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Perspectives on European vs. US Casualty Costing

INTMD-2 International Pricing Approaches --- Casualty,

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Robert Bender: Biography/ CV

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- Most of my actuarial career has been spent pricing US casualty lines (including Workers Compensation).
- While I am currently employed by a very large global reinsurance company with a home office in Switzerland, I continue to focus on US Casualty.
- I am not an expert in International Insurance or Reinsurance
- I do interact with my European colleagues and have noticed some material differences regarding how we approach the quantification of expected loss.
- We have learned from each other and, in this presentation, I will share some of my observations.
- First Difference:

US:

My Observations May Not Be Typical For Other Companies or Countries
The Views Expressed are My Own. I Do Not Speak for My Employer.

EU:

Differences 1/4 – in no particular order

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- Education
  - US: Emphasis is placed upon formal study/examinations by the CAS with corresponding credentials. Increasingly, we see people with undergraduate degrees in Actuarial Science (also mathematics or science).
  - Europe: Less emphasis on formal actuarial examinations and credentials. More likely to find people with graduate degrees in mathematics and science (particularly physics - at least for our Analytical Tools and Services Team) and of course Actuarial Science
- Underwriters *vs.* Actuaries
  - US: Separate disciplines (although some individuals may have both skill sets)
  - US: Actuaries can be found performing "costing," reserving, planning, and other roles
  - Europe: Underwriters are expected to be more analytical, hence "actuaries" tend to be reserving actuaries. Underwriters perform "costing."
  - Europe: Research and Development is performed by Analytical teams not Actuarial teams
  - Note: *Analytical* Tools and Services, not *Actuarial* Tools and Services

Differences 2/4 – in no particular order

- Terminology: Experience Rating

  Adjust the client's historical experience to an "as if prospective period" basis and develop historical layer losses to form:

  - US: Estimates of the burning cost (expected loss/subject premium) for the reinsurance to be quoted and sold.
  - Europe: Burning cost estimates for successive working layers so that they can determine the claim frequency at some loss threshold and the **Pareto** (usually) severity model that is consistent with these layer loss costs or counts. Then they use the frequency/severity model to "cost" the layer to be quoted and sold.

- Terminology: Pareto distribution (one less B)
  - US:

    $F(x) = 1-[B/(B+x)]^{\alpha}$ (which is defined at zero)

  - Europe:

    $F(x) = 1-[B/x]^{\alpha}$ (which is **not** defined at zero)

  More about this difference, later.

Differences 3/4 – in no particular order

- Terminology: Exposure Rating
  - US: Use a severity curve that is based upon industry data (promulgated by a rating bureau or other organization).
  - US: Use the severity curve to form limited expected values so as to stratify the expected full limits loss into layers.
  - US: The full limits loss is based upon either an industry loss cost per exposure unit, times the client's exposure, or a ground up loss ratio times subject premium.
  - Europe: What's a rating bureau? Develop Market Curves based upon your own experience for large segments of the market (which you must write in order to have data) and fit a frequency/severity model to the data, *i.e.*, use a "market curve"
  - Europe: Exposure Rating sounds very much like Experience Rating, with the model based upon the experience of *many* clients.
- Credibility/Blending
  - US: Estimate the burning cost using Experience and Exposure based methods, then find an appropriate weighting of the two estimates
  - Europe: Find an appropriate weighting of the two loss models such that we have a single frequency at the threshold and a survival curve that begins near the experience curve and approaches the exposure curve as losses increase.

Differences 4/4 – in no particular order

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- Data Issues
  - US: Client usually can provide historical paid loss as well as paid plus case O/S for all claims --- both individually and in the aggregate.
  - Europe: Individual claim history can be provided as paid and paid plus case O/S, but aggregate data frequently comes as paid or paid plus case plus bulk and IBNR. They tend to rely upon paid data when estimating loss ratios .
- Coverage Differences
  - US: Most coverage is limited (exception being WC) with relatively low limits offered on primary policies. Reinsurance typically focuses on primary and low umbrella limits.
  - Europe: Coverage can be unlimited and policyholders purchase very high limits. Reinsurance typically focuses on high attachment points and limits
- Loss Model Focus
  - US: Focus is on having the first part of the curve right (to $1M - $10M) with less concern about amounts in excess of $10M.
  - Europe: Focus in on the tail ($50M or more) with less concern about the first $10M

Which is why we use different versions of the Pareto

Rules of Thumb

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- US: Umbrella Rating – cost of first $1M is $Y. Cost of each successive $1M band is 60% (or some other number) of the cost of the previous band.
  - Can be expressed in terms of an exponential severity distribution
  - Not used very often, but it is encountered from time to time.
- Europe: The ILF Method (not to be confused with using ISO Increased Limits Factors to cost a layer).
- Premium for the basic limit is determined, *e.g.,* $Y for the first $1M of coverage
- Each time the limit is *doubled*, the cost increases by a factor of *1+f*, where *f* is the ILF factor.

	ILF Factor	f	0.30
Power	**Limit**	**Cost**	**Cost**
0	1,000,000	$\$Y*(1+f)^0$	1,000
1	2,000,000	$\$Y*(1+f)^1$	1,300
2	4,000,000	$\$Y*(1+f)^2$	1,690
3	8,000,000	$\$Y*(1+f)^3$	2,197
…	**…**	**…**	
n	$2^n*1,000,000$	$\$Y*(1+f)^n$	$1,000*1.30^n$

- Can be expressed in terms of a European Pareto severity distribution
- Used Frequently.

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Consider Three Probability Distributions

- European Pareto with Survival Function:

  $S(x) = [B/x]^{\alpha}$ (For x>B, 1.00 if x<B)

- US Pareto with Survival Function:

  $S(x) = [B/(B+x)]^{\alpha}$

- Exponential with Survival Function

  $S(x) = e^{-x/\lambda}$

- The Pareto α's are both equal to 1.75, all three means are $700K

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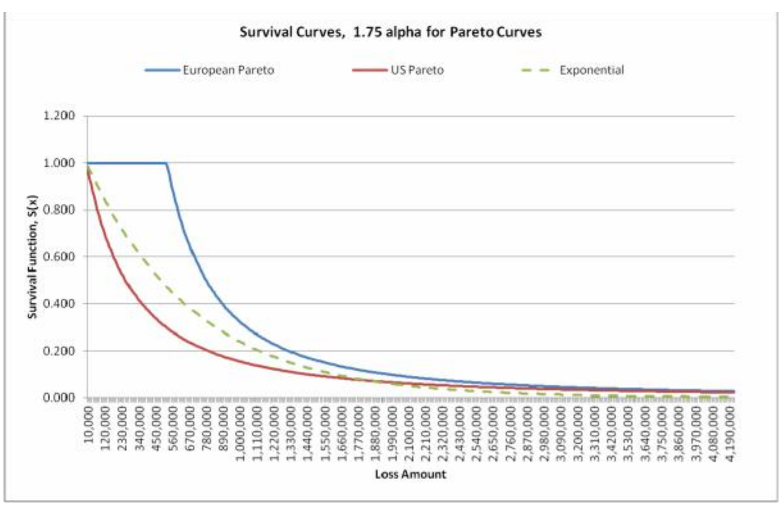
You Graph on What? – American View 1/2 - Linear Scales



I cannot see detail in the tail, change the scale

You Graph on What? – American View 2/2

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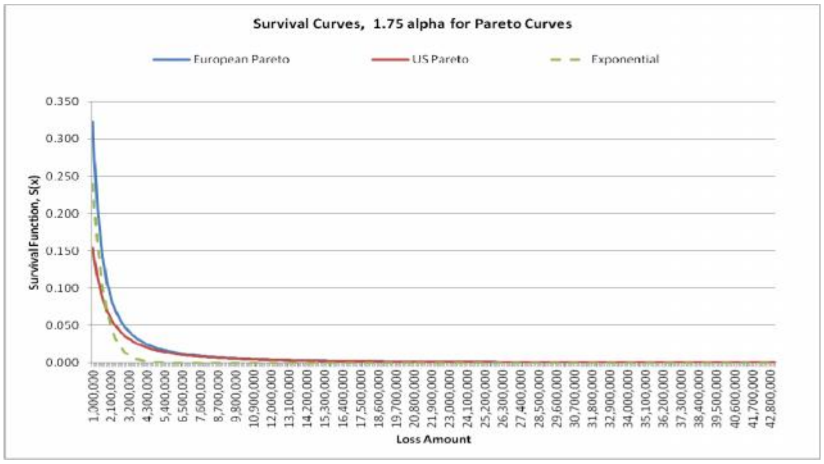

I cannot see detail in the tail, remove the first part
or
Graph a Series of Conditional Survival Curves

You Graph on What? – European View

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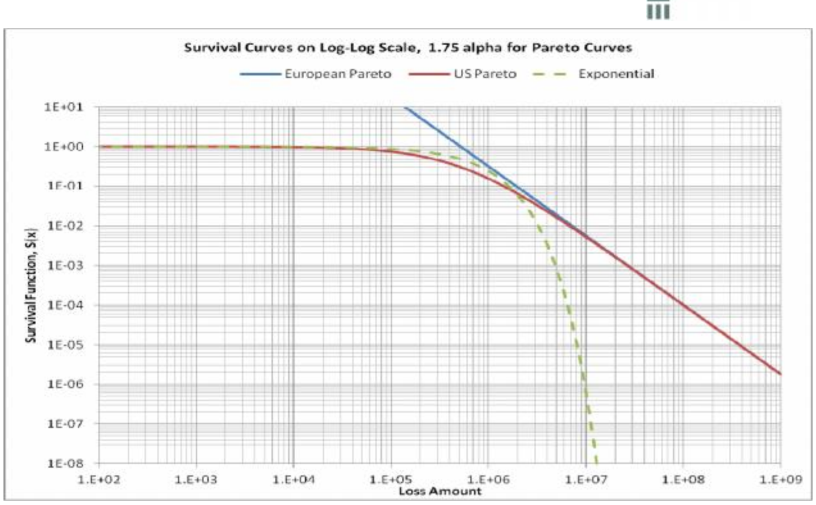

- I can see more detail
- The European Pareto is a straight line
- The US Pareto approaches the European Pareto Asymptotically

ISO Style Mixed Exponential
---on Log-Log Scale

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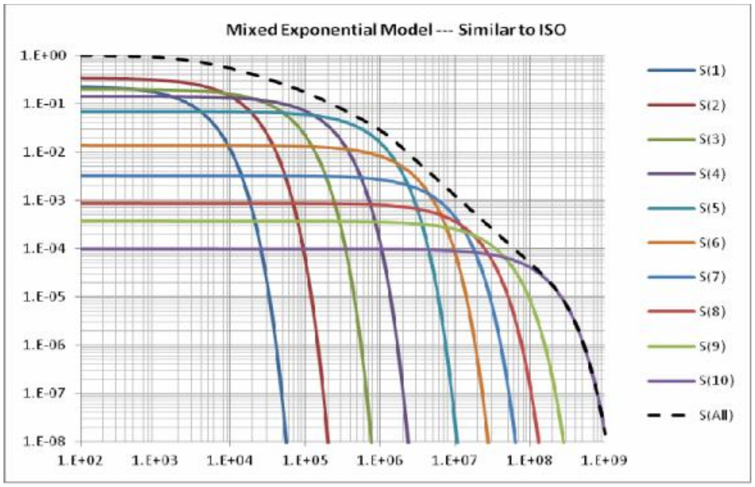

- Above 1M S(All) is essentially a European Pareto with α=1.4 until approximately $150M when it falls off more rapidly
- Only the last curve contributes materially to the tail

Mixed Exponential with a Pareto Splice

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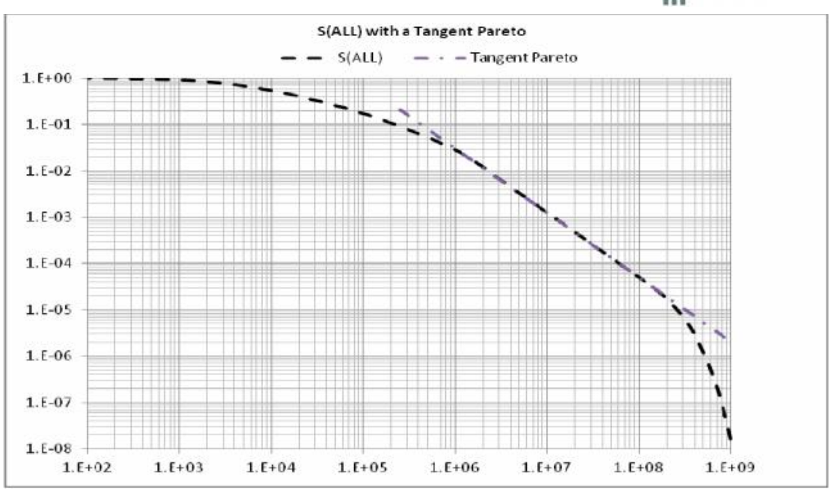

We do not necessarily recommend a *Pareto Splice*, but observe that using a log-log scale allows us to make decisions regarding the tail fit beyond $150M

Exponential/ US Pareto Mix – Alternative Tail Fitting

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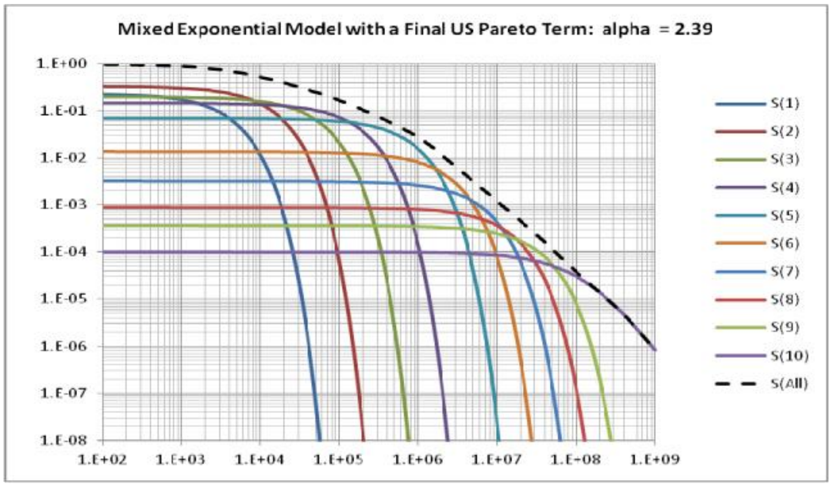

Last Exponential is replaced by a US Pareto with the same mean

Percentage of Loss in Layer–Typical Layers

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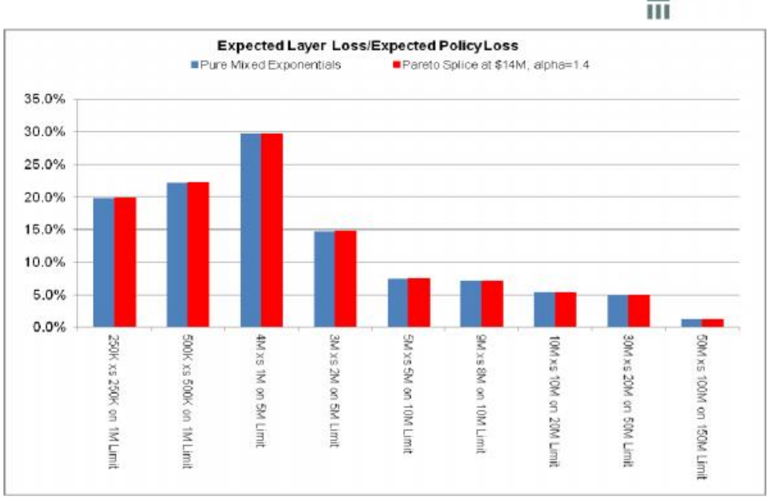

•Even with the conservative EU Pareto tail, typical reinsurance layers are unchanged

Percentage of Loss in Layer–Very High Layers

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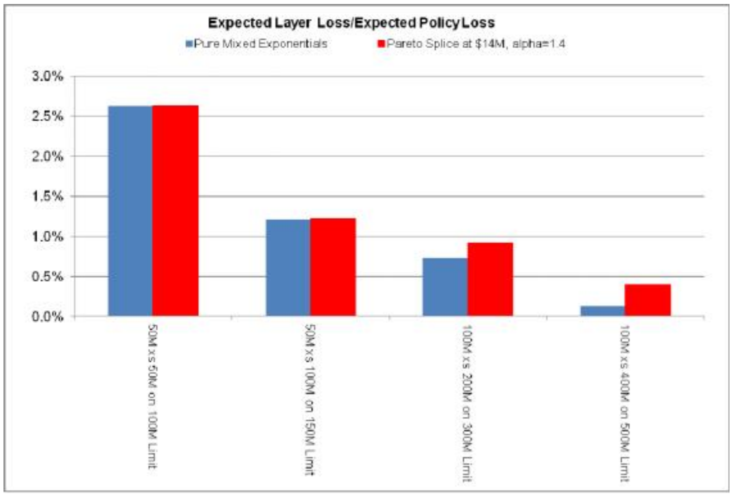

•For Very High Layers, consideration should be given to the tail selection because it makes a difference

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European ILF – A high level view

- The ILF methodology is widely used in Casualty exposure rating in Europe / Asia
- Parameter selection is largely based on best practice and expert judgement and not on calibration with real data
- If used over a wide range, the underwriter may elect to vary the parameter over the entire range.
- The methodology is quite simple but it lacks a sound explicit mathematical model
- The implicit mathematical model behind the ILF approach can easily be derived. It is a (European) Pareto distribution with a Pareto α between 0 and 1.
- The properties of this Pareto distribution allow us to analyze the limitations of the applicability of the ILF approach.

The ILF Method
Equivalence with a European Pareto

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- It can be shown that the loss costs produced by the ILF method with factor f can be reproduced by using a European Pareto Survival Curve

$$S(x) = [B/x]^{\alpha}$$

- with parameters

  $B = L_0$, the basic limits loss amount

  $\alpha = 1 - \ln(1+f)/\ln(2)$

- Underlying the (implicit) Pareto model:
  - The implicit Claims Severity can be represented by a Pareto Distribution
  - The loss-frequency curve* (LFC) diverges at the origin
  - The mean, however, exists for any finite limit
  - The mean does not exist for an infinite limit

* European for Survival Function, S(x)?

European ILF Method
EU-Pareto Representation

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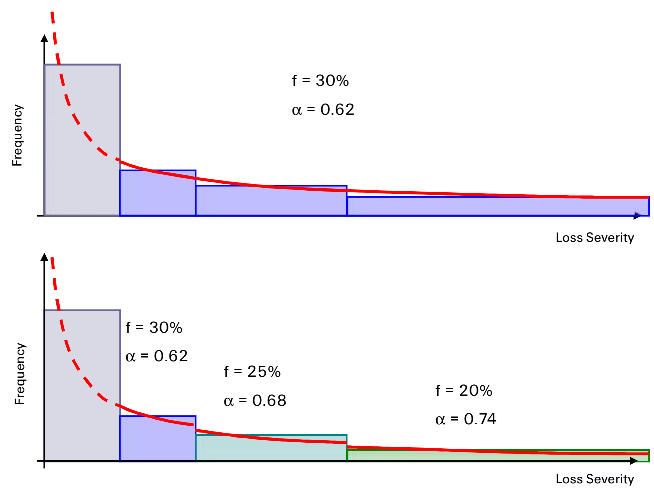
Constant ILF-factor.

LFC can be represented by a single Pareto distribution

ILF-factor varying by range.

LFC can be represented by discontinuous piecewise Pareto distributions.



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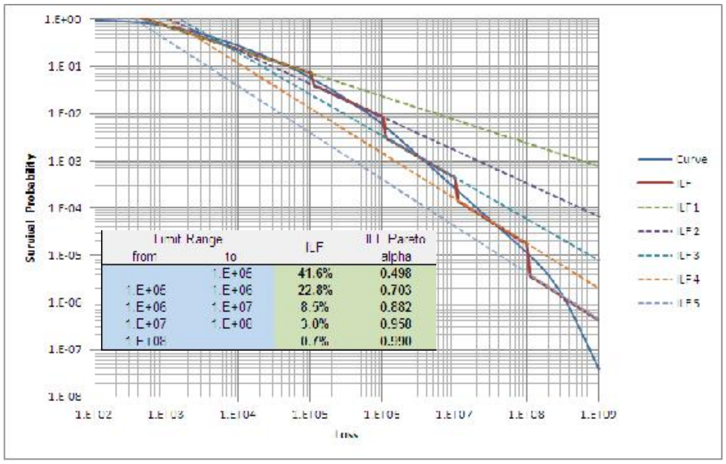
Example: ILF's for a Selected ISO curve

Implied ILF factors are derived for defined ranges



Limit Range from	to	ILF	ILF Pareto alpha
	1.E+05	41.6%	0.498
1.E+05	1.E+06	22.8%	0.703
1.E+06	1.E+07	8.5%	0.882
1.E+07	1.E+08	3.0%	0.958
1.E+08		0.7%	0.990

Remarks

An ILF implies a Pareto curve with slope alpha < 1

Multiple ILF factors are required to cover the entire dynamic range

The overall shape of the ILF representation is largely determined by the discontinuities (jumps) at the range boarders.

European ILF Method and Mixed Exponential Model are not so different, after all

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Technical Note: Demonstration that European ILF method can be expressed as a Pareto curve 1/2

Each loss limit can be expressed in terms of the basic limits loss, L_0 and a power of two:

$$L_n = L_0 2^n$$

or

$$n = \ln(L_n/L_0)/\ln(2)$$

From the ILF method, the relationship between the limited expected values is

$$E[x|L_n] = E[x|L_0]*(1+f)^n$$

Substituting the value of *n* in terms of the ratio of the limits,

$$E[x|L_n] = E[x|L_0](1+f)^{\frac{\ln\left(\frac{L_n}{L_0}\right)}{\ln(2)}}$$

which can also be written as

$$E[x|L_n] = E[x|L_0]\left(\frac{L_n}{L_0}\right)^{\frac{\ln(1+f)}{\ln(2)}}$$

If we assume that relationship between the limited expected value and the ratio of the limits holds for non-integral values of n, then we can write

$$E[x|L]/E[x|L_0] = \left(\frac{L}{L_0}\right)^{\frac{\ln(1+f)}{\ln(2)}}$$

Technical Note: Demonstration that European ILF method can be expressed as a Pareto curve 2/2

Now, assume that the there exists a European Pareto distribution that produces the same relationship between the ratio of limits and limited expected values. The distribution for CDF corresponding to the European Pareto is given by

$$F(x) = 1- (B/x)^{\alpha}$$

where B and α are, as yet, unknown parameters. We note that knowing F(x) gives us the desired survival function, S(x) = 1-F(x).

In terms of the probability,

$$E[x|L] = \propto B^{\propto} \left[\int_0^L x^{-\propto} dx + L \int_L^{\infty} x^{-(\propto+1)}\, dx \right]$$

which, for α <1 becomes,

$$E[x|L] = B^{\alpha}L^{1-\alpha}/(1-\alpha).$$

The ratio of Pareto expected values,

$$E[x|L]/E[x|L_0] = (L/L_0)^{1-\alpha},$$

is the same as the European ILF method if we set

$$\alpha = 1- \ln(1+f)/\ln(2),$$

which was to be demonstrated.

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