CARe Seminar on Reinsurance - Loss Sensitive Treaty Features

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What is a Loss Sensitive Treaty Feature?

- Definition: A provision within a reinsurance contract that causes the ceded premium, ceded loss, or commission to vary based on the loss experience of the contract.

- Why have such a feature?
  - Allows cedants to share in the ceded experience, aligning client and reinsurer incentives
  - Works to bridge the gap that may exist between the reinsurer's and cedant's view of the treaty's profitability

- The role of the reinsurance pricing actuary
  - How will this feature add to or subtract from expected profitability?
  - Does the feature make sense along with the rest of the deal structure?
  - Can you present more than one structure option to the cedant that has the same value to the reinsurer?

Types of Loss Sensitive Features

- Features that cause ceded **premium** to vary based on loss experience:
  - Reinstatement Provisions
  - Swing Rated Contracts
  - No Claims Bonus
- Features that cause ceding **commission** to vary based on loss experience:
  - Profit Commission
  - Sliding Scale Commission
- Features that cause ceded **loss** to vary based on loss experience:
  - Reinstatement Provisions
  - Annual Aggregate Deductibles (AADs)
  - Loss Ratio Corridors
  - Loss Ratio Caps

Which Reinsurance Structures Might Have these Features?

- Pro Rata / QS Treaties
  - Profit Commission
  - Sliding Scale Commission
  - Loss Corridor
  - Loss Ratio Cap
- Excess of Loss (XOL) Treaties
  - Profit Commission
  - Reinstatements
  - Swing Rating Provisions
  - No Claims Bonuses (most commonly on Cat XOLs)
  - Annual Aggregate Deductibles
  - Loss Ratio Cap

Profit Commission

- Very common loss sensitive feature for both Quota Shares and XOLs
- Cedant can receive a defined percentage of "profit" on the reinsurance contract, where **profit** is often defined as (Premium – Loss – Commission – Reinsurer's Margin)
- "50 after 10" PC with 30% ceding commission, has PC formula
  = 50% * (1-.3-.1-LR) = 50% * (.6 – LR)
- Therefore the cedant will achieve some sort of profit commission for any loss ratio result that is better than 60%
- If our expected LR is 60%, does that mean that the expected cost of the PC is zero?

Profit Commission (continued)

- Answer: No, just because there is no PC paid at the expected LR, that seldom means the expected cost of the profit commission is zero
- Put another way, the cost of the PC at the expected loss ratio is not equal to the expected cost of the PC
- Why? (A favorite question from underwriters)
  - 60% is the expected loss ratio, but that doesn't mean that every possible loss ratio outcome for this treaty is a 60%
  - There is a probability distribution of potential outcomes around that 60% expected loss ratio making it possible (and maybe even likely) that the loss ratio in any year could be far less than 60%
- Note that a PC only goes one way – the cedant receives money when the deal is running profitably; the cedant does not pay out more money when the deal isn't running profitably

Cost of PC Calculation – Specific Extreme Case

- EQ exposed California property QS
  - 40% Non-Cat Loss Ratio every year regardless of whether there is an EQ
  - 30% Cat (EQ) ELR
    - 90% chance of no EQ
    - 10% chance of EQ where resulting Cat LR = 300%
- Ceding commission = 30%
- PC terms are "50% after 10%"
- If there is NO EQ, LR = 40%, so PC value = .5*(1-.4-.3-.1) = 10%
- If there IS an EQ, LR balloons to 340%, so there is no PC
- So what is our expected cost of PC?
  - 10% PC, 90% of the time (no EQ), plus
  - 0% PC, 10% of the time (EQ)
  - Or....9% of Premium
- Because of the skewed nature of Cat, PCs are not common. If you are going to have a huge loss every 10 years, the reinsurer needs to keep as much premium as possible the other 9 years

Cost of Loss Sensitive Feature: General

- Build Aggregate Loss Distribution
  - Judgmentally select loss ratio outcomes and assign each a probability of happening
  - Fit data to either an aggregate distribution (like a lognormal) or fit frequency data separately from severity data and combine
  - Hardcore curve fitting? That's another presentation
- Apply loss sensitive terms to each point on the loss distribution or to each simulated year
- Calculate a probability weighted average cost (or savings) of the loss sensitive arrangement

Valuing the Cost of a PC of 50% after 10%, 30% Ceding Commission, 60% Expected LR

	Prob	LR	Cede	Cost of PC at LR	UW Ratio
1	4.0%	25.0%	30.0%	17.5%	72.5%
2	10.0%	35.0%	30.0%	12.5%	77.5%
3	20.0%	40.0%	30.0%	10.0%	80.0%
4	25.0%	50.0%	30.0%	5.0%	85.0%
5	20.0%	60.0%	30.0%	**0.0%**	90.0%
6	15.0%	70.0%	30.0%	0.0%	100.0%
7	2.0%	80.0%	30.0%	0.0%	110.0%
8	2.0%	145.0%	30.0%	0.0%	175.0%
9	1.0%	350.0%	30.0%	0.0%	380.0%
10	1.0%	450.0%	30.0%	0.0%	480.0%
Total	100.0%	60.0%	30.0%	**5.2%**	95.2%

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What if Your Loss Distribution is Shaped Like This?

	Prob	LR	Cede	Cost of PC at LR	UW Ratio
1	0.0%	25.0%	30.0%	17.5%	72.5%
2	1.0%	35.0%	30.0%	12.5%	77.5%
3	15.0%	40.0%	30.0%	10.0%	80.0%
4	25.0%	50.0%	30.0%	5.0%	85.0%
5	30.0%	60.0%	30.0%	**0.0%**	90.0%
6	20.0%	70.0%	30.0%	0.0%	100.0%
7	6.0%	80.0%	30.0%	0.0%	110.0%
8	3.0%	145.0%	30.0%	0.0%	175.0%
9	0.0%	350.0%	30.0%	0.0%	380.0%
10	0.0%	450.0%	30.0%	0.0%	480.0%
Total	100.0%	60.0%	30.0%	**2.9%**	92.9%

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Other Loss Sensitive Features on QSs

- Pro Rata / QS Treaties
  - Profit Commission (already covered)
  - Sliding Scale Commission
  - Loss Corridor
  - Loss Ratio Cap

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Sliding Scale Commission

- A ceding commission is set at a "provisional" level at the beginning of a contract.
- This provisional ceding commission corresponds to a certain loss ratio in the contract
- Ceding commission increases if contract's LR is lower than LR that corresponds to the provisional
- Ceding commission decreases if contract's LR is higher than LR that corresponds to the provisional
- A slide is particularly useful when the reinsurer and the insurer's loss picks differ

Sliding Scale Example

- Provisional Ceding Commission: 20%
- If the loss ratio is less than 65%, the commission increases by 1 point for each 1 point decrease in loss ratio (1:1) up to a maximum of 25% at a 60% LR
- If the loss ratio is greater than 65%, the commission decreases by 0.5 points for each 1 point increase in loss ratio (1/2:1) down to a minimum of 15% at a 75% LR

	Cede	@	LR	Cede + LR	Margin
Min	15%	@	75%	90%	10%
Prov	20%	@	65%	85%	15%
Max	25%	@	60%	85%	15%

Given a 60% ELR, is the Expected Ceding Commission 25%?

Valuing a Sliding Scale Commission

	Prob	LR	Cede	UW Ratio
1	4.0%	25.0%	25.0%	50.0%
2	10.0%	35.0%	25.0%	60.0%
3	20.0%	40.0%	25.0%	65.0%
4	25.0%	50.0%	25.0%	75.0%
5	20.0%	60.0%	**25.0%**	85.0%
6	15.0%	70.0%	17.5%	87.5%
7	2.0%	80.0%	15.0%	95.0%
8	2.0%	145.0%	15.0%	160.0%
9	1.0%	350.0%	15.0%	365.0%
10	1.0%	450.0%	15.0%	465.0%
Total	100.0%	60.0%	**23.3%**	83.3%

No, as with the Profit Commission, the expected commission is not equal to commission at the ELR.

Loss Ratio Corridor

- A loss ratio corridor is a provision that forces the ceding company to retain losses that would otherwise be ceded to the reinsurance treaty
- Useful when there is a difference in LR pick, but not nearly as common as a slide
- For example, the ceding company could keep 100% of the losses between a 75% and 85% loss ratio – or a "10 point corridor attaching at 75%"
  - Gross loss ratio = 75% -> Ceded loss ratio = 75% (no corridor attaches)
  - Gross loss ratio = 80% -> Ceded loss ratio = 75%
  - Gross loss ratio = 85% -> Ceded loss ratio = 75%
  - Gross loss ratio = 90% -> Ceded loss ratio = 80%

Loss Ratio Cap

- This is the maximum loss ratio that could be ceded to the treaty
- No impact if the loss ratio is below the cap
- Useful for new / start up operations where the limit to premium ratio may be unbalanced
  - New Umbrella program offering $10M policy limits but only plans on writing $3M in premium the first year
  - May be the only way for such a reinsurance treaty to get placed, particularly on start up business - while the cap is generally high, at least the deal downside is limited…

Determining an Aggregate Distribution – 3 Methods

- Judgmentally select loss ratio outcomes and corresponding probabilities whose weighted average equals your expected loss ratio
  - May be the easiest to explain to underwriters
  - May not properly reflect variability if based on experience
- Fit statistical distribution to on level loss ratios
  - Reasonable for Pro Rata (QS) Treaties
  - Lognormal is most common distribution actuaries use here
    - Loss Ratios are assumed to follow a lognormal distribution: natural log of loss ratios are normally distributed
    - Reflects skewed distribution of loss ratios
    - Central Limit Theorem suggests if underlying factors interact multiplicatively, results will be lognormally distributed
- Determine an aggregate distribution by modeling the frequency and severity pieces separately and either convolute them or simulate them together
  - Typically used for excess of loss (XOL) treaties
  - Lognormal doesn't make sense if you can have zero losses
  - Lognormal likely not skewed enough anyway; XOL can be "hit or miss"

Lognormal Distribution

- Fitting the lognormal
  - $\sigma^2 = \ln(CV^2 + 1)$
  - $\mu = \ln(\text{mean}) - \sigma^2 / 2$
  - Mean = Selected Expected Loss Ratio
  - CV = Standard Deviation over the Mean of the loss ratio (LR) distribution Parameter
- Prob (LR $\leq$ X) = Normal Dist{(ln(x) - μ)/ σ}
  - I.e., look up { ln(x) - μ)/ σ} on a standard normal distribution table

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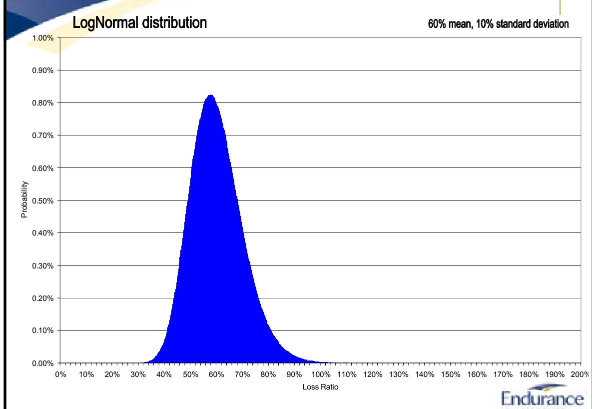

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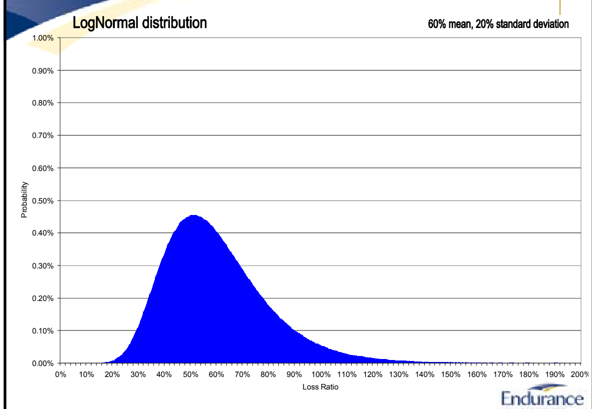

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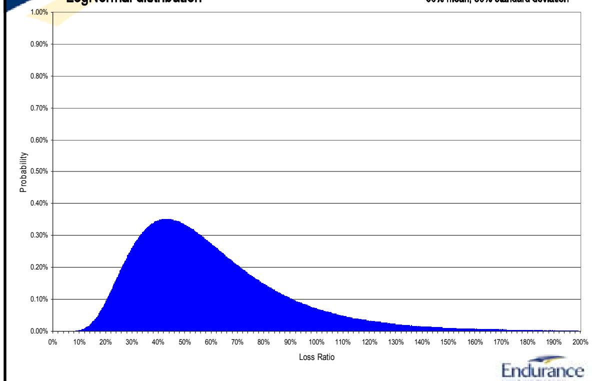
LogNormal distribution

60% mean, 30% standard deviation



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Is the Resulting Distribution Reasonable?

- Compare resulting distribution to historical results
  - On-level loss ratios should be the focus, but don't completely ignore untrended ultimate loss ratios
  - Consider comparing modeled 10th and 90th Percentile events to corresponding actual results
- On-Level Loss Ratios may not reflect cat or shock loss potential
- Are historical results are predictive of future results?
- Discuss distribution with Underwriters

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Process and Parameter Uncertainty?

- Process Uncertainty is the random fluctuation of results around the expected value
  - Unbiased, but often skewed to the downside
- Parameter Uncertainty is the fluctuation of results because our ELR selection is imperfect
  - Potential errors in trend, rate change, and loss development assumptions
  - For this book, are past results a good indication of future results?
    - Changes in mix of business
    - Changes in management or philosophy
    - Is the book growing? Shrinking? Stable?
- Selected CV should generally be above historical
  - 5 to 10 years of data does not reflect a full range of possibilities
  - Survivorship bias

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Addressing Parameter Uncertainty: One Approach

- Instead of just choosing one Expected Loss Ratio, choose several
- Assign weights to the new ELRs so that they all weight back to your original ELR
  - For example, if your ELR is a 60%, assign a 1/3 chance your true mean is 50%, 1/3 chance it is 60%, and 1/3 chance it is 70%
    - Simulate the true mean by randomly choosing between the 50%, 60%, and 70%.
    - Once you've randomly chosen the mean (either 50%, 60%, or 70%) then model using the lognormal with your selected CV
    - Note the CV accounts for process variance

Creating Distributions With Cat Exposure

- If you have a treaty with significant catastrophe exposure, model the non-cat loss ratio separately from the cat loss ratio
  - Non-cat loss can be modeled as above (e.g., using a lognormal)
  - Cat loss is usually much more skewed
    - Commercial catastrophe models produce distributions useable for simulations
  - Simulate cat and non-cat separately
  - In the case of non-cat and cat, it's very difficult to find one distribution to address all your needs
    - Lognormal with high CV?

Using a Lognormal to Model Convoluted Cat and Non-Cat Distribution

- A Lognormal with a high CV will produce high LR events, as needed for an account with cat exposure
  - But, a high CV will also lead to unrealistically high probabilities of low loss
  - Truncating the LR below a certain minimum (say 30%) would prevent this
    - Need to lower mean LR in lognormal distribution so that aggregate distribution balances back to selected ELR
  - Review Resultant distribution to make sure it fits your prior expectations

Loss Sensitive Features on XOLs

- Excess of Loss (XOL) Treaties
  - Profit Commission (already covered)
  - Swing Rating Provisions
  - No Claims Bonuses (if anywhere, Cat XOLs)
  - Reinstatements
  - Annual Aggregate Deductibles
  - Loss Ratio Cap

Swing Rating Provisions

- Ceded premium is dependent on loss experience
  - Reinsurer receives initial premium based on a provisional rate
  - That rate swings up or down depending on the loss experience in accordance to the terms of the contract
- Typical Swing Rated Terms
  - Provisional Rate = 10%
  - Minimum Rate/Margin = 3%; Maximum Rate = 15%
  - "Losses Loaded" at = 1.1
  - Ceded Rate = Min/Margin + (Ceded Loss / SPI) * (1.1), subject to the max rate of 15%
- Common on medical malpractice XOLs, but not really seen anywhere else

Swing Rating Example

Swing Rated Contract

Min/Margin = 3%; Losses Loaded at 1.1; Max = 15%

	Prob	Burn	Burn x 1.1	Final Rate	LR
	48.5%	0.0%	0.0%	3.0%	0.0%
	20.0%	5.0%	5.5%	8.5%	58.8%
	19.5%	7.5%	8.3%	11.3%	66.7%
	7.0%	25.0%	27.5%	15.0%	166.7%
	5.0%	35.0%	38.5%	15.0%	233.3%
Total	100.0%	6.0%	6.6%	7.1%	83.4%

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Annual Aggregate Deductible

- The annual aggregate deductible (AAD) refers to layer losses that the cedant retains that would otherwise be ceded to the treaty
- Example: Reinsurer provides a $500,000 xs $500,000 excess of loss contract. The cedant retains an AAD of $750,000
  - The cedant keeps the first $750,000 of layer losses
  - For example, if total Loss to Layer = $500,000
    - Cedant retains entire $500,000
    - No loss is ceded to reinsurers
  - If Total Loss to Layer = $1M
    - Cedant retains entire AAD of $750,000
    - Reinsurer pays $250,000
  - If the cedant requests a $500,000 AAD for a treaty, would the expected layer losses decrease by $500,000?

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Valuing an AAD

$1M x $1M Layer
AAD = $500,000

	Prob	Loss to Layer	After AAD	AAD Savings
1	48.5%	-	-	-
2	20.0%	1,000	500	500
3	19.5%	2,000	1,500	500
4	7.0%	3,000	2,500	500
5	5.0%	4,000	3,500	500
Total	100.0%	1,000	743	258

As with all of these examples, different shaped distributions will result in different savings

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No Claims Bonus

- A No Claims Bonus provision can be added to an excess of loss contract
  - Any pro rata or QS contract is very likely to have loss ceded to it because these structures cover losses of all sizes – not just large losses – so a no claims bonus doesn't make sense
- Very binary – if there are no losses, cedant can receive a small % of premium back
- Not a typical feature to see – might see a small no claims bonus on Property Catastrophe XOLs – but usually around a 10% bonus

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Limited Reinstatement Provisions

- Many excess of loss treaties have reinstatement provisions. Such provisions dictate how many times the cedant can use the risk limit of the treaty.
  - Reinstatements can be free or paid – but choosing to reinstate is almost always mandatory
    - Reinstatement premium can vary and is usually between 50% and 150% of the initial reinsurance premium
- Limited reinstatements are an implied treaty aggregate limit, or treaty cap.
- Example: $1M xs $1M layer with one reinstatement
  - After the cedant uses up the first $1M limit, they get a second limit
  - Treaty Aggregate Limit = $1M * (1+1) = $2M
  - Contract will indicate any additional premium to be paid when the limit is reinstated
- Reinstatement premium can simply be viewed as additional premium that reinsurers receive depending on loss experience

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Limited Reinstatement Example 1

$1M xs $1M Layer
1 reinstatement paid at 100%: Pro-rata as to amount, 100% as to Time
Upfront Ceded Premium = $200,000

	Simulated Year 1			Simulated Year 2		
Loss #	Ground Up Loss Amount	Ceded Loss Amount	Reinst Prem	Ground Up Loss Amount	Ceded Loss Amount	Reinst Prem
1	$2M	$1M	$200K	$1.5M	$500K	$100K
2	$2M	$1M	-	$1.5M	$500K	$100K
3	$2M	-	-	$1.5M	$500K	-

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Limited Reinstatement Example 2

$1M xs $1M Layer
1 reinstatement paid at 50%; 1 at 100%: Pro-rata as to amount, 100% as to Time
Upfront Ceded Premium = $200,000

	Simulated Year 1			Simulated Year 2		
Loss #	Ground Up Loss Amount	Ceded Loss Amount	Reinst Prem	Ground Up Loss Amount	Ceded Loss Amount	Reinst Prem
1	$3M	$1M	$100K	$1.5M	$500K	$50K
2	$2M	$1M	$200K	$1.5M	$500K	$50K
3	$2M	$1M	-	$1.5M	$500K	$100K
4	$2M	-	-			

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Valuing a Limited Reinstatement Provision

$1M x $1M Layer
1 reinstatement paid at 100% - Pro rata as to amount, 100% as to time
Upfront Ceded Premium = $300,000

	Prob	Loss to Layer	Losses after limitation	Upfront Premium	Reinst. Premium	Total Prem	LR
1	75.0%	-	-	300	-	300	
2	15.0%	1,000	1,000	300	300	600	
3	5.0%	2,000	2,000	300	300	600	
4	3.0%	3,000	2,000	300	300	600	
5	2.0%	4,000	2,000	300	300	600	
Total	100.0%	420	350	300	75	375	93%

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Rating on a Multi Year Block

- Each of the structures presented thus far covers a single year
- For a PC, the cedant can have a great 1st year and receive a large profit commission in return. If the 2nd year is much worse, the cedant will pay no PC.
  - Over the 2 years, the cedant may have made a significant profit while the reinsurer has lost money.
- Loss sensitive features can be evaluated using the total treaty experience across multiple years instead. This allows for a smoothing of results – and a smoothing of profit commission paid.
- This is called rating on a Multi Year Block
- Process risk decreases with more years
- Parameter risk increases
  - More difficult to rate years further into the future
  - Individual years are likely to be correlated

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Deficit / Credit Carryforward Provision

- Another way to effect loss sensitive smoothing, is to use a Deficit or Credit Carryforward Provision
- If the loss ratio is so good (bad) that the cedant receives the max (min) ceding commission, the amount that the LR is better than the loss ratio at the max rolls into the next year's calculation. This is a credit (deficit) carryforward.
- Similar to a multi year block, this provision works to smooth out loss sensitive results
- Apply the expected impact of a Carryforward with caution
  - Treaty terms may change or treaty may be terminated before the benefit of the deficit carry forward is felt by the reinsurer
  - The reinsurer with a deficit could be replaced by new reinsurer.

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Excess of Loss Contracts: Separate Modeling of Frequency and Severity

- Used primarily for modeling excess of loss contracts as Loss Ratio distribution is usually inappropriate for XOL contracts
  - Generally understates the probability of zero loss
  - May understate the potential of losses much greater than the expected loss
- Most aggregate distribution approaches assume that frequency and severity are independent
- Different Approaches
  - Simulation (Our focus)
  - Numerical Methods (Beyond the scope of this presentation)
    - Heckman Meyers – Fast calculating approximation to aggregate distribution
    - Panjer Method –
      - Select discrete number of possible severities (i.e. create 5 possible severities with a probability assigned to each)
      - Convolutes discrete frequency and severity distributions

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Common Frequency Distributions

- Poisson is an easy-to-use distribution to model expected claim count
  - Poisson distribution assumes the mean (lambda) and variance of the claim count distribution are equal
  - Discrete distribution – number of claims = 0, 1, 2, 3, etc…
- Despite the Poisson's ease of use, Negative Binomial preferred
  - Same form as the Poisson expect that lambda is no longer considered fixed but rather has a gamma distribution around lambda
  - Variance is greater than the mean (unlike Poisson where they are equal)
  - Reflects some parameter uncertainty regarding the true mean claim count
  - The extra variability of the Negative Binomial is more in line with historical experience

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Poisson Distribution

- Poisson
  - $f(x|\lambda)$ is the probability of x losses, given a mean claim count of λ
  - $f(x|\lambda) = \lambda^x * e^{-\lambda} / x!$

    where λ = mean of the claim count distribution and x = claim count = 0,1,2,...
  - Poisson distribution assumes the mean and variance of the claim count distribution are equal.

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Fitting a Poisson Claim Count Distribution

- Trend claims from ground up and slot to reinsurance layer
- Estimate ultimate claim counts by developing trended claims to layer
- Multiply trended claim counts by frequency trend factor to bring them to the frequency level of the upcoming treaty year
- Adjust for change in exposure levels
  - Prospective premium in treaty year to On-Level Premium in historical year
- Indicated Poisson parameter λ equals the mean of the ultimate, trended, adjusted claim counts from above

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Example of Indicated Claim Count

Year	SPI at 2011 Rate Level	Trended Counts to Layer	Count Devel Factor	Est Ult Trended Count	Annual Freq Trend	Freq Trend to 2011	Trended Ult Claim Count	(Note) Exposure Adj Factor	2011 Level Claim Count
2001	10,000	2.0	1.0	2.0	0.0%	1.104	2.21	1.60	3.53
2002	10,500	1.0	1.0	1.0	0.0%	1.104	1.10	1.52	1.68
2003	11,025	1.0	1.0	1.0	0.0%	1.104	1.10	1.45	1.60
2004	11,576	1.0	1.1	1.1	0.0%	1.104	1.16	1.38	1.60
2005	12,155	3.0	1.1	3.3	0.0%	1.104	3.64	1.32	4.80
2006	12,763	-	1.2	-	0.0%	1.104	-	1.25	-
2007	13,401	-	1.3	-	2.0%	1.082	-	1.19	-
2008	14,071	-	1.5	-	2.0%	1.061	-	1.14	-
2009	14,775	1.0	2.0	2.0	2.0%	1.040	2.08	1.08	2.25
2010	15,513	1.0	3.5	3.5	2.0%	1.020	3.57	1.03	3.68
2011	16,000				2.0%				
							Average:		1.92
							Variance:		2.82
Note: Exposure Adj Factor Yr i = 2011 SPI / SPI year i							Selected Variance:		3.11

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Modeling Frequency- Negative Binomial

- Negative Binomial: Same form as the Poisson distribution, but rather than a fixed λ, uses a gamma distribution around the selected λ
  - Claim count distribution is Negative Binomial if the variance of the count distribution is greater than the mean
  - The Gamma distribution around λ has a mean of 1
  - Reflects some parameter uncertainty regarding the true mean claim count
  - The extra variability of the Negative Binomial is more in line with historical experience
- Negative Binomial simulation
  - Simulate λ (Poisson expected count)
  - Using simulated expected claim count, simulate claim count for the year

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Negative Binomial Contagion Parameter

- Determine contagion parameter, c, of claim count distribution:
  - $(\sigma^2 / \mu) = 1 + c * \mu$
  - If the claim count distribution is Poisson, then c=0
  - If it is negative binomial, then c>0, i.e. variance is greater than the mean
- Solve for the contagion parameter:
  - $c = [(\sigma^2 / \mu) - 1] / \mu$

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Additional Steps for Simulating Claim Counts using Negative Binomial

- Simulate gamma random variable with a mean of 1
  - Gamma distribution has two parameters: α and β
    - $\alpha = 1/c$
    - $\beta = c$
    - c = contagion parameter
- Simulated Poisson parameter =
  - $=\lambda$ * Simulated Gamma Random Variable Above
- Use the Poisson distribution algorithm using the above simulated Poisson parameter, λ, to simulate the claim count for the year

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1 Instance of Simulated Negative Binomial Claim Count

(A)	Selected Mean Claim Count (Poisson Gamma)	1.92
(B)	Selected Variance of Claim Count Distribution	3.11
(C)	Contagion Parameter [(Variance / Mean -1) / Mean]	0.32
(D)	Gamma Distribution Alpha	3.08
(E)	Gamma Distribution Beta	0.32
(F)	Simulated Gamma CDF	0.412
(G)	Simulated Gamma Random Variable	0.78
(H)	Simulated Poisson Mean (A) X (G)	1.50

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1 Instance of Simulated Negative Binomial Claim Count

Simulated Poisson Mean					1.50
Simulated Poisson CDF:					0.808
Year 1 Simulated Claim Count:					2

Claim Count	Poisson Probability	Prob Count <= X	Clain Cour	Poisson Probability	Prob Count <= X
0	22.39%	22.39%	5	1.40%	99.56%
1	33.51%	55.90%	6	0.35%	99.91%
2	25.07%	80.97%	7	0.07%	99.98%
3	12.51%	93.48%	8	0.01%	100.00%
4	4.68%	98.16%	9	0.00%	100.00%

Endurance

Modeling Severity – Common Severity Distributions

- Lognormal
- Mixed Exponential
  - Currently used by ISO
  - Focus of our examples
- Pareto
- Truncated Pareto

Endurance

Algorithm for Simulating Severity to the Layer

- For each loss to be simulated, choose a random number between 0 and 1. This is the simulated CDF
- Transformed CDF for losses hitting layer (TCDF) =
  = Prob(Loss < Reins Att. Pt) +
  + Simulated CDF * Prob (Loss > Reins Att. Pt)
  - If there is a 95% chance that loss is below attachment point, then the transformed CDF (TCDF) is between 0.95 and 1.00
- Find simulated ground up loss, x, that corresponds to simulated TCDF
- From simulated ground up loss calculate loss to the layer

Endurance

Mixed Exponential

- Exponential Distribution
  - $F(x) = 1 - e^{-x * \lambda}$
  - Mean = $1/\lambda$
- Mixed Exponential Distribution
  - $F(x) = 1 - \sum_{i=1}^{n} w_i * e^{-x \lambda_i}$
  - w_i = weight to exponential i
- For our example, we'll use the following simple mixed exponential
  - $w_1 = .2$; $w_2 = .6$; $w_3 = .2$
  - $\mu_1 = \$10,000$; $\mu_2 = \$100,000$; $\mu_3 = \$1,000,000$

Endurance

1 Instance, 1st Loss Simulated Severity to the Layer

	1	2	3
Weight	20%	60%	20%
Lambda	0.0001	0.00001	0.000001
Mean	$10,000	$100,000	$1,000,000
Reinsurance Layer	$750,000	xs	$250,000
Probability of Loss < Attachment Point			79.5%
Simulated CDF			0.4029
Transformed CDF for Losses Simulated to the Excess Layer			0.8776
Simulated Loss			$ 518,699
Simulated Loss to Layer			$ 268,699

Endurance

1 Instance, 2nd Loss Simulated Severity to the Layer

	1	2	3
Weight	20%	60%	20%
Lambda	0.0001	0.00001	0.000001
Mean	$10,000	$100,000	$1,000,000
Reinsurance Layer	$750,000	xs	$250,000
Probability of Loss < Attachment Point			79.5%
Simulated CDF			0.8400
Transformed CDF for Losses Simulated to the Excess Layer			0.9672
Simulated Loss			$ 1,807,835
Simulated Loss to Layer			$ 750,000

Endurance

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Simulation Summary

	Claim Count	Loss to Layer
Instance 1 Simulation	2	268,699
		750,000
	Total:	1,018,699
Instance 2 Simulation	3	576,745
		281,323
		54,726
	Total:	912,794

Run about 1,000 more years and we have our aggregate distribution to the excess of loss layer

Endurance

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Additional Issues & Uses of Aggregate Distributions

- Correlation between lines of business
  - Often higher than you might think due to directives from upper management influencing multiple lines of business
- Reserving for loss sensitive treaty terms
- Some companies use aggregate distributions to measure risk & allocate capital
  - For example, a company could set the capital assigned to a contract at the 99th percentile of Discounted Loss * Correlation Factor
- Fitting Severity Curves: Don't Ignore Loss Development
  - Increases average severity
  - Increases variance – claims spread as they settle
  - See "Survey of Methods Used to Reflect Development in Excess Ratemaking" by Stephen Philbrick, CAS 1996 Winter Forum

Endurance

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Risk Transfer – Governing Regulations

- Topic 944 (formerly known as FASB 113): A reinsurance contract should be booked using deposit accounting unless:
  - "The reinsurer assumes significant insurance risk"
    - Insurance risk not significant if "the probability of a significant variation in either the amount or timing of payments by the reinsurer is remote"
  - "It is reasonably possible that the reinsurer may realize a significant loss from the transaction"
    - 10/10 Rule of Thumb: Is there a 10% chance that the reinsurer will have a loss of at least 10% of premium on a discounted basis
      - Calculation excludes brokerage and reinsurer internal expense
- Statutory Statements
  - SSAP 62 is governing document: requirements are similar to above
  - Also requires CEO's and CFO's attestation under penalty of perjury that
    - No side agreements exist that alter reinsurance terms
    - For contracts where risk transfer is not self-evident, documentation concerning economic intent and risk transfer analysis is available
    - Reporting entity in compliance with SSAP 62 and proper controls are in place

Endurance

Report of 2005 CAS Working Party on Risk Transfer – Key Findings

- Three step risk transfer testing process
  - Does contract transfer substantially all risk of ceding company? If so, no testing is required
    - Is reinsurer's risk position the same as the ceding companies?
  - Is risk transfer reasonably self evident? If yes, stop
    - Facultative, Cat XOL, XOL contracts without significant loss sensitive features, and contracts with immaterial premium (less than $1 mil of premium or 1% of GEP)
  - Remaining contracts: Perform risk transfer testing
    - Calculate recommended risk metric and compare to critical threshold
    - Aggregate distribution should contemplate process and parameter uncertainty
    - Recommend that 10/10 rule be replaced with Expected Reinsurer Deficit Calculation (ERD)
      - 10/10 inappropriate for low frequency high severity treaties like Cat XOLs
- Above are only CAS's working party recommendations. Actual procedures and methods are determined by company management and accounting firm

Expected Reinsurer Deficit (ERD) Example

Reinsurance Layer: 50 xs 50		
Ceded Premium:		10
(amounts in millions)		
Loss to Layer	Prob	Present Value of Reinsurer Result
-	93.0%	10
50	3.5%	(35)
100	2.0%	(80)
150	1.5%	(125)

- ERD = p * T / Premium
  - p = Probability of loss to reinsurer = 7%
  - T = Average Severity of Discounted Loss given a loss occurred
    = (3.5% * 35% + 2% * 80 + 1.5% * * 125) / 7% = 67.1
  - ERD = 7% * 67.1 / 10 = 47%
- CAS Working Party implied a standard that ERD must be above 1%, which at a minimum equates to the 10/10 rule, though this standard is less conservative

Concluding Remarks

- Aggregate distributions are a critical element in evaluating the profitability of business and cost/savings can vary greatly depending on their shape
- Distributions are frequently produced by (re)insurers as a risk management tool
- Critical to effectively communicate the difficulties in projecting aggregate distributions of future results
  - Regulators, Accountants, and Underwriters need to be aware of the degree of parameter uncertainty, especially when unmodeled