Casualty Actuaries in Reinsurance
2012 Seminar
CAT Pricing Methods

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- Writing CAT covers is risky – results may be catastrophic to your bottom line.
- Examples are for illustrative purposes only. Do not use in any example in real-world applications.
- There may be a quiz at the end – so take good notes!

Agenda
-A Mix of Theory and Practice

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- CAT Context
- Pricing Overview
- Statistics from Ordered Random Trials
- Basic Equations
- Required Capital Paradigms
- Order Dependence and Reference Portfolios
- Risk Measures
  - Definition and Properties
  - Take your pick
  - Ranking definitions of Var and TVaR
  - TVaR different from CTE
  - VaR Subadditivity –epic fail
- Real Allocation Approaches
  - Co-VaR instability
  - Co-TVaR not subadditive
- Conclusions

Context

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- CAT Pricing is part of the process of writing CAT business, but not the only part.
- Pricing models give indications – the market sets the price.
- Business bunched –lots of 1/1s.
- Authorized share vs bound share.
- Real time- by time a treaty gets bound, the portfolio has changed.
- Methods may not extend to direct/large volume/small risk business

Portfolio Management

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- Risk Management sets limits on PMLs and TIV/Limit Aggregations by peril/zone .
  - Compliance monitoring essential
- Selection problem is constrained optimization: Reinsurers looks to get most profitable portfolio with smallest risk.
- Does pricing help optimize/solve the selection problem?
- Have faulty pricing methods led to de-worsification??

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Pricing Overview

- UNL traditional product vs CAT Bonds vs ILWs
- Why do reinsurers have different pricing indications ?
- Emerald City Pricing: Don't look at the man behind the curtain
  - Different vendor models/different switches
  - Delay in adopting new versions
  - Differences in data quality
  - Loading factors and adjustment factors
- Non-modeled CAT events (Thai flood): Not always priced
  - Ostrich Excuse - "It was not in the model"
  - Hiding-in-Plain-Sight Swan - May not show up on risk management radar – obvious after the fact.
- Pricing Method Flavors: Different ways of translating model stats into indicated prices.
  - Can't we just all agree?

Equations and Properties

CAT PRICING

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Basic Equations

Å P= E[X]+ RL(X)

P = Indicated premium prior to expense loading

X = CAT Loss

RL(X) = Risk Load

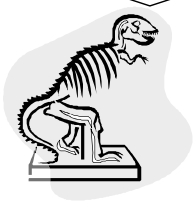
Å RL(X) = r_{target}*C(X)

Å C(X) = Required Capital

Å RORAC Approach

ï Universally used in actual CAT Treaty pricing

What is the right way to compute Required CAT Capital?



Required Capital Paradigms

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- Standalone: $C(X) = \rho(X)$, where is $\rho(X)$ is a risk measure.
- Incremental: Let T be the existing portfolio
  $C(X|T) = \rho(T+X) - \rho(T)$,
- Real Allocation
  $C(X|T) = A(X,T) * \rho(T+X)$

Order Dependence and Reference Portfolios

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- Order Dependence – Pricing depends on the order in which accounts are priced (Mango)
- A major problem for Incremental methods
- A small problem for Allocation methods
- Not a problem for Standalone
- Reference Portfolio Cure
  - Portfolio fixed over a given period
  - How often should it be updated??

Risk Measure: Definitions and properties

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- A **risk measure**, ρ, is a monotonic function that maps a real-valued random variable, X, to a non-negative number, ρ (X), such that:
- Risk Measure Basic Properties

1. *Non-negative: $\rho(X) \geq 0$*
2. *Monotonic Premium: If $X_1 \leq X_2$, then $E[X_1]+\rho(X_1) \leq E[X_2]+\rho(X_2)$*

- A risk measure is **pure** if it maps constants to zero: $\rho(c) = 0$

Risk Measure: Coherence properties

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1. Scalable: $\rho(\lambda X) = \lambda \cdot \rho(X)$
2. Translation Invariant: $\rho(X+ \alpha) = \rho(X)$
3. Subadditive: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

Å Some academicians refuse to refer to a function as a risk measure unless it is coherent

Å Most academicians uses reverse signs (X represents the value of assets instead of CAT losses)

Risk Measures: Take Your Pick

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1. *Variance: $Var(X) = E[(X-\mu)^2]$*
2. *Semivariance: $Var^+(X)$: $E[(X-\mu)^2 \mid X \geq \mu] * Prob(X \geq \mu)$*
3. *Standard Deviation: $\sigma = Var^{1/2}(X)$*
4. *Semi Standard Deviation: $\sigma^+ = Var^{+1/2}(X)$*
5. *Value at Risk: for $0 < \theta < 1$, $VaR(\theta) = \sup\{x \mid F(x) \leq \theta\}$*
6. *Tail Value at Risk: $TVaR(\theta)$ = conditional mean for all x values associated with the tail, $1-\theta$, of probability*
7. *Excess Tail Value at Risk: $XTVaR(\theta) = TVaR(\theta) - \mu$*
8. *Distortion Risk Measure: (Wang) $E^*[X] = E[X^*]$ where $F^*(x) = g(F(X))$ for g a distortion function*
9. *Excess Distortion Risk Measure: $E^*[X] - E[X]$*

CALCULATIONS AND COUNTEREXAMPLES

Random Trials

Trial Year	Event 1	Event 2	Event 3	...	Largest Event over the Year	Total Annual Loss
1	40,000	-	-	-	40,000	40,000
2	2,100	3,500	450	- -	3,500	6,050
3	-	-	-		0	0
4	5,500	27,550	-		27,550	33,050
5	700	400	50		700	1,150
6	1,250	900	25		1,250	2,175
7	8,750	-	-		8,750	8,750
8	75	45	70,000		70,000	70,120
9	-	-	-		0	0
10	15	3,500	45		3,500	3,560
⋮	⋮	⋮	⋮		⋮	⋮
9998	25	-	-		25	25
9999	550	7,750	-		7,750	8,300
10000	650	-	-		650	650

AEP and OEP PML from Ordered Trials [18]

Trial Year Rank		Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		125,000	165,000
4		100,000	137,500
5		100,000	135,000
6		100,000	130,000
7		90,000	125,000
8		90,000	115,000
9		90,000	110,000
10		90,000	110,000
.		.	.
.		.	.
.		.	.
99	100/10000 = 1.0% 100 year return period AEP PML =36,675 OEP PML= 21,000	21,250	37,500
100		21,000	36,675
101		21,000	35,950
.		.	.
.		.	.
.		.	.
9998		-	0
9999		-	0
10000		-	0

- PML = Probable Maximum Loss =VaR
- AEP = Annual Exceeding Probability
- OEP = Occurrence Exceeding Probability

Ranking Definition of VaR and TVaR on Random Sample Data

- Let $X_1 \geq X_2 \ldots \geq X_n$ *be an ordering of n trials of X*
- *Suppose k = (1 - θ)n, then*

$$VaR(\theta) = X_k$$

$$TVaR(\theta) = \frac{1}{k}\sum_{j=1}^{k} X_j$$

- Note TVaR is not necessarily equal to the Conditional Tail Expectation (CTE) when the data is discrete.
- *CTE(θ) = E[X|X>VaR(θ)]*

TVaR and CTE -not the same

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.80	26.00	28.80
VaR	*2.00*	*33.00*	*34.00*
TVaR	5.00	34.80	35.40
CTE	5.75	36.00	35.75

Loss Data by Trial			
Trial	A	Ref	A+Ref
1	8.00	12.00	20.00
2	0.00	37.00	37.00
3	0.00	36.00	36.00
4	0.00	35.00	35.00
5	1.00	33.00	34.00
6	2.00	17.00	19.00
7	7.00	16.00	23.00
8	2.00	33.00	35.00
9	4.00	27.00	31.00
10	4.00	14.00	18.00

Separately Ordered Loss Data			
Rank	A	Ref	A+Ref
1	8.00	37.00	37.00
2	7.00	36.00	36.00
3	4.00	35.00	35.00
4	4.00	33.00	35.00
5	*2.00*	*33.00*	*34.00*
6	2.00	27.00	31.00
7	1.00	17.00	23.00
8	0.00	16.00	20.00
9	0.00	14.00	19.00
10	0.00	12.00	18.00

VaR Subadditivity-Epic Fail

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.80	26.00	28.80
VaR	*2.00*	*33.00*	*37.00*
TVaR	5.00	34.80	39.40

Loss Data by Trial			
Trial	A	Ref	A+Ref
1	0.00	12.00	12.00
2	0.00	37.00	37.00
3	8.00	36.00	44.00
4	7.00	35.00	42.00
5	4.00	33.00	37.00
6	2.00	17.00	19.00
7	0.00	16.00	16.00
8	4.00	33.00	37.00
9	2.00	27.00	29.00
10	1.00	14.00	15.00

Separately Ordered Loss Data			
Rank	A	Ref	A+Ref
1	8.00	37.00	44.00
2	7.00	36.00	42.00
3	4.00	35.00	37.00
4	4.00	33.00	37.00
5	*2.00*	*33.00*	*37.00*
6	2.00	27.00	29.00
7	1.00	17.00	19.00
8	0.00	16.00	16.00
9	0.00	14.00	15.00
10	0.00	12.00	12.00

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Real Allocation Approaches

1. *Stand-alone Risk Measure as Allocation Base*
2. *Marginal Risk Measure as Allocation Base*
   - ï *Adjusted for Order Dependence (Mango)*
3. *Game theory –(LeMaire) Allocation of Portfolio Consolidation Benefit*
4. *Co-Measures – (Kreps)*
5. *Percentile Allocation (Bodoff)*

Co-VaR Instability

Rank	VaR Percentage	Portfolio Loss	Risk A Loss
1			
98	99.02%	$422	$6
99	99.01%	$408	$0
100	99.00%	$405	$20
101	98.99%	$395	$0
102	98.98%	$390	$4
10,000			

Co-TVaR A

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A+Ref	A	Ref
Mean	27.50	2.50	25.00
VaR	*35.00*	*2.00*	*30.00*
TVaR	37.00	4.40	35.00
Co-TVaR	37.00	2.00	35.00

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref	Co- A	Co-Ref
1	2.00	8.00	10.00	1	8.00	39.00	39.00	0.00	39.00
2	0.00	38.00	38.00	2	7.00	38.00	38.00	0.00	38.00
3	7.00	30.00	37.00	3	3.00	35.00	37.00	7.00	30.00
4	0.00	35.00	35.00	4	2.00	33.00	36.00	3.00	33.00
5	3.00	33.00	36.00	5	2.00	30.00	35.00	0.00	35.00
6	2.00	15.00	17.00	6	2.00	25.00	33.00	8.00	25.00
7	0.00	39.00	39.00	7	1.00	16.00	18.00	2.00	16.00
8	2.00	16.00	18.00	8	0.00	15.00	17.00	2.00	15.00
9	8.00	25.00	33.00	9	0.00	11.00	12.00	1.00	11.00
10	1.00	11.00	12.00	10	0.00	8.00	10.00	2.00	8.00

Co-TVaR B

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	B+Ref	B	Ref
Mean	27.50	2.50	25.00
VaR	*34.00*	*2.00*	*30.00*
TVaR	37.80	4.20	35.00
Co-TVaR	37.80	2.80	35.00

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	B	Ref	B+Ref	Rank	B	Ref	B+Ref	Co- B	Co-Ref
1	0.00	8.00	8.00	1	7.00	39.00	44.00	5.00	39.00
2	1.00	38.00	39.00	2	5.00	38.00	39.00	1.00	38.00
3	4.00	30.00	34.00	3	4.00	35.00	37.00	2.00	35.00
4	2.00	35.00	37.00	4	3.00	33.00	35.00	2.00	33.00
5	2.00	33.00	35.00	5	2.00	30.00	34.00	4.00	30.00
6	3.00	15.00	18.00	6	2.00	25.00	32.00	7.00	25.00
7	5.00	39.00	44.00	7	1.00	16.00	18.00	3.00	15.00
8	1.00	16.00	17.00	8	1.00	15.00	17.00	1.00	16.00
9	7.00	25.00	32.00	9	0.00	11.00	11.00	0.00	11.00
10	0.00	11.00	11.00	10	0.00	8.00	8.00	0.00	8.00

Co-TVaR A+B

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A+ B +Ref	A+ B	Ref
Mean	30.00	5.00	25.00
VaR	*38.00*	*5.00*	*30.00*
TVaR	40.40	8.20	35.00
Co-TVaR	40.40	7.40	33.00

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	A+ B	Ref	A+ B +Ref	Rank	A+ B	Ref	A+ B +Ref	Co- A+ B	Co-Ref
1	2.00	8.00	10.00	1	15.00	39.00	44.00	5.00	39.00
2	1.00	38.00	39.00	2	11.00	38.00	41.00	11.00	30.00
3	11.00	30.00	41.00	3	5.00	35.00	40.00	15.00	25.00
4	2.00	35.00	37.00	4	5.00	33.00	39.00	1.00	38.00
5	5.00	33.00	38.00	5	5.00	30.00	38.00	5.00	33.00
6	5.00	15.00	20.00	6	3.00	25.00	37.00	2.00	35.00
7	5.00	39.00	44.00	7	2.00	16.00	20.00	5.00	15.00
8	3.00	16.00	19.00	8	2.00	15.00	19.00	3.00	16.00
9	15.00	25.00	40.00	9	1.00	11.00	12.00	1.00	11.00
10	1.00	11.00	12.00	10	1.00	8.00	10.00	2.00	8.00

Co-TVaR Subadditivity Fail

Results	A	B	A+B	Ref	A+B +Ref
Mean	2.50	2.50	5.00	25.00	30.00
VaR	2.00	2.00	5.00	30.00	38.00
TVaR	4.40	4.20	8.20	35.00	40.40
Co-TVaR	2.00	2.80	7.40	33.00	40.40

Loss Data by Trial					
Trial	A	B	A+ B	Ref	A+ B +Ref
1	2.00	0.00	2.00	8.00	10.00
2	0.00	1.00	1.00	38.00	39.00
3	7.00	4.00	11.00	30.00	41.00
4	0.00	2.00	2.00	35.00	37.00
5	3.00	2.00	5.00	33.00	38.00
6	2.00	3.00	5.00	15.00	20.00
7	0.00	5.00	5.00	39.00	44.00
8	2.00	1.00	3.00	16.00	19.00
9	8.00	7.00	15.00	25.00	40.00
10	1.00	0.00	1.00	11.00	12.00

ISSUES AND OBSERVATIONS

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Key Issues-Conceptual

Issue	Alternatives/Observations
Distribution Segment Focus	Far Tail vs Adverse Deviation vs Full distribution
Portfolio Dependence	1. Portfolio Independent : using stand-alone or equilibrium Market price vs 2. Portfolio Dependent : using incremental or allocation
Pure Risk Measure	1. Seems to make sense in theory 2. Many measures used are not pure
Coherence	1. Absolutely necessary 2. Desirable, but not critical

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Key Issues-Practical

Issue/observation	Practical solution/Observation
Order Dependence	Use Reference portfolio
Scale (Share) dependence of portfolio methods	Price initially at highest authorized share
Co-VaR instability	Average over events in neighborhood
Portfolio dependent methods may promote de-worsification	Consider alternative approaches or minimum ROL risk load charges.
"Pure" risk measures may lead to very small risk loads.	Use impure risk measure – for example Co-TVaR instead of XC0-TVaR or add minimum ROL risk load.

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Observations and Conclusions

- Å Target return on required capital is the basis for reinsurer pricing indications.
- Å Debate is over required capital.
- Å A profusion of methods and approaches
- Å Tail focus and portfolio dependence are key areas where methods differ
- Å Some of the key methods used in practice do not satisfy all the desired conceptual properties and may have led to deworsification.