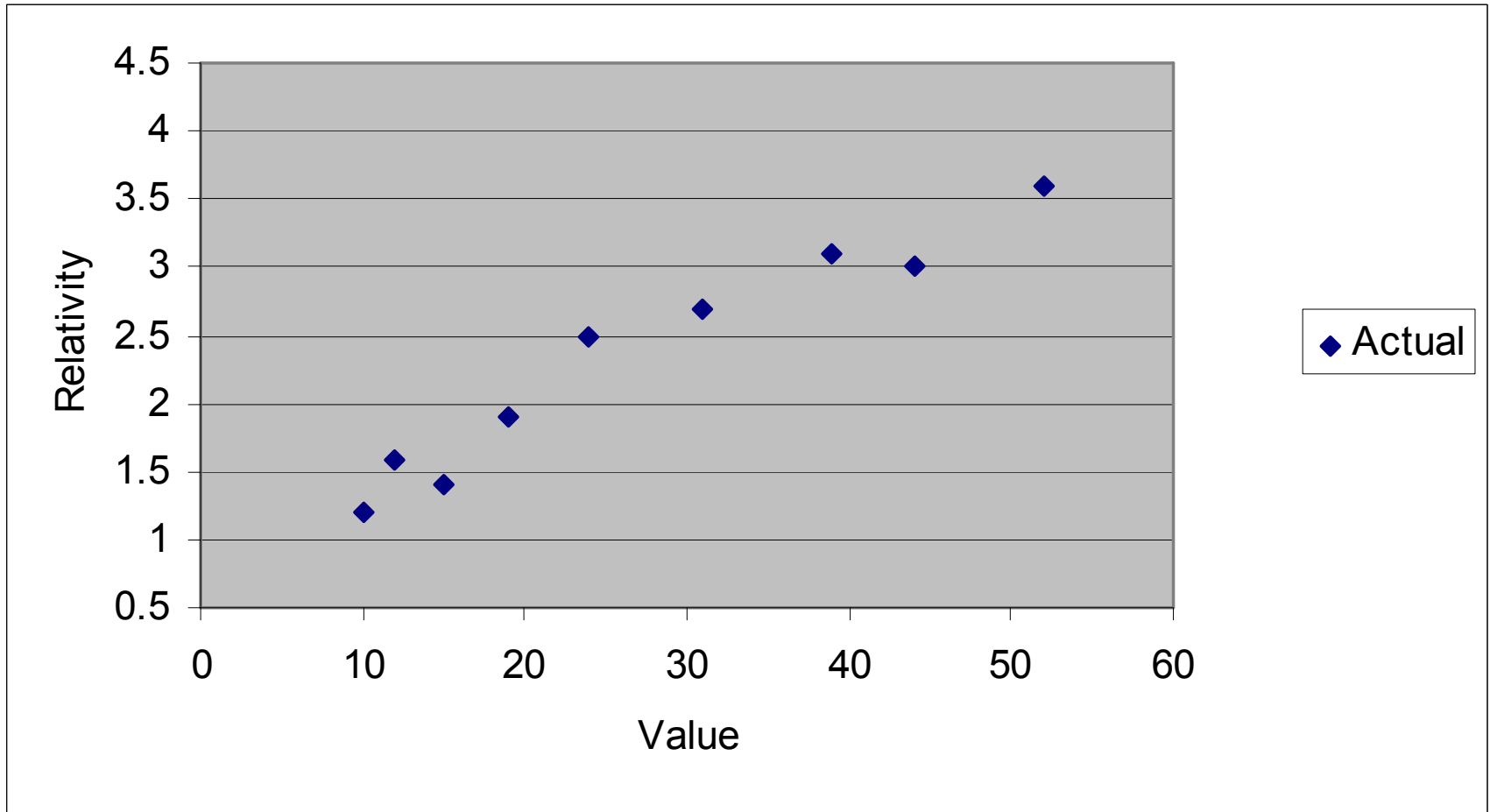
"Traditional" Regression

"Traditional" Regression

$R^2 = 0.94$

"Traditional" Regression

GLM output

GLM output

Indicated Relativities

Vehicle Group	Relativity
1	0.5036
2	0.5563
3	0.6209
4	0.6565
5	0.6456
6	0.7221
7	0.8561
8	0.9031
9	0.9322
10	1.0000
11	1.0960
12	1.1722
13	1.2403
14	1.3564
15	1.4191
16	1.5192
17	1.6640
18	1.8287
19	1.9933
20	1.9517

Policy Tenure	Relativity
0	1.0000
1	0.8835
2	0.9115
3	0.9150
4	0.8493
5	0.8137
6	0.8185
7	0.8275
8	0.7685
9	0.7675
10	0.7338
11	0.6775
12	0.7313
13	0.7401
14	0.6252
>= 15	0.6381

Gender	Relativity
Female	0.6997
Male	1.0000

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GLM output

Indications and Confidence Estimates

Vehicle Group	Lower 95% CI	Indicated Relativity	Upper 95% CI	Selected Relativity
1	0.4373	0.5036	0.5800	
2	0.4913	0.5563	0.6298	
3	0.5559	0.6209	0.6936	
4	0.5926	0.6565	0.7272	
5	0.5856	0.6456	0.7118	
6	0.6609	0.7221	0.7890	
7	0.7898	0.8561	0.9280	
8	0.8362	0.9031	0.9754	
9	0.8657	0.9322	1.0037	
10	1.0000	1.0000	1.0000	
11	1.0223	1.0960	1.1750	
12	1.0936	1.1722	1.2565	
13	1.1571	1.2403	1.3295	
14	1.2648	1.3564	1.4546	
15	1.3214	1.4191	1.5240	
16	1.4114	1.5192	1.6353	
17	1.5428	1.6640	1.7948	
18	1.6903	1.8287	1.9783	
19	1.8347	1.9933	2.1655	
20	1.7767	1.9517	2.1441	

Policy Tenure	Lower 95% CI	Indicated Relativity	Upper 95% CI	Selected Relativity
0	1.0000	1.0000	1.0000	
1	0.8401	0.8835	0.9291	
2	0.8646	0.9115	0.9609	
3	0.8652	0.9150	0.9677	
4	0.7995	0.8493	0.9022	
5	0.7622	0.8137	0.8687	
6	0.7651	0.8185	0.8756	
7	0.7716	0.8275	0.8874	
8	0.7142	0.7685	0.8270	
9	0.7135	0.7675	0.8256	
10	0.6793	0.7338	0.7927	
11	0.6244	0.6775	0.7350	
12	0.6752	0.7313	0.7920	
13	0.6800	0.7401	0.8054	
14	0.5721	0.6252	0.6831	
>= 15	0.5846	0.6381	0.6964	

Example job

Run 5 Model 3 - Small interaction - Third party material damage, Numbers

P level = 0.0%
Rank 6/6
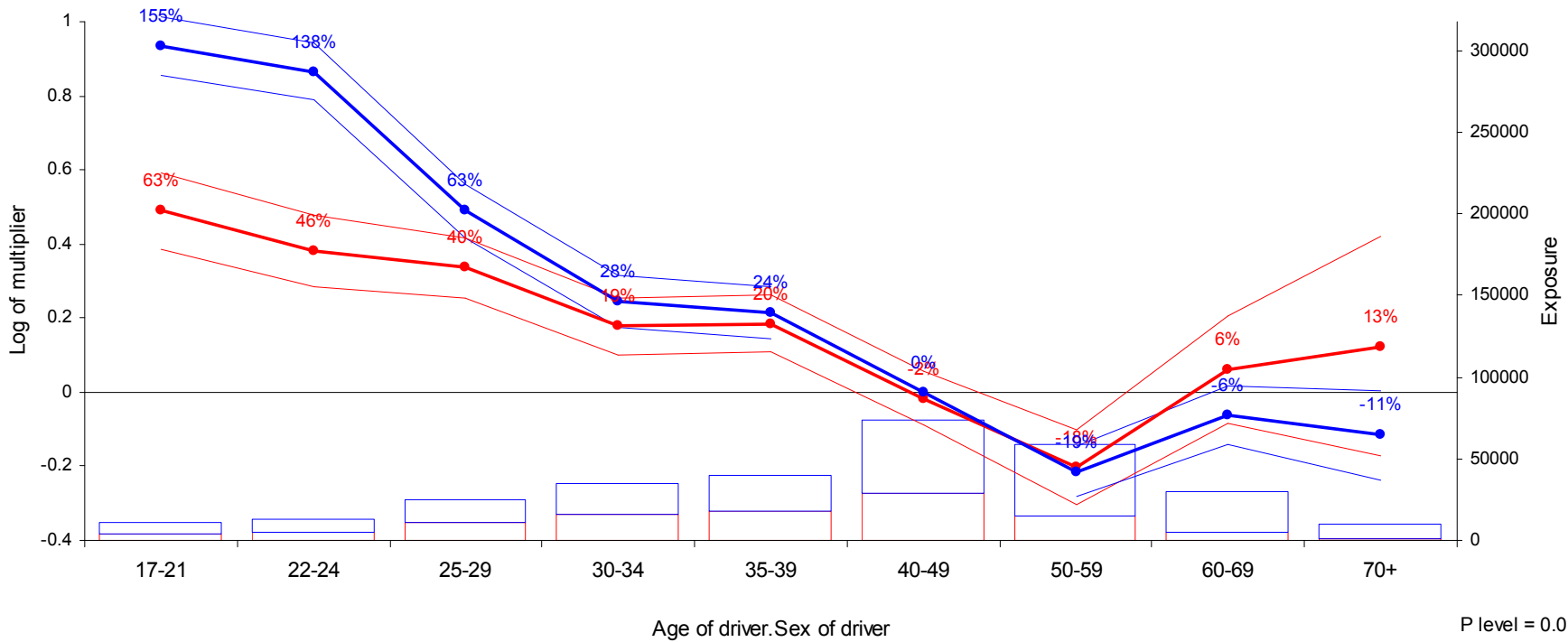
Age of driver.Sex of driver

Legend:
- Approx 2 SEs from estimate, Sex of driver: Female
- Approx 2 SEs from estimate, Sex of driver: Male
- Unsmoothed estimate, Sex of driver: Female
- Unsmoothed estimate, Sex of driver: Male
- Smoothed estimate, Sex of driver: Female
- Smoothed estimate, Sex of driver: Male

GLM output

Indications and CI for an Interaction

Male Age	Lower 95% CI	Relativity	Upper 95% CI
17-21	2.2950	2.5500	2.8050
22-24	2.1420	2.3800	2.6180
25-29	1.4670	1.6300	1.7930
30-34	1.1520	1.2800	1.4080
35-39	1.1210	1.2400	1.3640
40-49	1.0000	1.0000	1.0000
50-59	0.8280	0.9200	1.0120
60-69	0.8210	0.9400	1.0450
70+	0.7650	0.8900	1.0310

Female Age	Lower 95% CI	Relativity	Upper 95% CI
17-21	1.4670	1.6300	1.7930
22-24	1.3140	1.4600	1.6060
25-29	1.3050	1.4500	1.5950
30-34	1.0710	1.1900	1.3090
35-39	1.0800	1.2000	1.3200
40-49	0.8820	0.9800	1.0780
50-59	0.7380	0.8200	0.9020
60-69	0.9400	1.0600	1.2420
70+	0.9810	1.1300	1.3670

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Model validation
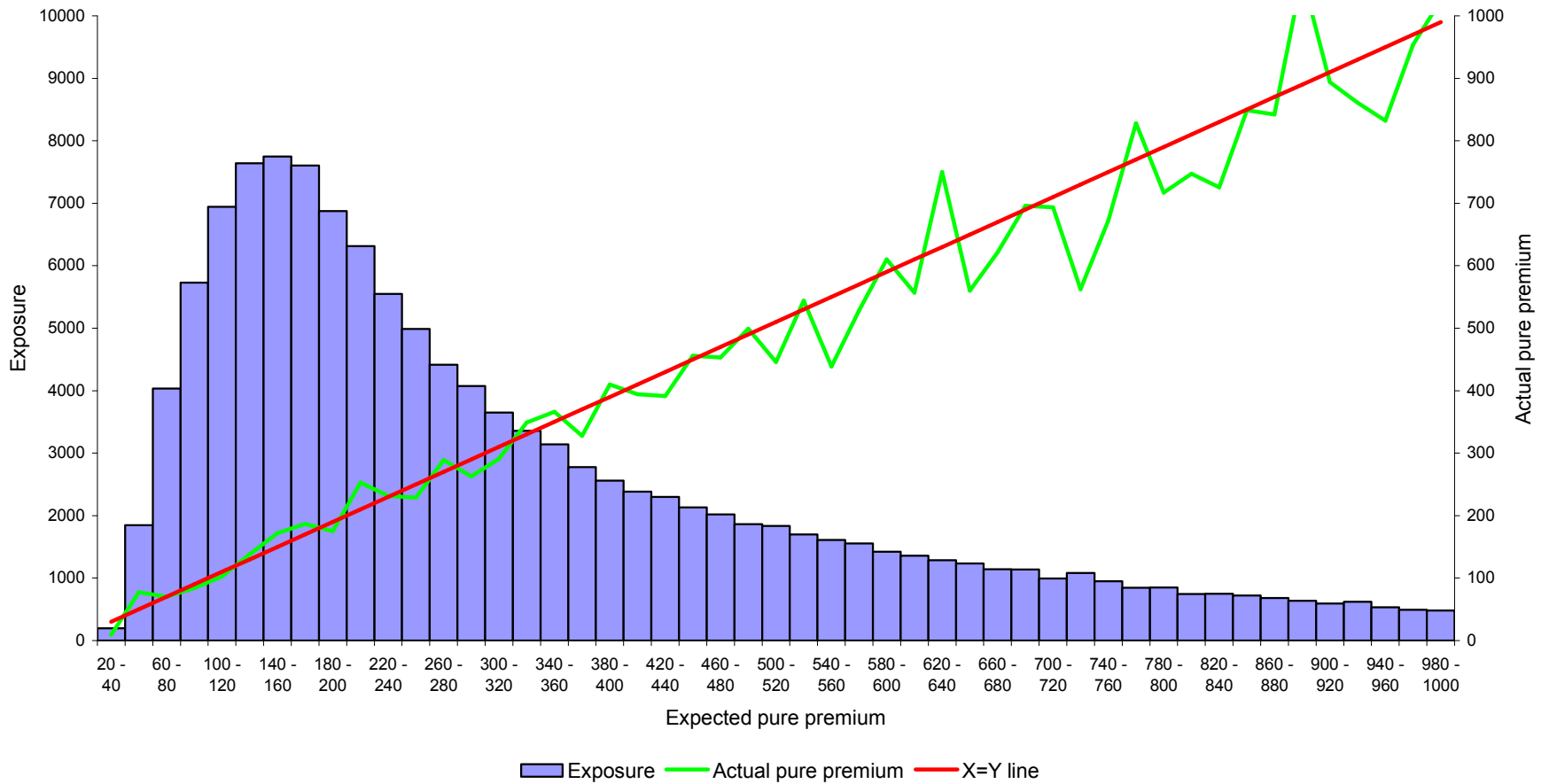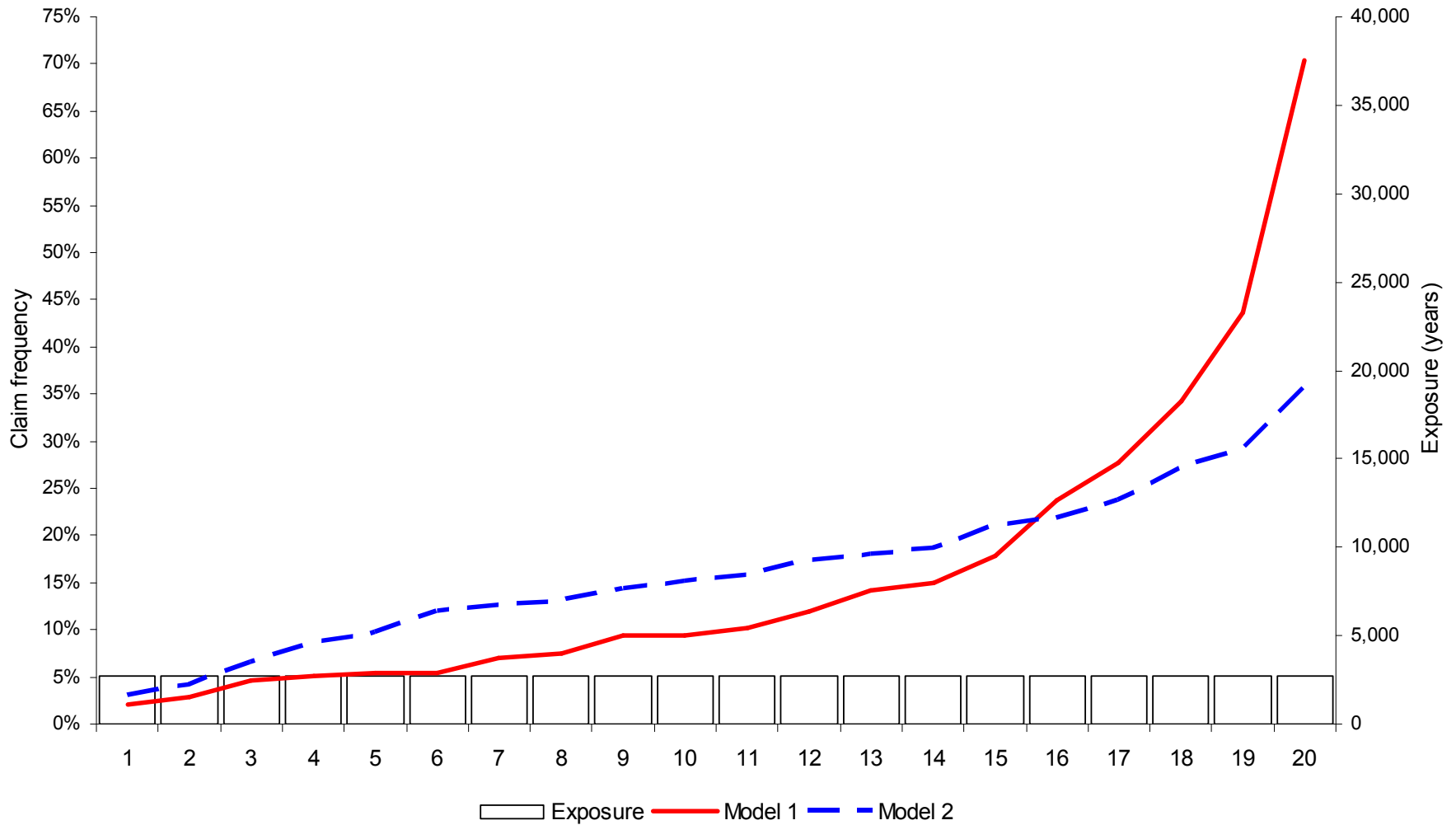
Lift curves

Demonstration job

Run 10 Model 2 - Third party material, standard risk premium run - Unsmoothed standard risk premium model
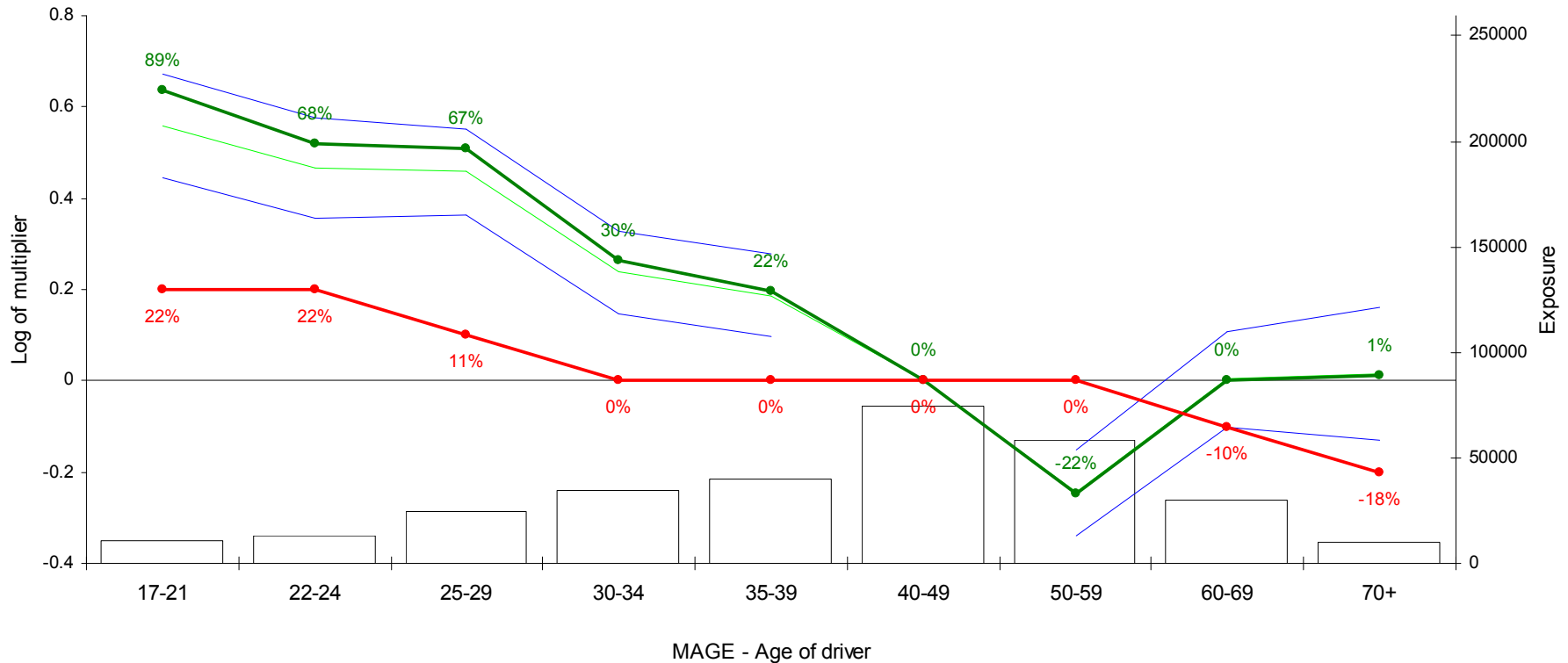
Example job
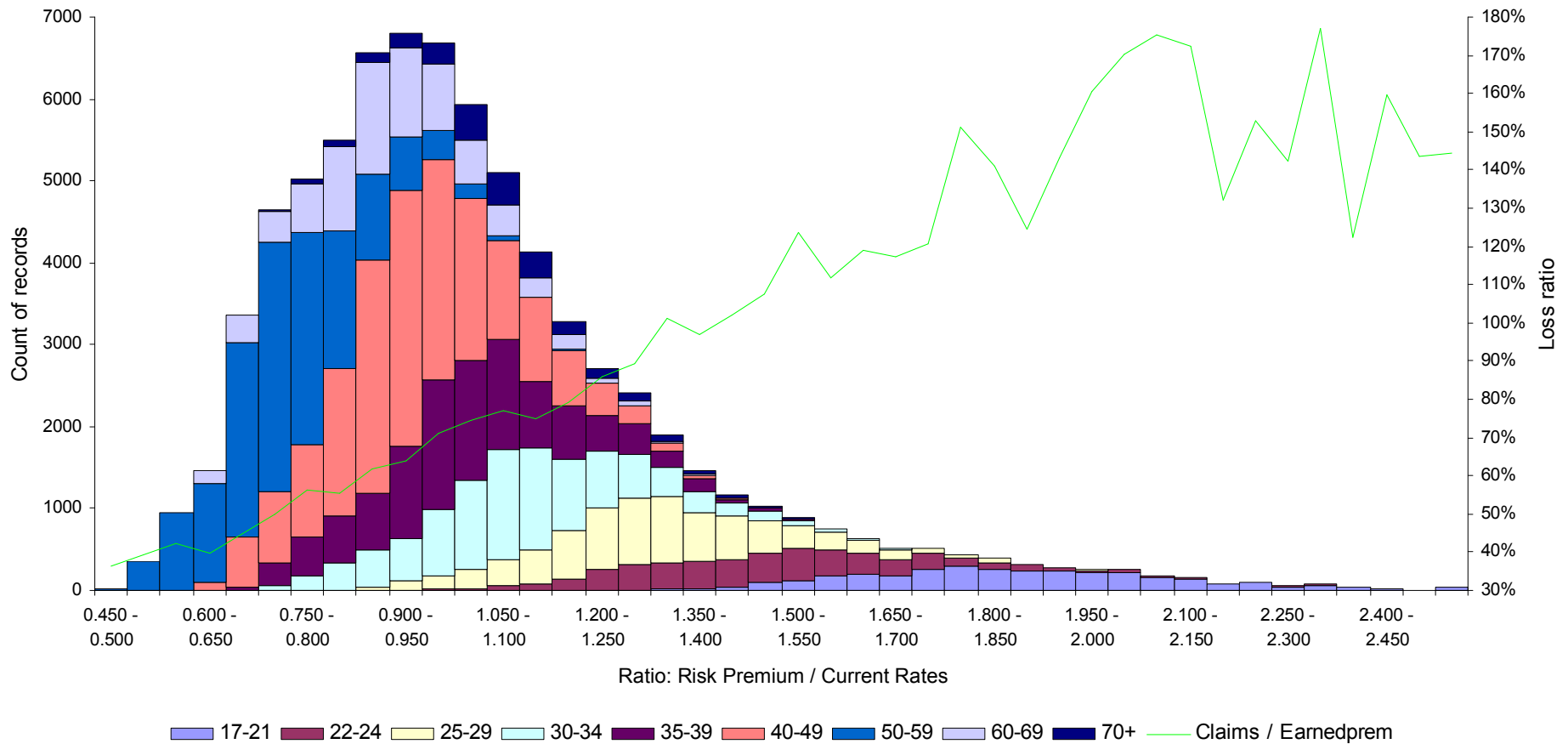
Age of driver

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- **Know the regulations**

Know the regulations!

- Are any variables banned?
- Confidentiality (What is required? Can you get it?)
- Minimum information
- Difference between rating & underwriting

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Know the regulations!

- New vs renewal business implications
- DOI staff
- Process (Should we initiate a conversation before everything is finalized? Who on our team should be involved?)
- Strike a balance

Credit or Insurance Score

- Can I use it? How?
- Are there any restrictions as to how I can model it?
- New vs renewal business
- How is "multivariate model" defined? Does my comply?

- "Please provide a description of how **GLM** works."

- "Please clarify which rating elements were included in your **multivariate analysis**."

- "As stated before, such variables appear to be in **violation** of RCW 48.19.035(2)(a) and (d)."

- "Was the insurance score variable randomly assigned to the individual risks for the **multivariate analysis**? If so, we do not see why we should accept the results of the **multivariate analysis**. If not, we do not see why these rating elements were assigned randomly in determining your base rates."

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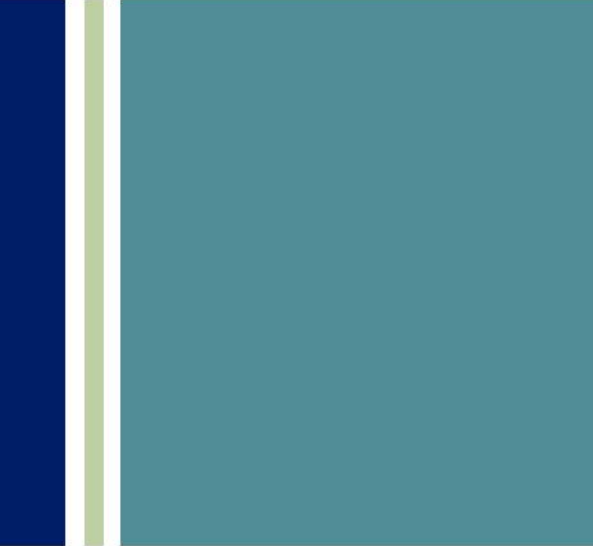
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