An Analysis of the Market Price of Cat Bonds

Neil Bodoff, FCAS and Yunbo Gan, PhD

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Agenda

- Brief introduction to cat bonds
- Description of problem
- Some current models of cat risk pricing
- Motivation for new model
- Proposed model
- Results
- Summary
- Caveats
- Areas for future research

Brief Intro to Cat Bonds

- (Re)insurance company wants to hedge its cat exposure
- Buys reinsurance from SPV
- SPV holds capital equal to the coverage limit ("fully collateralized")
- SPV raises this capital from investors by selling cat bonds
  - often in several layers or "tranches"
- Investors earn coupon rate on the contributed money
- Coupon rate usually defined as LIBOR + "spread" %

Description of Problem

- If the bonds had no exposure to cat loss, then coupon rate should equal LIBOR
- With cat exposure, coupon rate is LIBOR + spread
- Implies that “spread” is the “price of cat risk”
  - thus spread can be considered similar to the RoL of traditional reinsurance contracts
- Problem:
  - how can we describe the price of cat risk in the cat bond market?
  - how can we model the spreads on various cat bonds?

Current Models of Spreads

- Model #1
- Spread % = (expected loss %) x (multiple)
  - practitioner model
  - used to describe, predict, and benchmark various cat bond spreads
  - key parameter is the "multiple"
  - problem: multiple tends to vary
    - when expected loss is large, multiple is small
    - when expected loss is small, multiple is large
  - therefore the model is not complete for describing spreads

Current Models of Spreads

- Model #2
- Spread = function of (probability of loss, conditional severity)
- Example: Spread % = a * probability^b * conditional severity^c
- Suggested by Morton Lane, ASTIN Bulletin 2000
  - winner of CAS Hachemeister Prize, 2001
- Problems
  - no variation of parameters for different perils and/or correlation
  - Gatumel (ASTIN Colloquium, 2008) notes that not all of Lane's parameters are statistically significant

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Current Models of Spreads

- Model #3
- Spread = function of expected loss and standard deviation
- Example: spread % = expected loss % + alpha * standard deviation
- Popular in the traditional reinsurance market
- Often attributed to paper by Kreps
  - but Kreps explicitly states:
    - standalone standard deviation is only upper bound
  - true price depends on the risk within a portfolio, not standalone
- Other problems
  - reality: loading as a % of sd is not constant, so the sd loading itself tends to vary from low layers to high layers
    - need a “model” of a parameter of a model?
  - skewness matters (PCAS paper by Kozik and Larson)
  - violates Venter’s “no arbitrage” criterion
    - unhelpful when structuring, layering, and tranching

Current Models of Spreads

- Model #4
- Spread = expected loss % + margin %
- Used in the corporate bond market
  - “spread over risk free” = expected default loss + margin
    - “Credit Spread Puzzle”
      - > market pricing: spreads are higher than needed to cover the expected default loss; why need margin?
      - > puzzle even more pronounced for corporate bonds with higher expected default
- Problems
  - cat bond data not consistent with this model
  - rather, when cat bond expected loss increases, so does margin
  - conjecture: increase in expected loss leads to increase in margin because of uncertainty in the estimated expected loss
    - conversely, other explanations of the “credit spread puzzle”, such as correlation with equities, do not work well for cat bonds

Motivation for New Model

- Unlike existing models, we seek a model that
  - does not violate portfolio theory
    - riskiness must be measured within a portfolio, not standalone
  - is consistent with the empirical data
  - is practical and easy to explain to others
  - does not violate Venter's principle (ASTIN, 1991) of "reinsurance without arbitrage"
    - use the pricing model to calculate the price of the cat cover (all layers combined)
    - then slice the cat cover into various layers ("tranches")
    - use the model to price the layers; add up the prices of the layers
    - does sum of the prices for the various layers equal the price of the total program in one large layer?
    - if not, the formula violates "no arbitrage"

Proposed Model

- Spread % depends upon the covered peril

- Spread % =

  peril specific flat margin %

  + expected loss % * peril specific loss multiplier

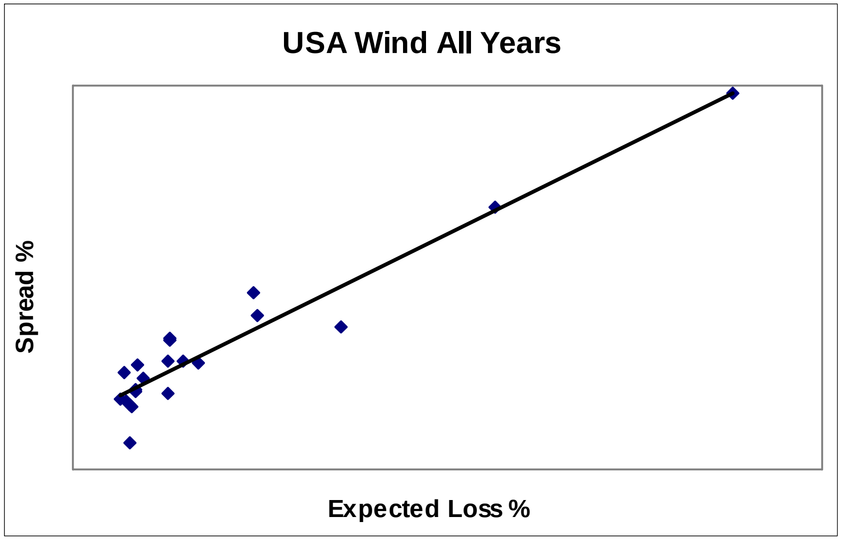
- For each peril, we have a linear function:

  - Spread % = constant % + loss multiplier * expected loss %

Data

- Years ending June 30, 1998 – 2008
- Example: "2008 Year" = July 1, 2007 through June 30, 2008
- Single peril bonds only
  - can use multi peril bonds as well
    - but need granular data about the various perils that contribute to the expected loss
- Perils classified based on broadly defined buckets
  - USA Wind
  - Europe Wind
  - California EQ
  - Japan EQ
  - etc.

Results: USA Wind



Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Wind	USA	All years	Full cycle	Constant %	3.33%	0.45%	2.38%	4.27%
Wind	USA	All years	Full cycle	Loss Multiplier	2.40	0.17	2.05	2.76

Parameters are statistically significant

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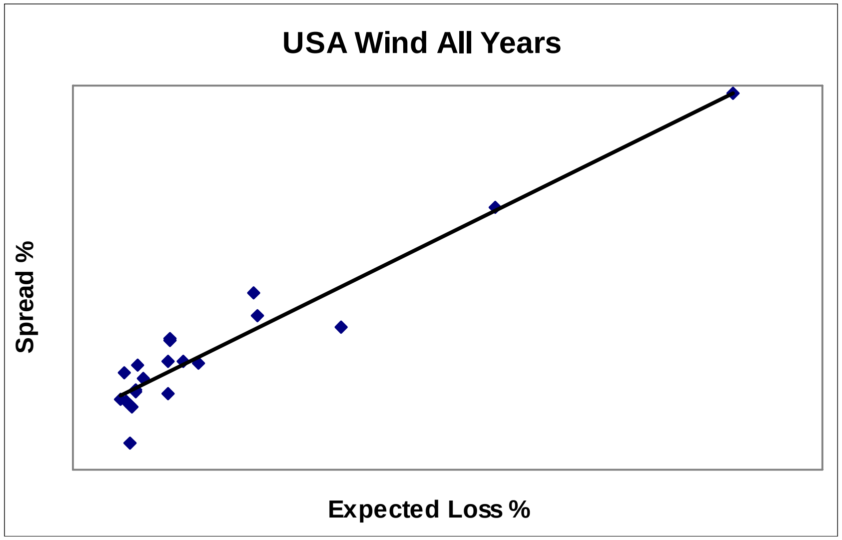
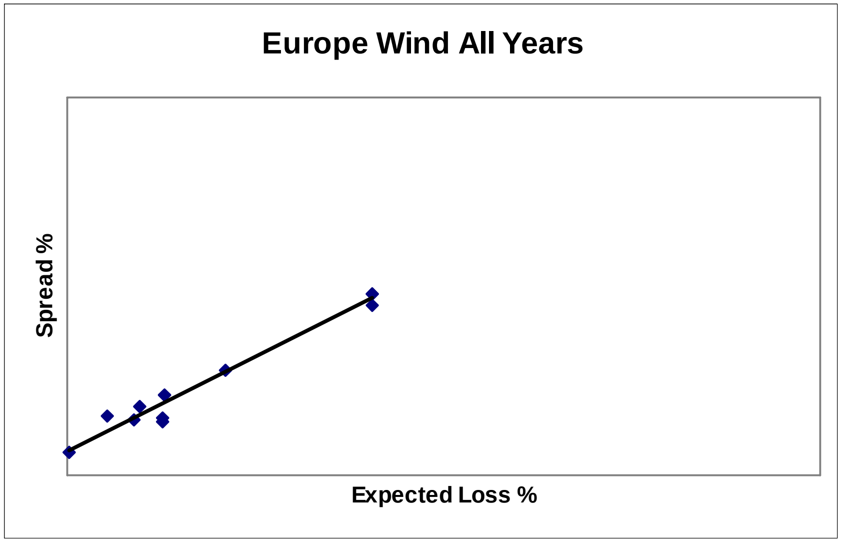
Wind: USA vs. Europe





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Wind	USA	All years	Full cycle	Loss Multiplier	2.40	0.17	2.05	2.76
Wind	Europe	All years	Full cycle	Constant %	1.61%	0.33%	0.88%	2.33%
Wind	Europe	All years	Full cycle	Loss Multiplier	2.49	0.14	2.17	2.81

Intercept for Europe is <u>lower</u> than USA; slope is <u>similar</u>.

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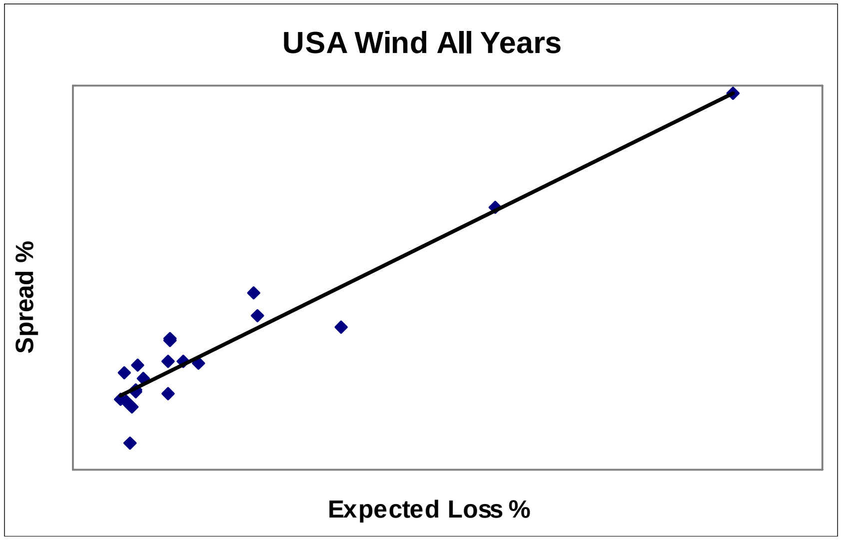
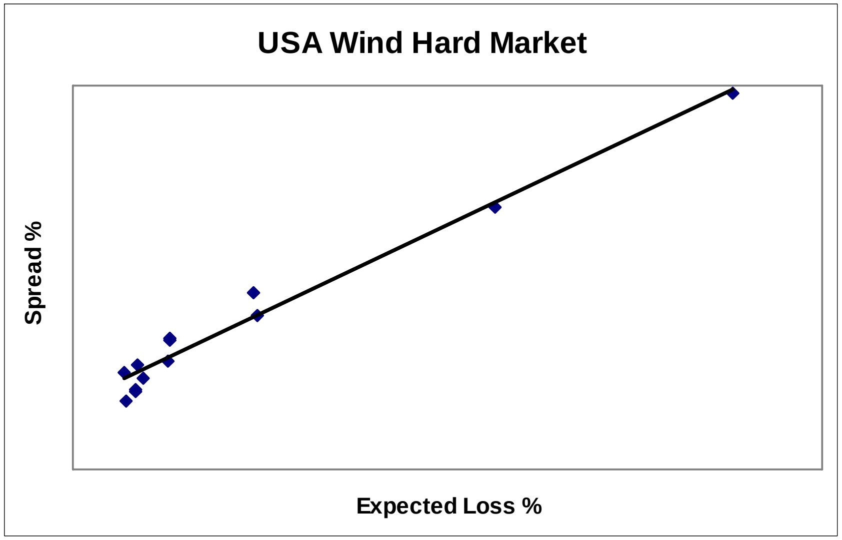
Wind: All Years vs Hard Market





Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Wind	USA	All years	Full cycle	Constant %	3.33%	0.45%	2.38%	4.27%
Wind	USA	All years	Full cycle	Loss Multiplier	2.40	0.17	2.05	2.76
Wind	USA	2006 - 2007	Hard Market	Constant %	4.28%	0.37%	3.47%	5.09%
Wind	USA	2006 - 2007	Hard Market	Loss Multiplier	2.33	0.12	2.07	2.58

Intercept for USA Wind using hard market data is <u>higher</u> than using all years; slope is <u>similar</u>.

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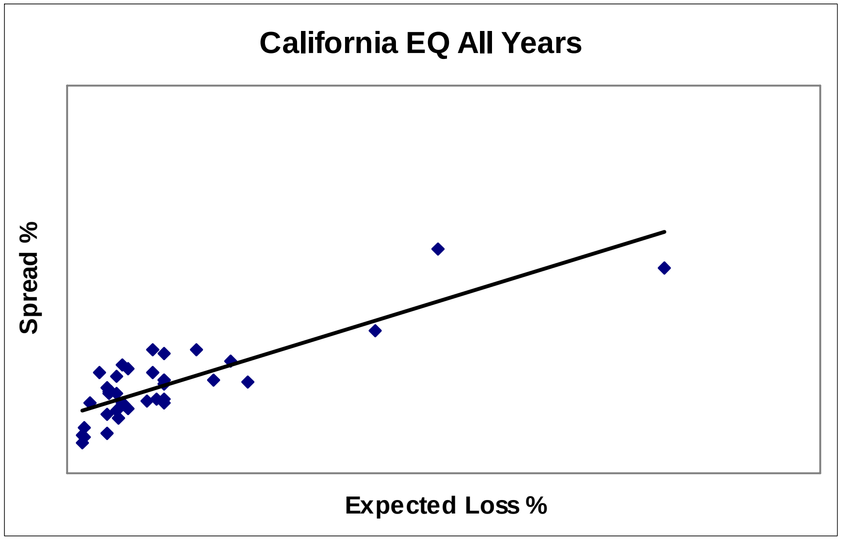
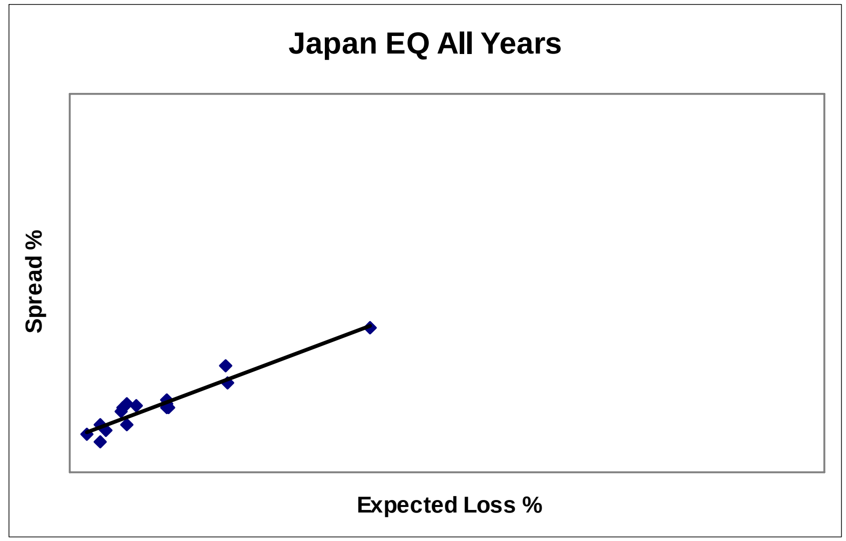
EQ: California vs Japan





Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Earthquake	California	All years	Full cycle	Constant %	3.78%	0.29%	3.19%	4.36%
Earthquake	California	All years	Full cycle	Loss Multiplier	1.48	0.16	1.16	1.79
Earthquake	Japan	All years	Full cycle	Constant %	2.28%	0.20%	1.85%	2.70%
Earthquake	Japan	All years	Full cycle	Loss Multiplier	1.85	0.12	1.60	2.10

Intercept for Japan EQ is lower than California; slope for Japan EQ is somewhat higher.

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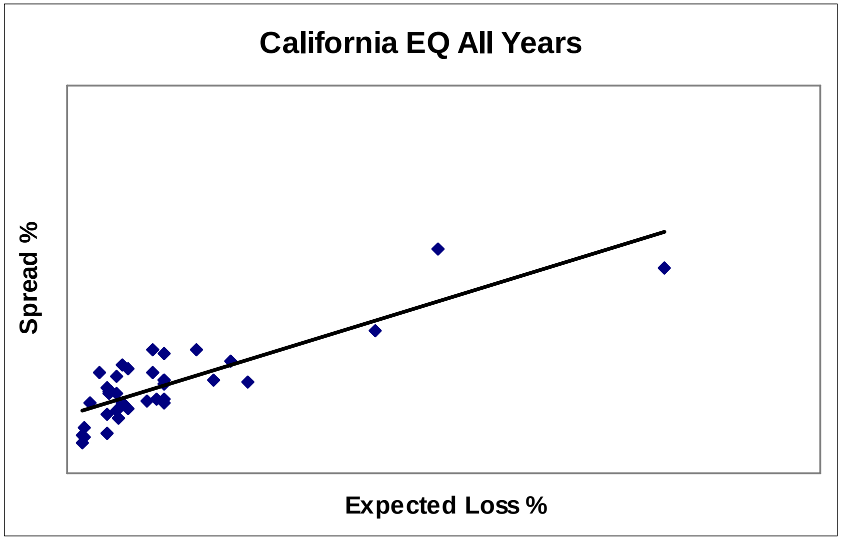
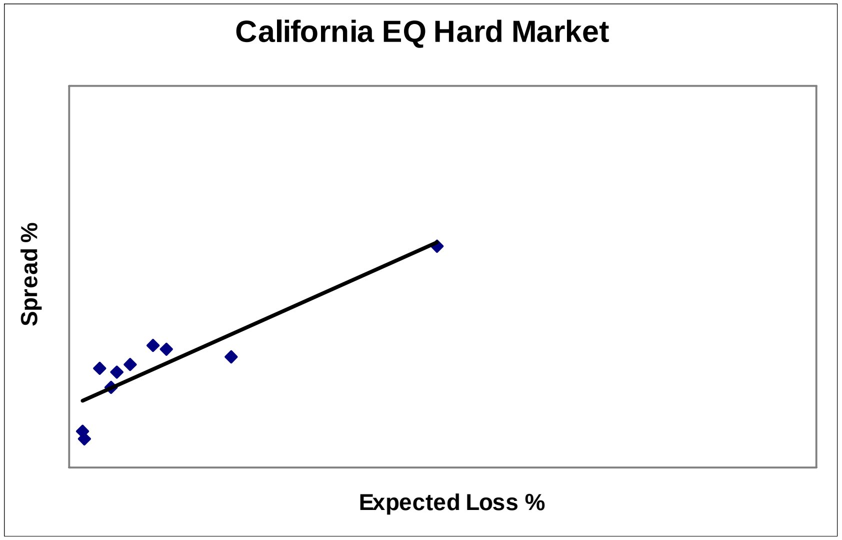
EQ: All Years vs Hard Market





Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Earthquake	California	All years	Full cycle	Constant %	3.78%	0.29%	3.19%	4.36%
Earthquake	California	All years	Full cycle	Loss Multiplier	1.48	0.16	1.16	1.79
Earthquake	California	2006 - 2007	Hard Market	Constant %	4.40%	0.55%	3.12%	5.67%
Earthquake	California	2006 - 2007	Hard Market	Loss Multiplier	2.04	0.30	1.34	2.73

Intercept for California EQ is higher using hard market data; slope is higher as well.

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Tables of Fitted Parameters

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Wind	USA	All Years	Full Cycle	Constant %	3.33%	0.45%	2.38%	4.27%
Earthquake	California	All Years	Full Cycle	Constant %	3.78%	0.29%	3.19%	4.36%
Wind	Europe	All Years	Full Cycle	Constant %	1.61%	0.33%	0.88%	2.33%
Earthquake	Japan	All Years	Full Cycle	Constant %	2.28%	0.20%	1.85%	2.70%

Intercept is similar based on whether exposure is “peak” (USA Wind, California EQ) or “non-peak” (Europe Wind, Japan EQ).

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Wind	USA	All Years	Full Cycle	Loss Multiplier	2.40	0.17	2.05	2.76
Wind	Europe	All Years	Full Cycle	Loss Multiplier	2.49	0.14	2.17	2.81
Earthquake	California	All Years	Full Cycle	Loss Multiplier	1.48	0.16	1.16	1.79
Earthquake	Japan	All Years	Full Cycle	Loss Multiplier	1.85	0.12	1.60	2.10

Slope is similar based on whether physical peril is Wind or EQ, but not based on “peak” versus “non-peak”.

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Enhancing Parsimony

- Currently we have used 8 parameters
  - 4 equations with 2 parameters each

- Similarity of some parameters suggests opportunity for enhancing parsimony

- Create “combined multiperil model”

Combined Multiperil Model

Spread % =

$constant_{All}$ %

+ $constant_{Peak}$ % * peak peril indicator variable

+ loss $multiplier_{EQ}$ * expected $loss_{EQ}$ %

+ loss $multiplier_{Wind}$ * expected $loss_{Wind}$ %

Multiperil Model Results

Using Data from All Years

Peril	Zone	Years	Market Condition	# of Observations	R Square	Adjusted R Square
Multiple	Multiple	All years	Full cycle	93	87.3%	86.9%

Healthy R Square

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Multiple	Multiple	All Years	Full Cycle	Constant_{All} %	2.31%	0.26%	1.79%	2.83%
Multiple	Multiple	All Years	Full Cycle	Additional Constant_{Peak} %	1.24%	0.28%	0.70%	1.79%
Multiple	Multiple	All Years	Full Cycle	Loss Multiplier_{EQ}	1.63	0.11	1.41	1.85
Multiple	Multiple	All Years	Full Cycle	Loss Multiplier_{Wind}	2.32	0.10	2.12	2.52

All parameters are significant

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Multiperil Model Results

Using Data from Hard Market 2006 - 2007

Peril	Zone	Years	Market Condition	# of Observations	R Square	Adjusted R Square
Multiple	Multiple	2006 - 2007	Hard Market	32	95.7%	95.3%

More homogenous data, higher R Square

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
Multiple	Multiple	2006 - 2007	Hard Market	$\text{Constant}_{\text{All}}$ %	2.07%	0.41%	1.23%	2.91%
Multiple	Multiple	2006 - 2007	Hard Market	Additional $\text{Constant}_{\text{Peak}}$ %	2.30%	0.38%	1.51%	3.09%
Multiple	Multiple	2006 - 2007	Hard Market	Loss $\text{Multiplier}_{\text{EQ}}$	1.94	0.14	1.65	2.24
Multiple	Multiple	2006 - 2007	Hard Market	Loss $\text{Multiplier}_{\text{Wind}}$	2.34	0.09	2.15	2.53

All parameters are significant

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Extending to Other Perils

- What about other perils such as Australia EQ, Mexico EQ, Japan Wind, Mediterranean EQ, etc.?
- Extend the “multiperil” combined model to an “all peril combined model”
- Assign perils to 3 buckets
  - Peak: USA Wind, California EQ
  - Non-peak (but major): Europe Wind, Japan EQ
  - Diversifying: Australia EQ, Mexico EQ, etc.

All Perils Model

Spread % =

constant_{All} %

+ constant_{Peak} % * peak peril indicator variable

+ $\text{constant}_{Diversifying}$ % * diversifying peril indicator variable

+ loss multiplier_{EQ} * expected loss_{EQ} %

+ loss multiplier_{Wind} * expected loss_{Wind} %

All Perils Model Results

Using Data from All Years

Peril	Zone	Years	Market Condition	# of Observations	R Square	Adjusted R Square
All	All	All years	Full cycle	115	87.4%	87.0%

Healthy R Square

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
All	All	All Years	Full Cycle	$\text{Constant}_{\text{All}}$ %	2.35%	0.25%	1.85%	2.85%
All	All	All Years	Full Cycle	$\text{Additional Constant}_{\text{Peak}}$ %	1.28%	0.27%	0.76%	1.81%
All	All	All Years	Full Cycle	$\text{Additional Constant}_{\text{Diversifying}}$ %	-1.09%	0.35%	-1.79%	-0.39%
All	All	All Years	Full Cycle	$\text{Loss Multiplier}_{\text{EQ}}$	1.60	0.10	1.40	1.81
All	All	All Years	Full Cycle	$\text{Loss Multiplier}_{\text{Wind}}$	2.29	0.10	2.10	2.48

All parameters are significant

Diversifying Perils' intercept equals "$\text{constant}_{\text{All}}$ %" plus the additional amount of "$\text{constant}_{\text{Diversifying}}$ %", which is negative.

Thus Diversifying Perils have a lower intercept than other perils.

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All Perils Model Results

Using Data from Hard Market 2006 - 2007

Peril	Zone	Years	Market Condition	# of Observations	R Square	Adjusted R Square
All	All	2006 - 2007	Hard Market	43	95.5%	95.1%

Healthy R Square

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
All	All	2006 - 2007	Hard Market	$Constant_{All}$ %	2.20%	0.40%	1.38%	3.02%
All	All	2006 - 2007	Hard Market	Additional $Constant_{Peak}$ %	2.31%	0.38%	1.54%	3.08%
All	All	2006 - 2007	Hard Market	Additional $Constant_{Diversifying}$ %	-1.66%	0.45%	-2.56%	-0.76%
All	All	2006 - 2007	Hard Market	Loss $Multiplier_{EQ}$	1.87	0.13	1.60	2.14
All	All	2006 - 2007	Hard Market	Loss $Multiplier_{Wind}$	2.31	0.09	2.12	2.50

All parameters are significant

All Years vs Hard Market

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
All	All	All Years	Full Cycle	Constant_{All} %	2.35%	0.25%	1.85%	2.85%
All	All	All Years	Full Cycle	Additional Constant_{Peak} %	1.28%	0.27%	0.76%	1.81%
All	All	All Years	Full Cycle	Additional $\text{Constant}_{Diversifying}$ %	-1.09%	0.35%	-1.79%	-0.39%
All	All	All Years	Full Cycle	Loss Multiplier_{EQ}	1.60	0.10	1.40	1.81
All	All	All Years	Full Cycle	Loss Multiplier_{Wind}	2.29	0.10	2.10	2.48

Peril	Zone	Years	Market Condition	Parameter Name	Parameter Value	Standard Error	Confidence Interval (95%) Lower Bound	Confidence Interval (95%) Upper Bound
All	All	2006 - 2007	Hard Market	Constant_{All} %	2.20%	0.40%	1.38%	3.02%
All	All	2006 - 2007	Hard Market	Additional Constant_{Peak} %	2.31%	0.38%	1.54%	3.08%
All	All	2006 - 2007	Hard Market	Additional $\text{Constant}_{Diversifying}$ %	-1.66%	0.45%	-2.56%	-0.76%
All	All	2006 - 2007	Hard Market	Loss Multiplier_{EQ}	1.87	0.13	1.60	2.14
All	All	2006 - 2007	Hard Market	Loss Multiplier_{Wind}	2.31	0.09	2.12	2.50

These parameters increased in absolute magnitude during the hard market (additional constant for diversifying perils became even more negative)

These parameters did not change much during the hard market

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Summary

- A linear model with peril-specific parameters:
  - compactly describes an array of data points
  - fits the historical data well
  - is straightforward to explain
  - aligns with portfolio theory
  - reflects tail downside risk
  - satisfies Venter's "no arbitrage" criterion
  - will produce the same overall price for a reinsurance tower no matter how you split into "layers" or "tranches"
  - illuminates the "credit spread puzzle"
  - measures how risk aversion waxes and wanes across the cycle

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Caveats

- Limited data points / small sample size
- Did not perform “out of sample” testing
- Only used spread data for bonds “when issued”
- Only used data for single peril bonds
- Slotting bonds into “buckets” of perils is somewhat arbitrary
- Only used standard regression and error structure

Areas for Future Research

- Expand choices of linear model and error structure (generalize the linear model)

- Include multiperil bonds in the analysis
  - do multiperil bonds suffer price penalty?
  - which choice is preferable: sponsoring one bond covering multiple perils versus sponsoring multiple bonds, each covering one peril?

- Time series model of the parameters of the linear model
  - Additional constant_{Peak} % (time t+1) =

    function {Additional constant_{Peak} % (time t), actual cat loss (time t), etc.}?

- Would similar linear model work for describing the market price of traditional reinsurance contracts?
  - need to handle reinstatement of limit and reinstatement of premium
  - how would parameters for traditional reinsurance compare / contrast to parameters for cat bonds?
  - would the different parameters highlight that certain exposures are more efficiently handled via reinsurance versus cat bonds and vice versa?
    - implications for optimizing capital structure

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Questions?

Send email to:

neil.bodoff@willis.com and yunbo.gan@willis.com