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LARGE LOSS TREND VIA PARAMETRIC MODEL

CAS Seminar on Reinsurance 2012

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Agenda

1. Introduction
2. Parametric Model for Large Loss Trend
3. Advantages & Disadvantages
4. Example

Introduction: Trend on Aggregate Losses

Trend is usually calculated on aggregate losses by dividing total losses by total claim counts.

- Calendar Year basis – all losses closed in a given year
- Report Year basis – all losses reported in a given year
- Accident Year basis – all losses occurring in a given year

If we are estimating a long-term average trend, then all the methods should produce similar results.

We also generally assume that the same inflation trend applies to all sizes of loss.

Question is easy to ask: *What trend factor should be applied to large losses?*

But it is difficult to answer!

- Credibility of large losses due to high skewness
- Data quality
- Impact of policy limits and excess attachment points
- Handling case reserves and loss IBNER
- Interaction of frequency trend and severity trend

Outline of the approach:

We will assume that losses come from a continuous distribution, the shape of which is constant over time but the scale changes with an inflation trend.

The parameters of this continuous distribution, including this trend, are what we will try to estimate.

Continuous Distributions

Pareto: $$F(x) = 1 - \left(\frac{B}{B+x}\right)^{Q} = 1 - \left(1 + \frac{x}{B}\right)^{-Q}$$

*Scale parameter **B**, in dollar units.*

*Shape parameters, **Q** and **ω**, unaffected by change in scale.*

Weibull: $$F(x) = 1 - exp\left(-\left(\frac{x}{B}\right)^{\omega}\right)$$

Continuous Distributions with trend on Scale Parameter

Allow the “scale” parameter to change each year based on a constant trend factor, “*g*”, while holding the “shape” of the distribution the same over time.

Pareto: $$F_k(x) = 1 - \left(1 + \frac{x}{B \cdot g^k}\right)^{-Q}$$

Weibull: $$F_k(x) = 1 - exp\left(-\left(\frac{x}{B \cdot g^k}\right)^{\omega}\right)$$

We do not have every possible loss from the distribution.

We only have large losses, above some threshold *T*.
(truncated from below)

Some losses are capped at historical Policy Limit (*PL*).
(censored from above)

$$F(x|x \geq T) = \begin{cases} 0 & 0 \leq x \leq T \\ 1 - \dfrac{1 - F(x)}{1 - F(T)} & T \leq x < PL \\ 1 & PL \leq x \end{cases}$$

We can use Maximum Likelihood Estimation to find the model parameters:

Q or *ω*	Shape parameter; constant for all years
B	Scale parameter for base year
g	Trend factor (*g*=1.06 means 6% annual trend)

Other inputs supplied by the user, for each loss record:

k	Year index k = 1, 2, 3, …
T_i	Truncation point or reporting threshold
PL_i	Policy Limit

Continuous Distributions
Advantages

Advantages of Parametric Approach:

- Can work with loss data as received, subject to reporting thresholds and policy limits
- Can produce standard errors for trend estimators
- Other diagnostics
  - Likelihood Ratio Tests
  - Q/Q Plots
  - Residual Plots

Continuous Distributions
Advantages – Standard Error Calculation

Large Loss Trend - Pareto Model

Truncation	50,000
B	20,000
Q	1.2500
Trend	6.50%

Number of Years	Number of Losses per Year: 10	20	25	50
5	53.23%	37.64%	33.67%	23.81%
10	17.14%	12.12%	10.84%	7.67%
15	8.73%	6.17%	5.52%	3.90%
20	5.36%	3.79%	3.39%	2.40%
25	3.65%	2.58%	2.31%	1.63%

Numbers for illustration only.

Disadvantages of Parametric Approach:

- Data Quality Issues
  - Changing case reserve adequacy
  - “Clustering” and precautionary reserves
- Small sample distortions on likelihood ratios and standard errors
- Dependence on curve form (Pareto problem...)

Standard Pareto:

$$F(x) = 1 - \left(\frac{B}{B+x}\right)^{Q}$$

Lower-Truncated Pareto:

$$F(x|x \geq T) = 1 - \left(\frac{B+T}{B+x}\right)^{Q}$$

Single Parameter Pareto:

$$F(x|x \geq T) = 1 - \left(\frac{T}{x}\right)^{Q}$$

No “scale” parameter; no way to estimate trend.

Similarly, for Lower-Truncated Weibull:

$$F(x|x \geq T) = 1 - exp\left(\left(\frac{T}{B}\right)^{\omega} - \left(\frac{x}{B}\right)^{\omega}\right)$$

This becomes a single parameter Pareto when the B,ω→0

$$\lim_{B,\omega \to 0} exp\left(\left(\frac{T}{B}\right)^{\omega} - \left(\frac{x}{B}\right)^{\omega}\right) = \left(\frac{T}{x}\right)^{Q}$$

The "scale" parameter disappears; so again no way to estimate trend.

Estimate of Trend is dependent on the form of the loss distribution.

If losses really do follow a single parameter Pareto, then there is <u>no way</u> to estimate inflation trend without additional information.

[N.B. This is not a problem with "Dave's method," it is a problem inherent in the nature of insurance losses.]

See article "When Inflation Causes No Increase in Claims Amounts" by Brazauskas, Jones & Zitikis; Journal of Probability and Statistics.

Example for Report Year Data

Sample of GL losses for report years 2001-2010.

Total of 713 claims at one time reserved > 25,000

Limitations:

- Small sample
- Some losses missing accident year or policy limit
- No split of paid and case reserve or status (open/closed)
- No split of loss and alae (harder to identify capped losses)

Example for Report Year Data

We fit the Pareto model to the large loss data and calculate parameters values and the covariance matrix for the parameters. This allows the estimate of the standard error on our estimated annual trend.

Large Loss Trend Fit

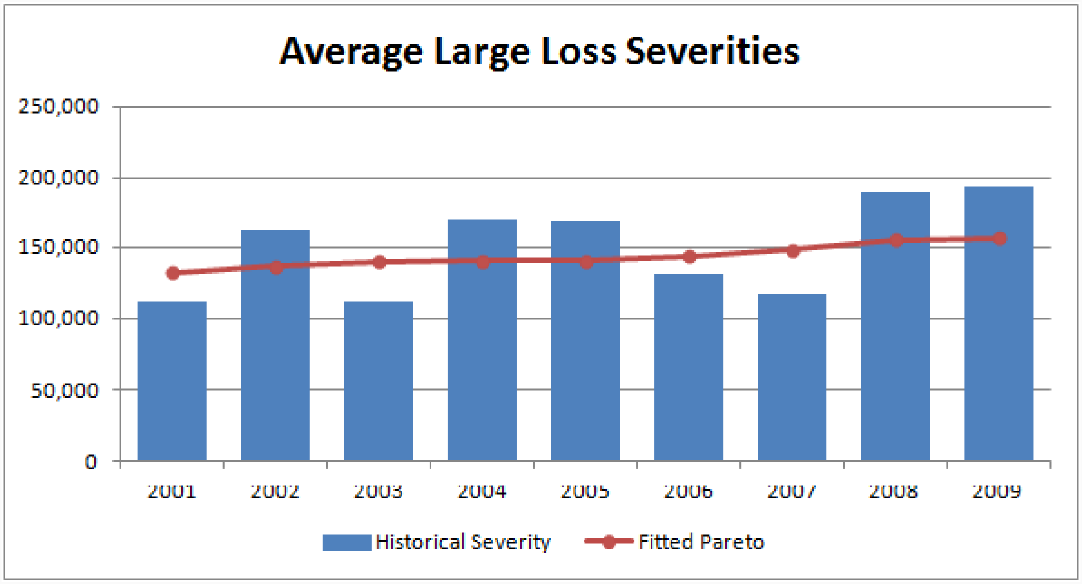
Parameter	Value	Std Error
Shape Q	1.1509	0.1059
Base Scale B	16,208	7,363
Annual Trend	6.5%	6.1%

Parameter Covariance Matrix

Shape Q	0.01121891	550.512052	-0.0008056
Base Scale B	550.512052	54210169.2	-319.21598
Annual Trend	-0.0008056	-319.21598	0.00370542

Numbers for illustration only.

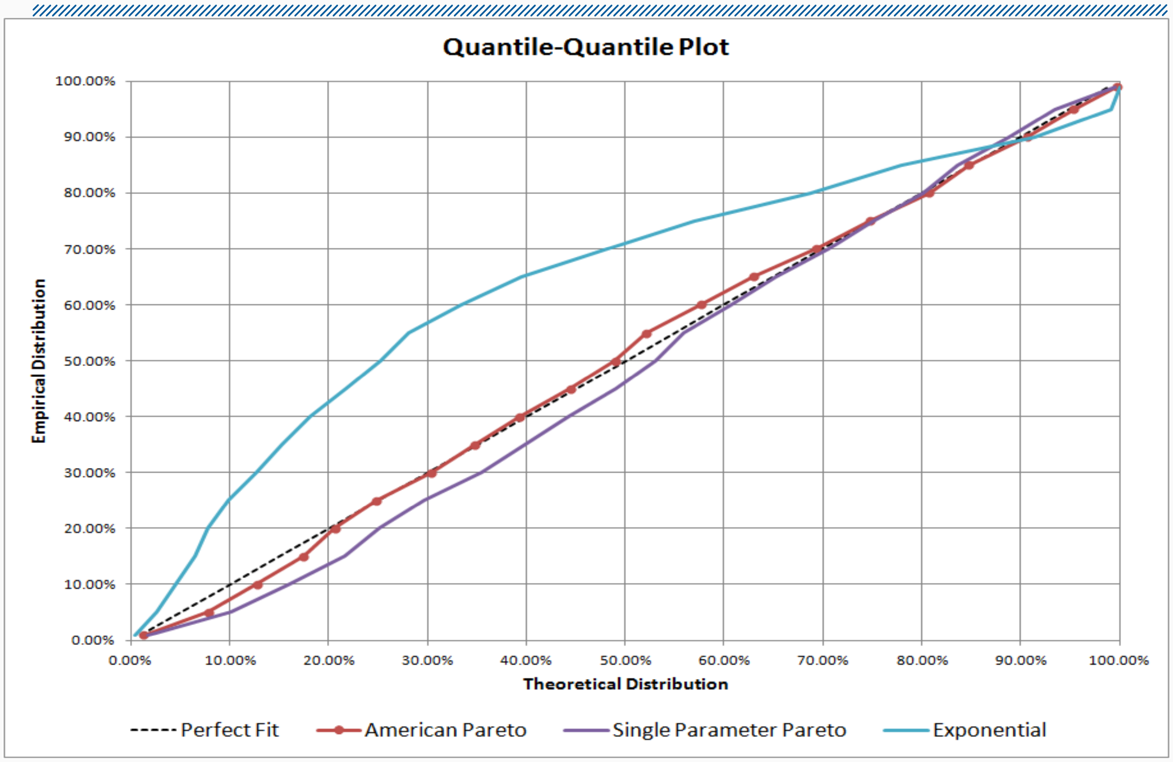
Example for Report Year Data



Number of Losses > 25,000 at second report year

2001	2002	2003	2004	2005	2006	2007	2008	2009	Total
47	93	90	79	55	45	51	54	55	569

Example for Report Year Data



THANK YOU FOR YOUR ATTENTION

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