

INTRODUCTION TO EXPOSURE  
RATING

Maria M. Morrill, PhD, FCAS

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## Acknowledgement

Thanks to Halina Smosna for authoring the original presentation

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## Reinsurance XOL pricing

- XOL treaties provide a limit of coverage in excess of a ceding company's retention (e.g. \$100K xs \$100K)
- Reinsurance pricing actuaries must calculate expected loss and ALAE in the layer
- Two standard approaches taken: experience rating and exposure rating
- Focus here on exposure rating
- The expected loss & ALAE in the layer must be loaded for internal expense, commission & brokerage, profit, contingencies, loss sensitive features to get a rate
- This reinsurance ceded premium is usually expressed as percent of the ceding company's prospective subject premium

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## The burn

- Reinsurance pricing actuaries must calculate expected loss and ALAE in the layer
- The expected loss & ALAE in the layer divided by the subject premium is called the burn
- $\text{Burn} = \text{ceded loss \& ALAE} / \text{subject premium}$

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## What is exposure rating?

- Exposure rating estimates expected loss to the layer for a prospective period
- Exposure rating uses severity curves, the ceding company's limits profile and expected ground up loss ratio
- Exposure rating does NOT consider the actual client experience in the layer
- Severity distributions based on industry data are used to calculate LEVs (limited expected values)
  - The LEVs are used to estimate losses to the reinsurance layer by spreading ground up loss into the desired layer
- ELF or Excess ratios used for Workers Comp
- PSOLD Curves for Property
- There are nuances to exposure rating by LOB. Suggest attending advanced sessions.

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## Exposure rating – what info do you need?

- Prospective gross loss ratio for subject business
- Prospective subject premium
- Limit & attachment point profile with premium

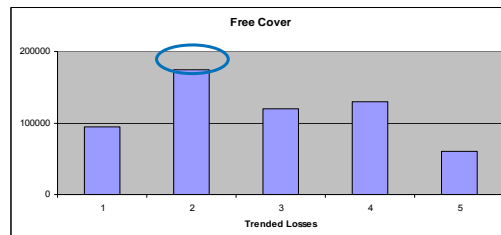
	Attachment Point	Limit	Premium
Policy A	0	300,000	10,500,000
Policy B	0	150,000	5,000,000
Policy C	0	50,000	21,500,000
<b>Total</b>			<b>37,000,000</b>

- Severity distribution/LEVs for the line of business reflecting hazard level of underlying risks (Table 123ABC, Auto) – see your UW
- In our example we assume PremOps Table 1
- Reinsurance submission data is rarely provided in the full detail corresponding to the ISO Table definitions
- The layer you are pricing—in our example \$100K xs \$100K
- No loss experience to the layer required

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## Why exposure rate?

- Complement of credibility for experience rate
- Price for 'free cover' (when the top of your layer exceeds the largest trended loss in your data)



- Experience rate is not credible
- Can use to adjust experience burns for limits drift
- Can use exposure burns to determine relativity based burns for higher layers

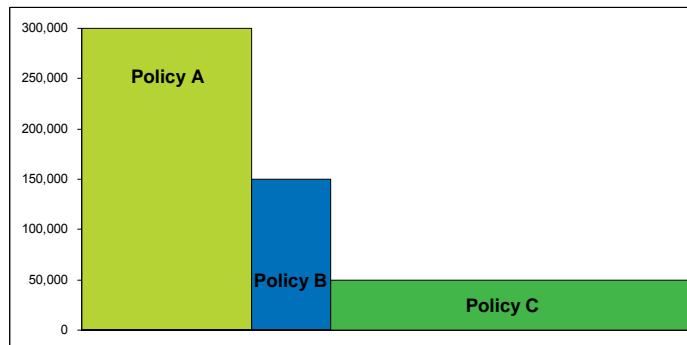
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## How does exposure rating work ?

- Calculate what percent of the total expected loss falls into your layer (exposure factor)
- This equates to:
  - Expected loss limited to the top of the layer (or policy)
  - Minus
  - Expected loss limited to the bottom of the layer
  - Divided by
  - Expected loss limited to the policy itself
- Or the ratio: ceded loss/gross loss

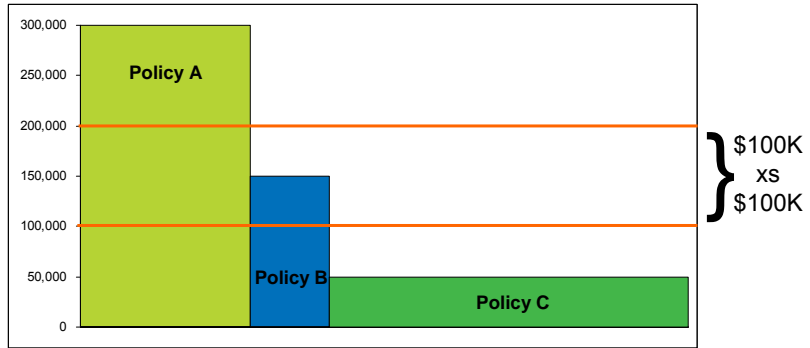
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## Visualization – limits profile



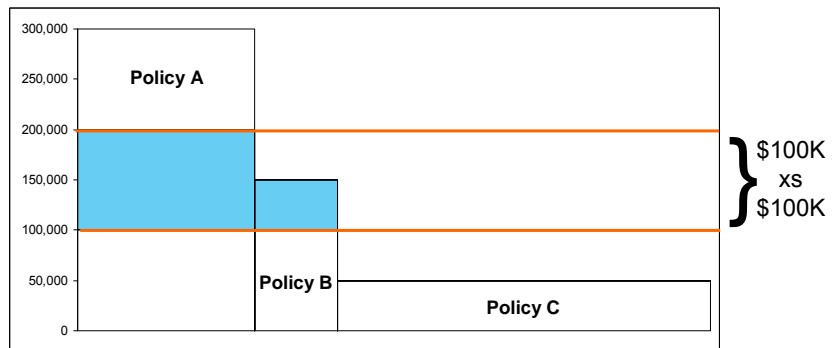
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## Visualization—reinsurance layer \$100K xs \$100K



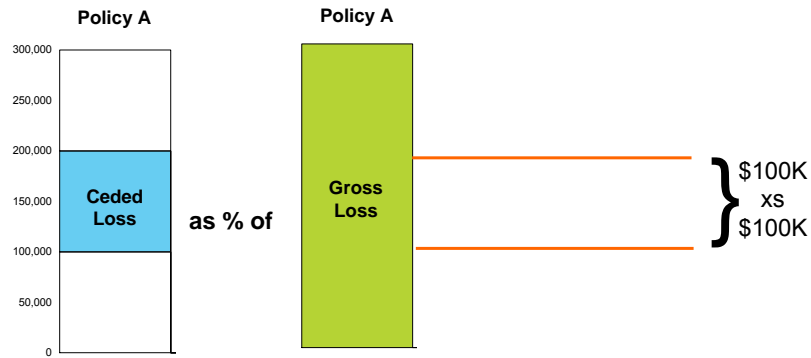
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## Visualization—ceded loss in \$100K xs \$100K layer



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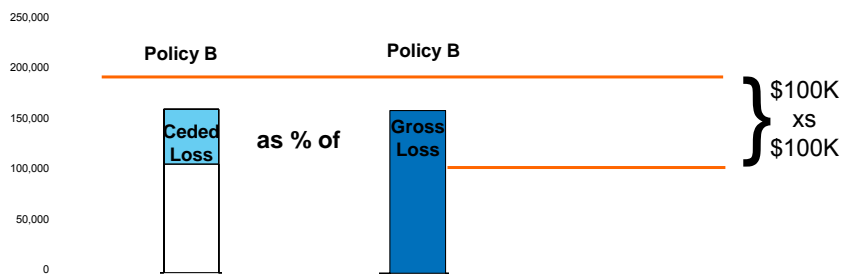
## Policy A visualization—ceded loss as % of gross loss



Ceded loss/gross loss = 20% for illustration only. Would be derived using LEVs which we will explain in a moment

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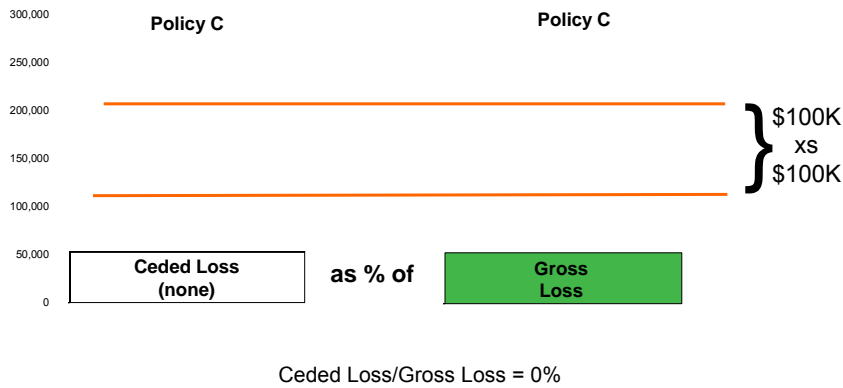
## Policy B visualization—ceded loss as % of gross loss



Ceded loss/gross loss = 16% for illustration only. Would be derived using LEVs which we will explain in a moment

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## Policy C visualization—ceded loss as % of gross loss



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## The exposure burn concept

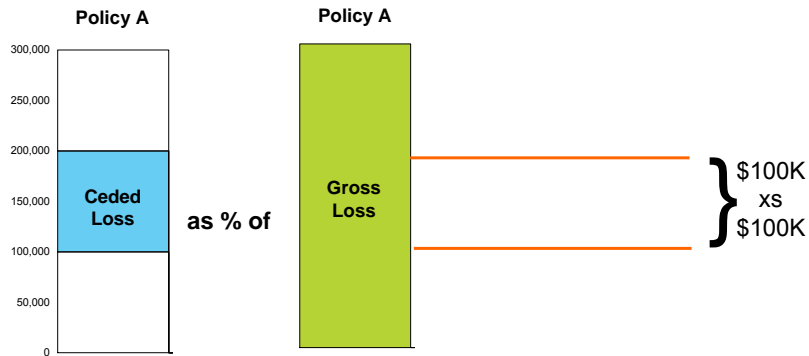
- Ceded loss/gross loss can be referred to as the **exposure factor**
- gross loss ratio x exposure factor =

$$\begin{aligned}
 & \frac{\text{gross loss}}{\text{subject premium}} \times \frac{\text{ceded loss}}{\text{gross loss}} \\
 = & \frac{\text{ceded loss}}{\text{subject premium}} = \text{exposure burn}
 \end{aligned}$$

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## Exposure burn—Policy A

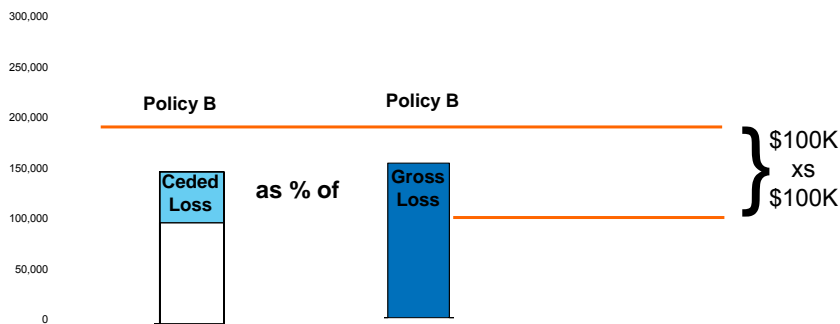


Exposure burn calculation--Policy A		
(1)	Ceded loss / gross loss =	20.0%
(2)	Gross loss ratio =	50.0%
(3)	Exposure burn = ceded loss / subject premium = (1) x (2) =	10.0%

Assume 50% projected gross loss ratio

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## Exposure burn—Policy B

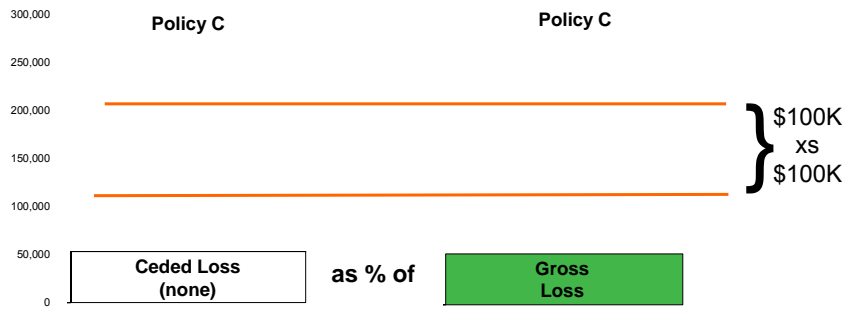


Exposure burn calculation--Policy B		
(1)	Ceded loss / gross loss =	16.0%
(2)	Gross loss ratio =	50.0%
(3)	Exposure burn = ceded loss / subject premium = (1) x (2) =	8.0%

Assume 50% projected gross loss ratio

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## Exposure burn—Policy C



Exposure burn calculation—Policy C		
(1)	Ceded loss / gross loss =	0.0%
(2)	Gross loss ratio =	50.0%
(3)	Exposure burn = ceded loss / subject premium = (1) x (2) =	0.0%

Assume 50% projected gross loss ratio

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## Exposure burn for the whole portfolio

	Burn	Premium	Loss cost
Policy A	10.00%	10,500,000	1,050,000
Policy B	8.00%	5,000,000	400,000
Policy C	0.00%	21,500,000	0
All	3.92%	37,000,000	1,450,000

Portfolio burn applied to projected treaty subject premium =  
 $3.92\% \times \$40M = \$1,568,000$

Notice the limits profile premium is \$37M because it is likely the in-force profile. The projected subject premium for the treaty is \$40M, so some growth is anticipated

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## From the burn to the rate

- So now that you have your exposure burn, what do you do?
- Credibility weight it with your experience burn or some alternate method burn to derive your selected burn
- Load your selected burn for internal expense, commission & brokerage, profit, contingencies, loss sensitive features
- This turns the burn into a rate
- Now you can quote your reinsurance rate
- Reinsurance rate \* subject premium = ceded premium

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## But wait...how did we get those exposure factors?

- Exposure factor = ceded loss / gross loss
- You need severity distributions based on industry data to calculate LEVs (limited expected values)
- The LEVs are used to estimate losses to the reinsurance layer by spreading ground up loss into the desired layer

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## What is an LEV?

- Limited Expected Value
  - The average size of loss when all losses are limited to a particular value
  - The expected loss from ground up to some limit (k)

$$LEV(k) = \int_0^k xf(x)dx + k[1-F(k)]$$

- $x$  is the severity of an individual claim
- $f(x)$  is the pdf of the severity
- $F(x)$  is the cdf of the severity

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## What is an LEV?

- Limited Expected Value

$$LEV(k) = \int_0^k xf(x)dx + k[1-F(k)]$$

- For any random variable  $x$ , one of two things can happen:
  1.  $x$  is  $\leq$  the limitation  $k$
  2.  $x$  is  $>$  the limitation  $k$
- The first part of the equation tackles (1) by calculating the expected loss limited to  $k$  when  $x \leq k$ .
- The second part of the equation tackles (2). For any  $x > k$ , you 'cap'  $x$  at  $k$ .
- The sum of (1) and (2) gives you the average ground up loss when all losses are limited to  $k$ .

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## Severity distributions—mixed exponential LEVs

- Obtained from ISO
  - For the mixed exponential (ME) you will receive the parameters, weights, and closed form formula - all you need to build your own exposure model
  - ME closed form formula:

$$LEV(x) = \sum w_j \lambda_j [1 - e^{-(x/\lambda_j)}]$$

s

ISO Mixed Exponential Parameters and Weights										
Selected Table	Weight	1	2	3	4	5	6	7	8	
2006 ISO PremOps Table 2 -- State Group B	100	Parameter $\lambda$	1,366	6,823	31,157	98,452	500,542	2,074,148	9,146,627	1
		Weight w	0.492762	0.316992	0.113027	0.056507	0.018238	0.002036	0.000438	0.000000

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## Severity distributions—5 parameter Pareto LEVs

- Also from ISO....
- For the 5 Parameter Pareto you need the parameters and closed form formula – and you can build your own exposure model
  - 5PP closed form formula:

$$LEV(x) = P*S + [(1-P)/(Q-1)] * [(B+Q*T)-(B+x)*\{(B+T)/(B+x)\}^Q]$$

for  $Q > 1$

ISO Five Parameter Pareto Parameters						
Selected Table	Weight	B	Q	P	S	T
2002 Products Table A	100	57,584	1.39	0.97	5,131	58,557

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## LEVs—Different curves for different LOBs, state groups, and/or hazard groups

### 5 Parameter Pareto

2002 Premops 1 - Multistate  
 2002 Premops 2 - Multistate  
 2002 Premops 3 - Multistate  
 2002 Premops 1 - CA  
 2002 Premops 2 - CA  
 2002 Premops 3 - CA  
 2002 Premops 1 - FL  
 2002 Premops 2 - FL  
 2002 Premops 3 - FL  
 2002 Premops 1 - IL  
 2002 Premops 2 - IL  
 2002 Premops 3 - IL  
 2002 Premops 1 - NJ  
 2002 Premops 2 - NJ  
 2002 Premops 3 - NJ  
 2002 Premops 1 - NY  
 2002 Premops 2 - NY  
 2002 Premops 3 - NY  
 2002 Premops 1 - OH  
 2002 Premops 2 - OH  
 2002 Premops 3 - OH  
 2002 Premops 1 - PA  
 2002 Premops 2 - PA  
 2002 Premops 3 - PA  
 2002 Premops 1 - TX  
 2002 Premops 2 - TX  
 2002 Premops 3 - TX  
 2002 Products A  
 2002 Products B  
 2002 Products C  
 2002 CCA Lt Grp 1  
 2002 CCA Hvy Grp 1  
 2002 CCA XHvy Grp 1

### Mixed Exponential

2003 ISO CCA Liab Lt - Grp 1  
 2003 ISO CCA Liab Hvy - Grp 1  
 2003 ISO CCA Liab XHvy - Grp 1  
 2003 ISO CCA Liab All Other - Grp 1  
 2003 ISO CCA Liab Lt - Grp 2  
 2003 ISO CCA Liab Hvy - Grp 2  
 2003 ISO CCA Liab XHvy - Grp 2  
 2003 ISO CCA Liab All Other - Grp 2  
 2003 ISO CCA Liab Lt - Grp 3  
 2003 ISO CCA Liab Hvy - Grp 3  
 2003 ISO CCA Liab XHvy - Grp 3  
 2003 ISO CCA Liab All Other - Grp 3  
 2003 ISO CCA Liab Lt - Grp 4  
 2003 ISO CCA Liab Hvy - Grp 4  
 2003 ISO CCA Liab XHvy - Grp 4  
 2003 ISO CCA Liab All Other - Grp 4  
 2003 ISO CCA Liab Lt - Grp 5  
 2003 ISO CCA Liab Hvy - Grp 5  
 2003 ISO CCA Liab XHvy - Grp 5  
 2003 ISO CCA Liab All Other - Grp 5  
 2003 ISO CCA Liab Lt - Grp 6  
 2003 ISO CCA Liab Hvy - Grp 6  
 2003 ISO CCA Liab XHvy - Grp 6  
 2003 ISO CCA Liab All Other - Grp 6  
 2003 ISO CCA Liab Zone Rated  
 2006 ISO Premops 1 - Multistate  
 2006 ISO Premops 2 - Multistate  
 2006 ISO Premops 3 - Multistate  
 2006 ISO Premops 1 - Group A  
 2006 ISO Premops 2 - Group A  
 2006 ISO Premops 3 - Group A  
 2006 ISO Premops 1 - Group B  
 2006 ISO Premops 2 - Group B  
 2006 ISO Premops 3 - Group B

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## Empirical LEV example

Claim #	Individual Loss Experience										Sum
	1	2	3	4	5	6	7	8	9	10	
Ground up loss	141,000	16,000	46,000	40,000	351,000	259,000	317,000	1,511,000	107,000	567,000	3,355,000
Loss limited at \$500K	141,000	16,000	46,000	40,000	351,000	259,000	317,000	500,000	107,000	500,000	2,277,000
Loss limited at \$1M	141,000	16,000	46,000	40,000	351,000	259,000	317,000	1,000,000	107,000	567,000	2,844,000
Loss in layer \$500K xs \$500K	0	0	0	0	0	0	0	500,000	0	67,000	567,000
LEV(\$500K)	average of losses limited to \$500K = 2,277,000 / 10 =							227,700			
LEV(\$1M)	average of losses limited to \$1M = 2,844,000 / 10 =							284,400			
LEV(\$1M) - LEV(\$500K)	284,400 - 227,700 =							56,700			
Expected value in \$500K xs \$500K	average of layer losses = 567,000 / 10 =							56,700			
Exposure factor for \$1M policy in \$500K xs \$500K layer	[ LEV(\$1M) - LEV(\$500K) ] / LEV(\$1M) = 56,700 / 2,844,000 =							20.0%			

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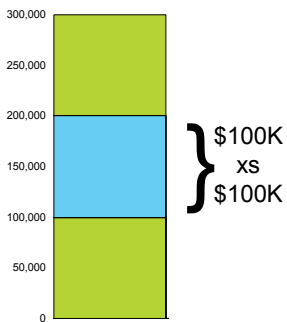
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  - Expected loss limited to the bottom of the layer
  - Divided by
  - Expected loss limited to the policy itself
- Or the ratio: ceded loss/gross loss

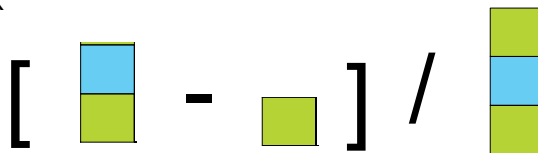
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## Policy A—\$300K policy limit

Policy A



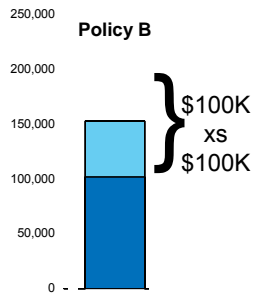
- Exposure factor = 
$$\frac{[ \text{LEV}(\$200\text{K}) - \text{LEV}(\$100\text{K}) ]}{\text{LEV}(\$300\text{K})} = \frac{[ 11,000 - 8,600 ]}{12,000} = 20.0\%$$



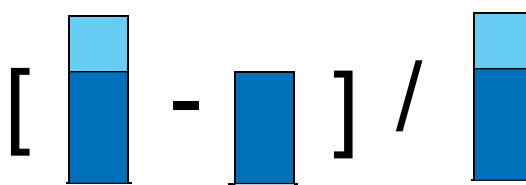
Numbers for illustration purposes only

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## Policy B—\$150K policy limit



- Exposure factor = 
$$\frac{[ \text{LEV}(\$150\text{K}) - \text{LEV}(\$100\text{K}) ]}{\text{LEV}(\$150\text{K})} = \frac{[ 10,240 - 8,600 ]}{10,240} = 16.0\%$$



Numbers for illustration purposes only

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## Appropriate LEVs – not that simple

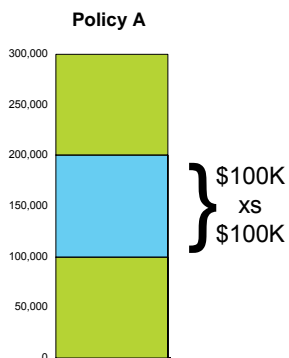
- Rare to receive data from the ceding company that maps perfectly to ISO severity distributions
- In the example below we are deriving LEVs for an Umbrella profile
  - Umbrella policies are exposed by underlying GL and AL policies
  - In exposure rating umbrella we want our LEVs based on a weighting of the underlying, varied exposures
- Your exposure rating model should have the capability of deriving LEVs based on weighting together the exposure factors from the various severity distributions

ISO Five Parameter Pareto Parameters						
Selected Table	Weight	B	Q	P	S	T
2002 PremOps Table 1 - Multistate	15	15,020	1.38	0.97	4,813	58,557
2002 PremOps Table 2 - Multistate	15	186,831	1.68	0.96	7,058	58,557
2002 CCA Hvy - State Group 2	20	378,277	1.56	0.98	6,814	18,178
2002 CCA XH - State Group 2	20	431,825	1.55	0.98	7,688	18,178
2002 Products Table B	15	271,585	1.65	0.93	10,474	58,557
2002 Products Table C	15	313,990	1.64	0.88	13,479	58,557

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## Using increased limits factors



- In absence of LEV exposure curves, ILFs can be used for exposure rating
- $ILF(x) = LEV(x) / LEV(\text{base limit})$
- Exposure factor =
  - [  $LEV(\$200K) - LEV(\$100K)$  ] /  $LEV(\$300K)$  =
  - [  $ILF(\$200K) - ILF(\$100K)$  ] /  $ILF(\$300K)$
- Caveat—ILFs may incorporate costs other than indemnity and ALAE
  - Risk load, ULAE, aggregate limits can distort values

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## Exposure rating recap

- Relevant parameters defaults/overrides for
  - LEVs (or ILFs, ELFs, PSOLD)
  - Gross loss ratio (on-level)
  - Policy profile (by LOB, hazard group)
  - Limit/attachment profile
- Adjust for expected changes in
  - Rating year policy limits/attachments
  - Rating year exposures expected to be written

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## Advantages of exposure rating

- The current risk profile is modeled
- Shifting limits and attachment points over time, which complicates the experience rating exercise, are irrelevant here
- Any excess layer can be priced
- For a new book of business (so no experience available) pro forma profiles can be used to project expected loss to the layer
- The exposure rating exercise is often easy to perform so UWs can determine an exposure rating based reinsurance rate as part of their triage process

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## Disadvantages of exposure rating

- Selected severity curves may not properly reflect the client's subject business
- Selected gross loss & ALAE ratio may not appropriately reflect exposed risks
- Data issues with the limits profile
  - Broad ranges
  - Limits profile separate from attachment point profile
  - Count based (to fix, multiply by the LEV at corresponding policy limit)
  - Premium from profile doesn't reconcile well with historical or projected premium
    - Do you have all business units, companies?
- Clash exposure not captured

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## Free cover

- In your experience rating no losses have trended into the highest portion of the layer you are pricing
- Example:
  - You are pricing a \$750K xs \$250K layer
  - Largest trended loss is \$500K from ground up
  - Your experience burn will be the same for your \$750 xs \$250K layer as for a \$250K xs \$250K layer

Loss Experience				
Trended Ground Up Claims	Loss to \$250K xs \$250K layer	Loss to \$750K xs \$250K layer	Loss to \$500K xs \$500K layer	
500,000	250,000	250,000	0	0
400,000	150,000	150,000	0	0
300,000	50,000	50,000	0	0
200,000	0	0	0	0
100,000	0	0	0	0

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## Free cover

- One solution:
  - Split the layer you are pricing (\$750K xs \$250K) into 2 pieces:
    - \$250K xs \$250K
    - \$500K xs \$500K
  - If deemed credible, use the experience burn as your selected burn for the lower part of your layer OR credibility weight your experience and exposure burns to derive your selected burn
  - Then use exposure burn relativities to derive the burn for the upper part of your layer
  - Sum the selected burns for the two parts of the layer you are pricing to derive the full layer selected burn

Layer	Experience Burn	Exposure Burn	Selected Burn
\$250K xs \$250K	10.0%	12.0%	11.0%
\$500K xs \$500K	0.0%	6.0%	5.5%
\$750K xs \$250K	10.0%	18.0%	16.5%

Where 5.5% = 11% x (6% / 12%)

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## Clash, ECO/XPL—not captured in exposure rating

- Casualty treaties often cover losses for which exposure rating does not provide an answer
  - A long-haul trucker collides with an auto
    - WC loss from trucker
    - CAL loss from individual in the auto
  - Treaty will define this as one occurrence
    - The WC claim is below the treaty retention
    - The CAL claim is below the treaty retention
    - Combined the WC & CAL claims pierce the treaty retention – Clash loss
- Exposure rating also does not estimate for ECO/XPL

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## Limits drift

- Your limits profile is shifting/driftng upwards
  - In 2010, 8% of your policies were at limits of 2M
  - Now, 13% of your policies are at 2M
  - You can use an exposure approach to adjust your AY experience burns for limits drift
- Need historical limits profiles

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## Limits drift

Historical Limits Profiles					
Year	Limit	Premium Weight	Expected loss to ILF \$1M xs \$1M Layer	Weighted Expected loss to \$1M xs \$1M layer	Limits drift factor
2010	100,000	2.0%	1.00	0.0%	1.63
	500,000	30.0%	1.10	0.0%	
	1,000,000	60.0%	1.25	0.0%	
	2,000,000	8.0%	1.70	26.5%	
2011	100,000	2.0%	1.00	0.0%	1.00
	500,000	20.0%	1.10	0.0%	
	1,000,000	65.0%	1.25	0.0%	
	2,000,000	13.0%	1.70	3.4%	

- Exposure factor =  $[ \text{ILF}(\$2\text{M}) - \text{ILF}(\$1\text{M}) ] / \text{ILF}(\$2\text{M}) = (1.7 - 1.25) / 1.7 = 26.5\%$
- Limits drift factor (2010) =  $\text{Expected loss (2011)} / \text{Expected loss (2010)} = .034 / .021 = 1.63$
- Adjust experience burn for 2010 by 1.63 factor to correct for fact that cedant is now writing more \$2M policies

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## Caveat

- Any pricing tool is only a first step towards determining adequate reinsurance premium
- Note when modeling or data assumptions are not met then try to adjust, correct, supplement

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