GUY CARPENTER

# Model Risk
## No More Pretending

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## Motivation




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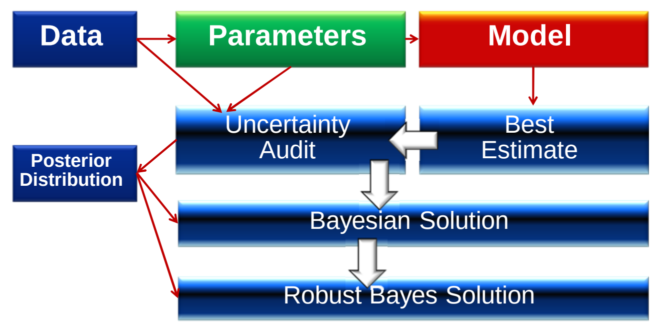
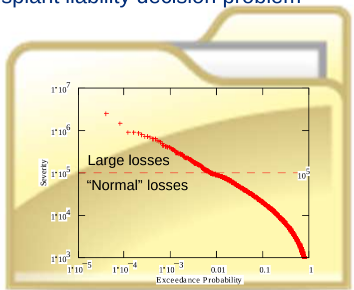
## Roadmap for future best practices in actuarial modeling




- Best Estimate = *"Give us your best guess"*
- Uncertainty Audit = *"How wrong might you be (and why)?"*
- Bayes = *"Treat uncertainty as another risk and update your answer"*
- Robust Bayes = *"How do we protect the firm from Murphy's Law?"*

## CASE STUDY: Year 2064 – a brain transplant liability decision problem

- Historical 25,000 claims over 7 years
  - Split into “normal” and large losses
- Decision alternatives:
  - Various per-risk excess (or bare)
  - Corporate provides a capital backstop for the remaining risk




- Constraint: Pr{Capital+Premium-Expenses-Losses<0} ≤ 0.4%
  - Required capital is a function of *actual* risk
- Objective: maximize Economic Underwriting Gain
  - EUG = Premium-Expenses – E[Ultimate Losses] – (17%*Capital)
  - 17% covers entire development period



## Best estimate (no-uncertainty) analysis


- **Give us your best guess**



## Fitting a collective risk model for aggregate losses

| | “Normal” (<$100k) losses | “Large” losses |
|---|---|---|
| Individual losses | λ=3,545<br>μ=$7,639<br>σ=$12,845 | λ=26.4<br>Pareto α=1.505<br>(up to policy limit $25mm) |
| Aggregate losses | Well-represented by a normal distribution<br>mean = $27.1mm<br>stdev = $0.89mm<br>VaR99.6 = $29.4mm | Must be evaluated numerically (e.g., simulation, FFT, …)<br>mean = $7.6mm<br>stdev = $4.0mm<br>VaR99.6 = $31.9mm |
| | **Combined, all losses** | |
| Aggregate losses | mean = $34.6mm<br>stdev = $4.1mm<br>VaR99.6 = $59.1mm | |
| Premiums & Expenses | Premiums = $34.6/60% = $57.7<br>Expenses = Premiums*30% = $17.3<br>Premium and expense assumptions are fixed; they do not vary with parameters | |



## Decision alternatives
## Per-risk excess layers, all paying to policy max (= $25mm)

All figures are $mm

| Retention | Aggregate Expected Payout | Aggregate R/I Premium |
|---|---|---|
| 1 | 1.380 | 7.57 |
| 2 | 0.869 | 5.42 |
| 5 | 0.411 | 3.48 |
| 10 | 0.191 | 2.11 |
| 20 | 0.038 | 0.58 |
| Bare | 0 | 0 |

## Evaluate alternatives at best estimate (MLE)

- How much capital is charged against the risk?
  - Pr{ Capital + NetPremium – Expenses - NetLosses < 0 } ≤ 0.4%
  - Capital = $VaR_{99.6\%}$ { Losses – RIrecovery } + RIprem - 40.4
- Evaluate over the alternatives

| Alternative (Retention) | Required Capital | Objective: Economic Underwriting Gain |
|---|---|---|
| $1mm | 3.93 | -1.582 |
| 2 | 4.83 | 0.167 |
| 5 | 6.87 | 1.400 |
| 10 | 9.51 | 2.131 |
| 20 | 15.41 | 2.512 Best |
| **Bare** | 18.68 | **2.497** Close 2nd |

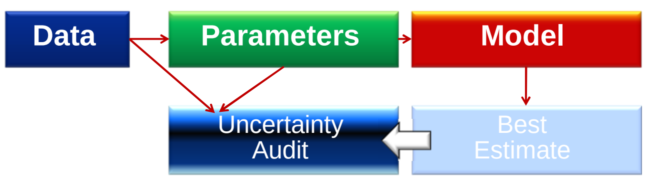


## Best estimate conclusions


- **Give us your best guess**
- **Excess-per-risk opportunities reduce the needed capital back-stop, but most of them are still too expensive to be worthwhile.**
- **The risk-adjusted profit maximizing alternative is the 5xs20 treaty**
  - **However, it is only $15,000 superior to retaining all risk.**



## Uncertainty audit – recognizing and quantifying uncertainty




- **How wrong might you be (and why)?**



## Sampling error vs. augmented uncertainties in basic model parameters

| "Normal" (<$100k) losses | "Large" losses |
|---|---|
| λ=3,545: s.e. = 22.5; Double for uncertainty in projecting historical rates ***s.e. = 45.0*** | λ=26.4: s.e. = 1.94; Double for uncertainty in projecting historical rates ***s.e. = 3.89*** |
| μ=$7,639: s.e. = 81.5; Double for uncertainty in trending and developing ***s.e. = 163.1*** | Pareto α=1.505: s.e. = 0.111; Double for uncertainty in trending and developing ***s.e. = 0.221*** |
| σ=$12,845: s.e. = 57.7 Double for uncertainty in trending and developing ***s.e. = 115.3*** ***Mutually independent*** | ***Mutually independent, and independent from normal loss parameters*** |

## How does parameter uncertainty translate to output uncertainty?
Simplest version of uncertainty propagation – the delta method

- Uncertain input parameters $\theta_1, \theta_2, \ldots, \theta_m$
  - covariance matrix $\mathrm{cov}(\theta_i, \theta_j) = \Sigma_{i,j}$
- Output quantity of interest $Y(\theta_1, \ldots, \theta_m)$
- Second-order Taylor expansion of variance

$$\mathrm{var}(Y) \approx (\nabla Y)^T \Sigma (\nabla Y)$$

$$\text{where} \quad \nabla Y = \left\langle \frac{\partial Y}{\partial \theta_1}, \frac{\partial Y}{\partial \theta_2}, \ldots, \frac{\partial Y}{\partial \theta_m} \right\rangle^T$$

- If uncorrelated, just look at the diagonal terms $\left(\frac{\partial Y}{\partial \theta_i}\right)^2 \cdot \sigma_i^2$



## Conclusions from uncertainty audit

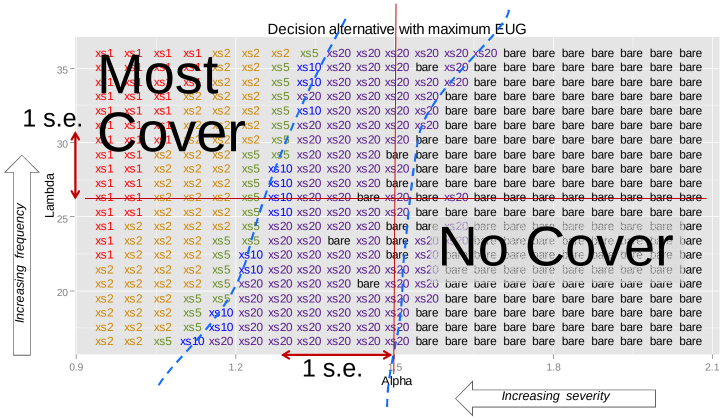
- **Conclusion 1:** Total standard error on EUG Bare is $2.93mm
  - ***This is larger than the 2.50 best estimate!*** (true of other XPRs, too)
- **Conclusion 2:** Normal loss parameters are relatively unimportant
  - Large loss parameter uncertainty covers 90% of the EUG variance (xs1: 73%)
  - In subsequent analysis, we will treat them as if they were known with certainty and only allow large loss parameters to vary.

| Parameter | Sensitivity | Uncertainty (std. err.) | Induced σ | % of total variance |
|---|---|---|---|---|
| Normal loss frequency | -0.010 X | 45.0 = | $0.47mm | 2.5% |
| " " severity mean | -0.005 | $163.1mm | 0.82 | 7.9 |
| " " severity std. dev. | ~0 | $115.3mm | ~0 | ~0 |
| Large loss frequency | -0.356 | 3.89 | 1.38 | 22.3 (89.6%) |
| " " Pareto tail index | (3.447-)* 10.856 | 0.221 | (0.76-)* 2.40 | 67.3 |

*(Low-retention XPR alternatives were less sensitive to changes in Pareto alpha)*

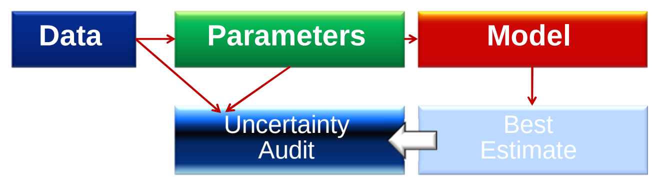


## **Conclusion 3:** No decision alternative dominates
The preferred alternative is a function of the parameters



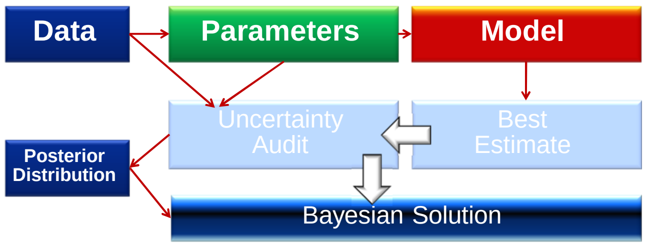



## Uncertainty audit conclusions




- **How wrong might you be (and why)?**
- Standard errors on EUG estimates exceed best estimates
- Best-EUG decision is very sensitive to model parameters
  - Shifts across multiple alternatives over a range of about 1.5 standard errors
- Most of uncertainty is attributable to the large loss model

## Bayesian solution – treating uncertainty as a kind of risk

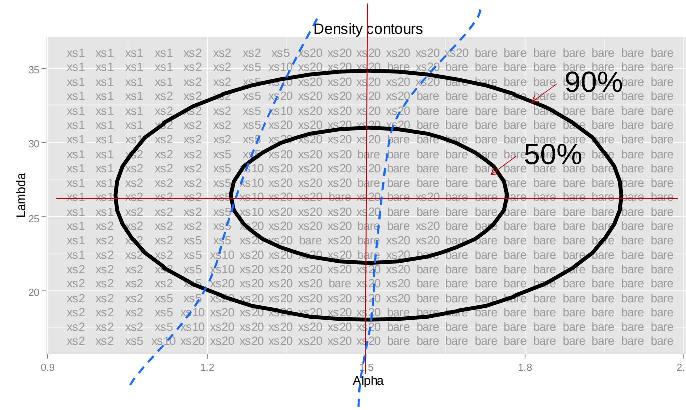



- **Treat uncertainty as another risk and update your answer**



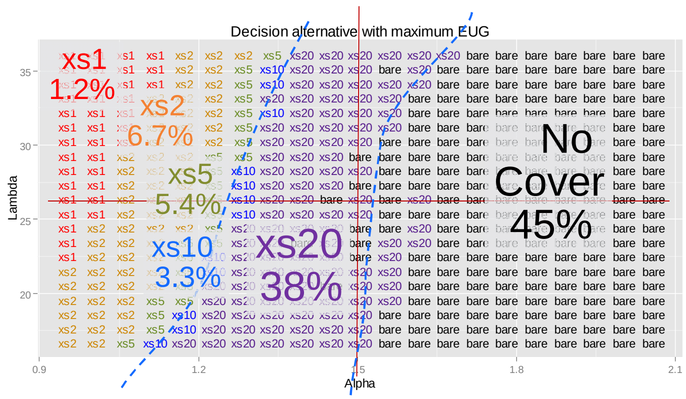
## Bayesian solution: impose a probability distribution on the parameters

Assume parameters follow a bivariate normal with specified mean and variance






## (1) Posterior probabilities associated with best alternatives






## (2) Posterior mean EUG

Average the EUG results over the parameter probability distribution

- Which decision alternative does the best "on average," that is, averaging over all parameter values according to their posterior probabilities?

| Decision (Retention) | Best Est. Economic Underwriting Gain | Posterior Mean EUG |
|---|---|---|
| $1mm | -1.582 | -1.680 |
| 2 | 0.167 | -0.112 |
| 5 | 1.400 | 0.934 |
| 10 | 2.131 | 1.477 |
| 20 | 2.512 Best | 1.773 Close 2nd |
| **Bare** | **2.497** Close 2nd | 1.855 Best |

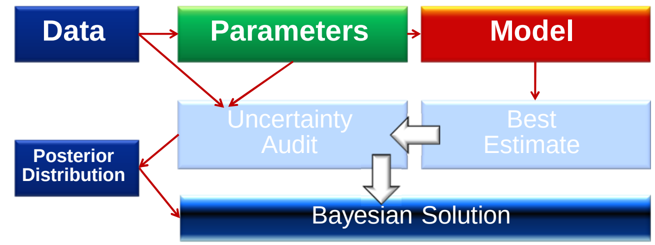
## (3) EUG performance against the predictive distribution
Mix the parameter-specific loss distributions over the parameter distribution

- Which alternative does best "overall," that is, treating uncertainty as part of the loss-generating process?

| Decision (Retention) | Best Est. EUG | Posterior Mean EUG | EUG of Predictive Distribution |
|---|---|---|---|
| $1mm | -1.582 | -1.680 | -1.778 |
| 2 | 0.167 | -0.112 | -0.214 |
| 5 | 1.400 | 0.934 | 0.815 |
| 10 | 2.131 | 1.477 | 1.315 |
| 20 | 2.512 | 1.773 | 1.626 Best |
| **Bare** | **2.497** | **1.855** | **1.566** Close 2nd |



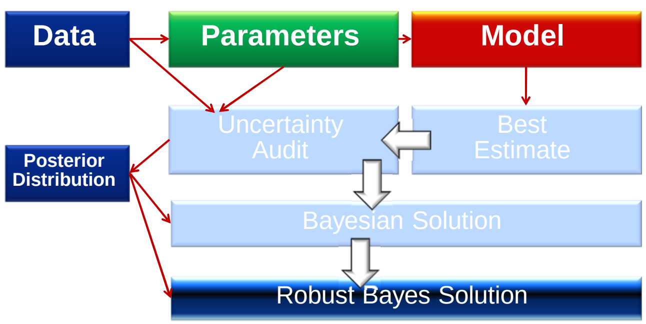
## Bayesian solution conclusions




- **Treat uncertainty as another risk and update your answer**
- Retaining all risk has a 45% chance of being the best decision
  - 5xs20 has a 38% chance; remaining 17% divided among lower retentions
- Retaining all risk has best average performance
- 5xs20 has best overall performance



## Robust Bayes solution – defensive measures




- **How do we protect the firm from Murphy's Law?**



## Game theory maximin principle – airport pickup example

You have a VIP to pick up in your limo

| | VIP arrives on time | VIP arrives late |
|---|---|---|
| **You arrive on time** | Everything's good; VALUE = TIP = $50 | You have to pay to wait; VALUE = TIP-$20 = $30 |
| **You arrive late** | VIP annoyed – NO TIP; VALUE = $0 | Everything's good; VALUE = TIP = $50 |

These are the worst outcomes for each of your decision alternatives
Netting $30 is better than netting $0. **$30 is the maximin.**

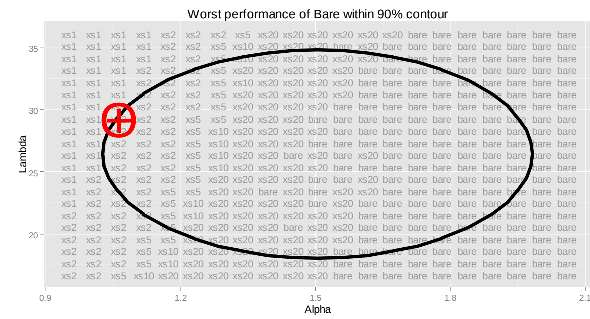
## Robustness: statistical decision as an adversarial game

- Gilboa & Schmeidler (1989) “Maxmin Expected Utility with Non-Unique Prior” Journal of Mathematical Economics
- “Nature” is your opponent
  - Opponent’s move: select a set of parameters for the stochastic process
- Your move: select a decision alternative (r/i program)
- Maximin: which selection has the best worst-case outcome across all the opponent parameter possibilities?
  - Wait a minute… *all* parameter possibilities?
  - No, you have to bound them somehow



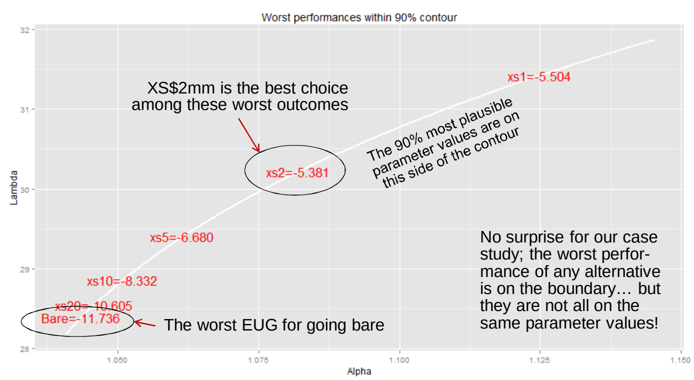
## Opponent’s moves (parameter choices) are considered bounded

- Boundary is usually based on implausibility relative to some observed data.
- Parameter values that are “too unlikely” are excluded from consideration.
- Say we only consider parameters within the 90% contour





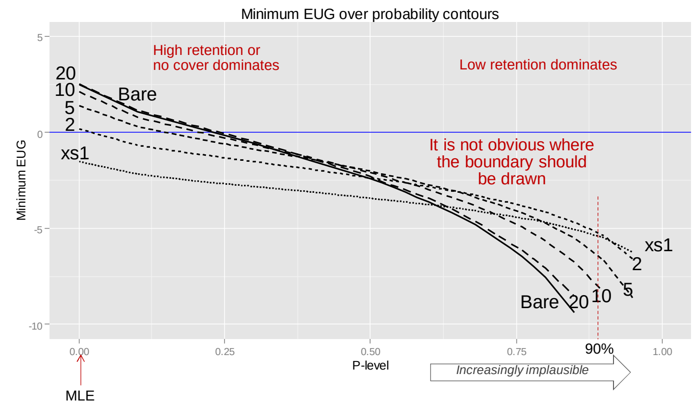

## A closer look at the worst-performance region



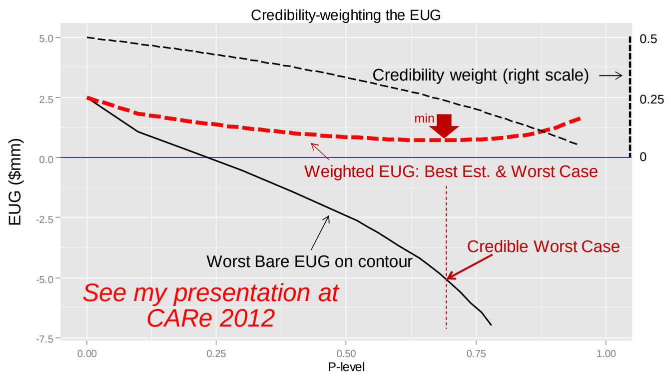



## Why 90%? Why not 60%, 80%, 95%, 99%,…?

Survey of worst-case outcomes over a large range of probability levels



## Credibility-weighting the best estimate EUG and the worst-case EUG by using the likelihood ratio between them




Unless the downside explodes exponentially, there *will* be a minimum



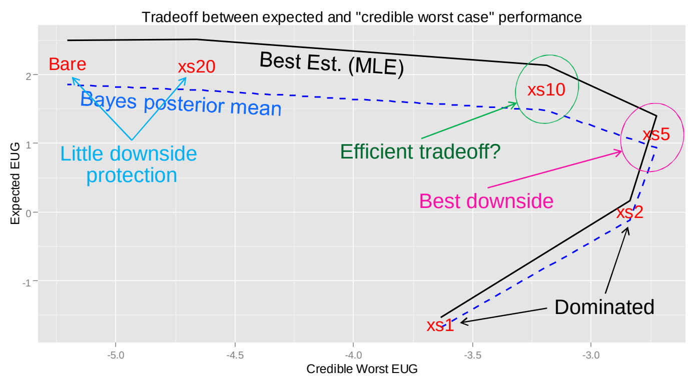
## Evaluate the alternatives at the natural stress test points

- Which alternative performs best under its worst credible circumstances?

| Decision | Best Est. EUG | Posterior Mean EUG | Predictive EUG | EUG of Credible Worst Case |
|---|---|---|---|---|
| $1mm | -1.582 | -1.680 | -1.778 | -3.634 |
| 2 | 0.167 | -0.112 | -0.214 | -2.838 Close 2nd |
| 5 | 1.400 | 0.934 | 0.815 | -2.727 Best |
| 10 | 2.131 | 1.477 | 1.315 | -3.188 |
| 20 | 2.512 | 1.773 | 1.626 | -4.661 |
| **Bare** | **2.497** | **1.855** | **1.566** | **-5.205** |
| | No uncertainty | Treating uncertainty like risk | | Uncertainty stress test |

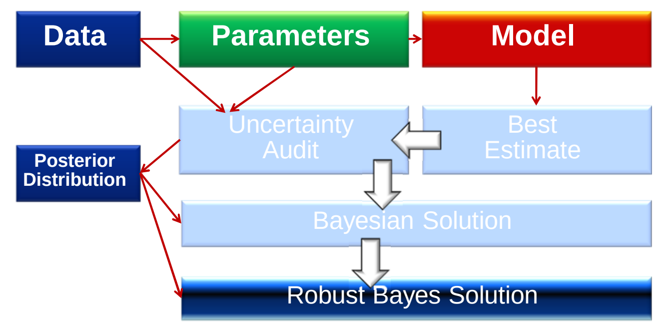


## Visualizing expected and worst-case performance






## Robust Bayes solution - conclusions




- **How do we protect the firm from Murphy's Law?**
- 20xs5 gives best protection against credible worst case
- 15xs10 looks like an efficient upside/downside tradeoff

## Summary

- The four stages:
  - Best Estimate = *"Give us your best guess"*
  - Uncertainty Audit = *"How wrong might you be (and why)?"*
  - Bayes = *"Treat uncertainty as another risk and update your answer"*
  - Robust Bayes = *"How do we protect the firm from Murphy's Law?"*
- You already know how to do the first three; the fourth isn't much harder



## Call to action

- **The point: "model risk" is alive and well**
  - **Your client deserves an effort to address it**
    - ASOPs 17, 36, 41, 43; modeling exposure draft
  - **No excuse for pretending or implying it doesn't exist**
    - "…silence would have made me feel guilty of complicity" - Einstein
- **Yes, it's going to take many model runs**
  - **…or at least many runs of a *surrogate* model**
  - **Plan to build out your capabilities accordingly**
- **Go forth and make the world a better place!**



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