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MODEL UNCERTAINTY

CAS Seminar on Reinsurance – May 21-22, 2014

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Agenda

1. What is Model Uncertainty?
2. Estimating Model Uncertainty
3. Examples for Reinsurance Pricing
   - Multiyear Contracts
   - Accident Year Stop Loss (AYSL)
   - Disguised Stop Losses
   - Per-Occurrence Excess Treaty
4. Conclusions

What is Model Uncertainty?

Context: Decision-making in the face of uncertainty.

- How much to quote on a reinsurance contract
- Whether or not to accept offered terms
- How much to set aside for reserves on a contract or portfolio

The "**uncertainty**" is that we do not know everything needed to guarantee the outcome of our decision.

The "**risk**" is that we make a bad decision because of this uncertainty (and therefore lose money).

Sources of Uncertainty

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- PROCESS VARIANCE (randomness, “stuff happens”)
- ESTIMATION UNCERTAINTY (sample size in estimating expected loss)
- MODEL UNCERTAINTY
  - Pricing Parameters treated as known
    1) Frequency and Severity trends
    2) Rate Change (market cycle)
    3) Loss development and tail factor
  - Selection of distribution forms, and independence assumptions
  - Social or Political Change
  - Management or Business Strategy Change
  - Data Quality and/or Bias in Data Available

Sources of Model Uncertainty

MODEL UNCERTAINTY exists because our estimate of expected loss is made using many assumptions.

MODEL RISK exists because getting these assumptions wrong can lead to a financial loss.

Some assumptions are formal: e.g., selection of inflation trend.

Some assumptions are informal: e.g., data quality, business strategy changes

Uncertainty in assumptions exists whether we have a formal statistical model or not.

There is model uncertainty in "expert judgment."

Estimating Uncertainty in Pricing Factors

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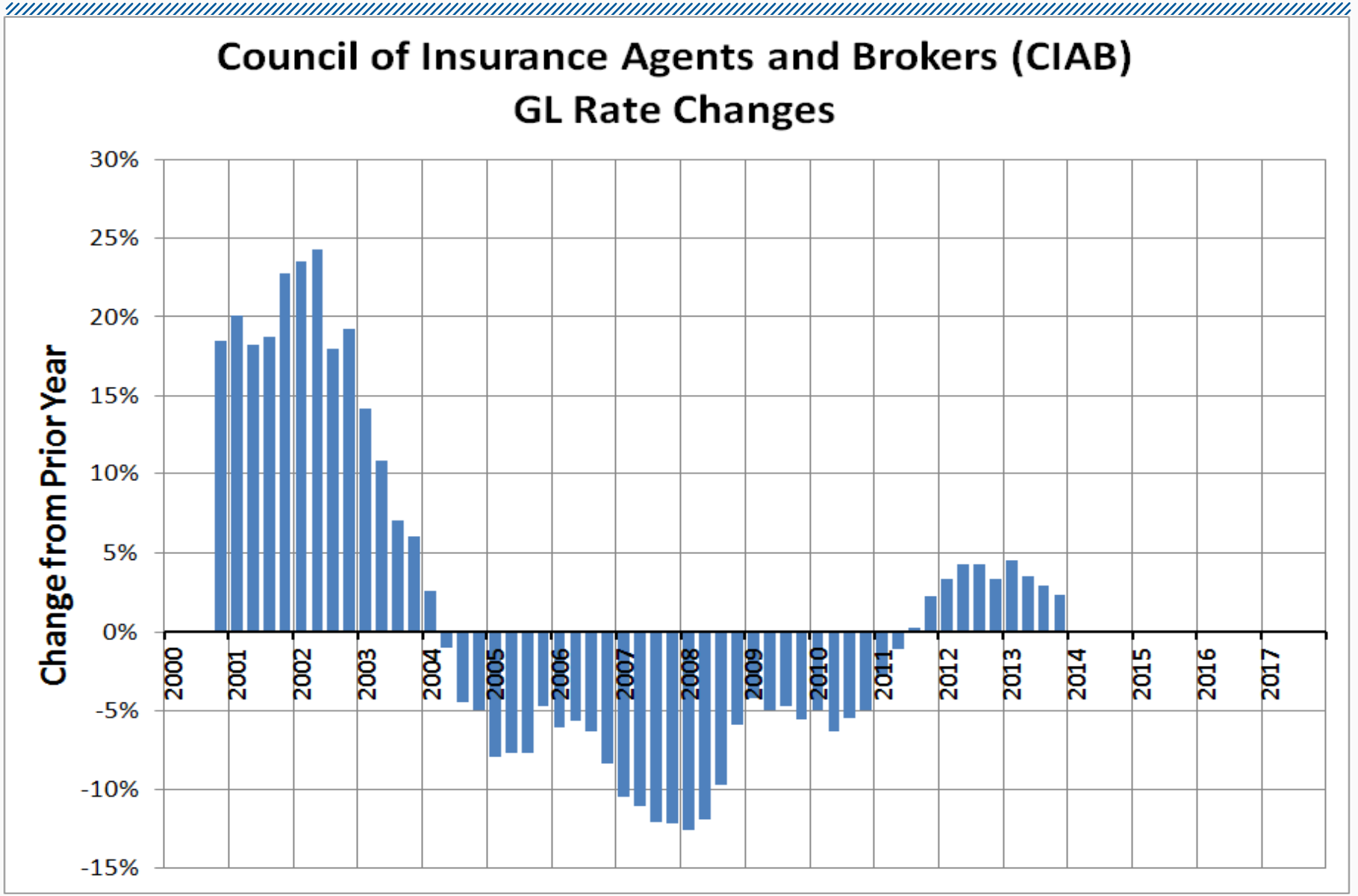

Estimating Uncertainty in Pricing Factors

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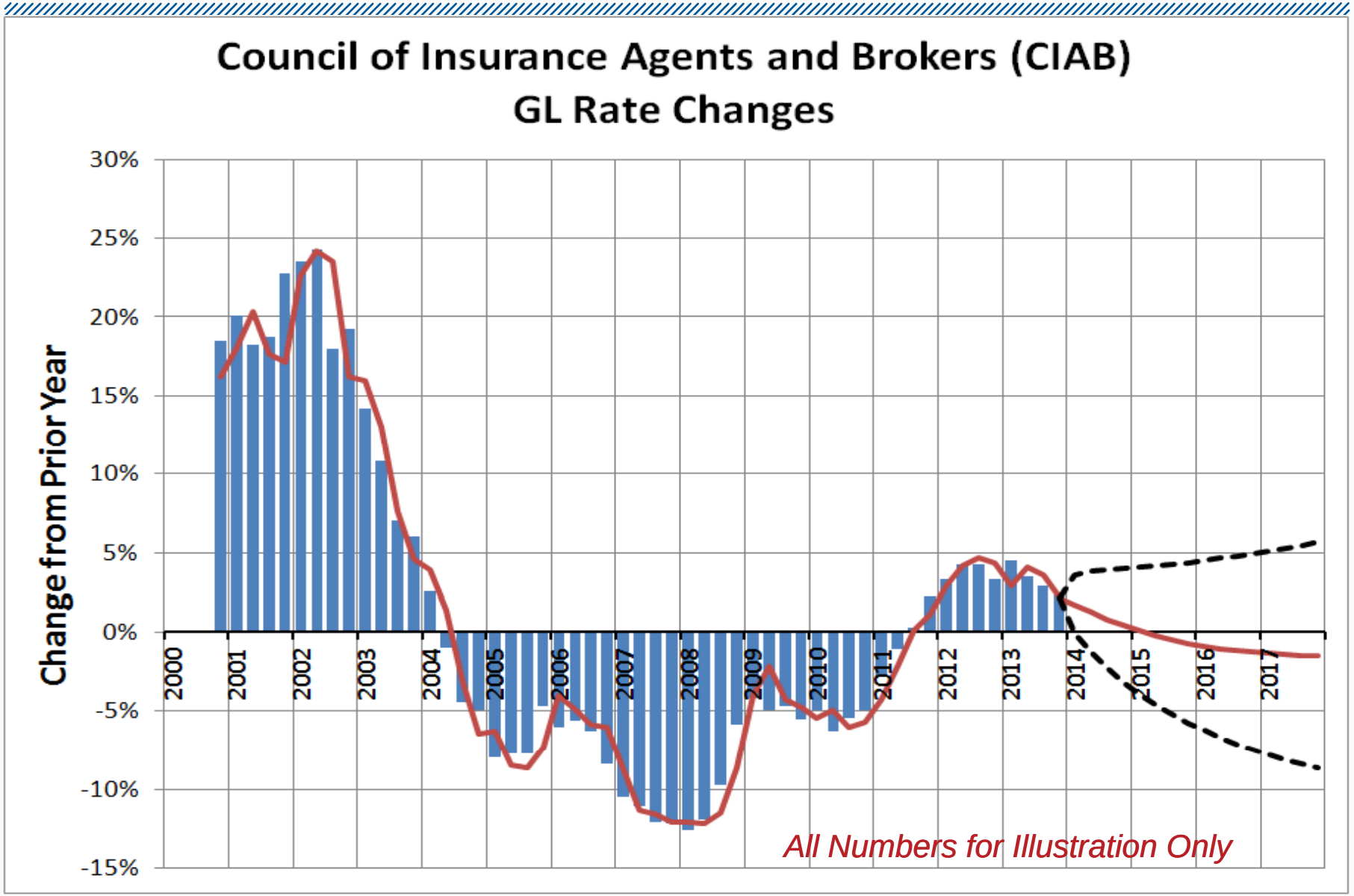

Estimating Uncertainty in Pricing Factors

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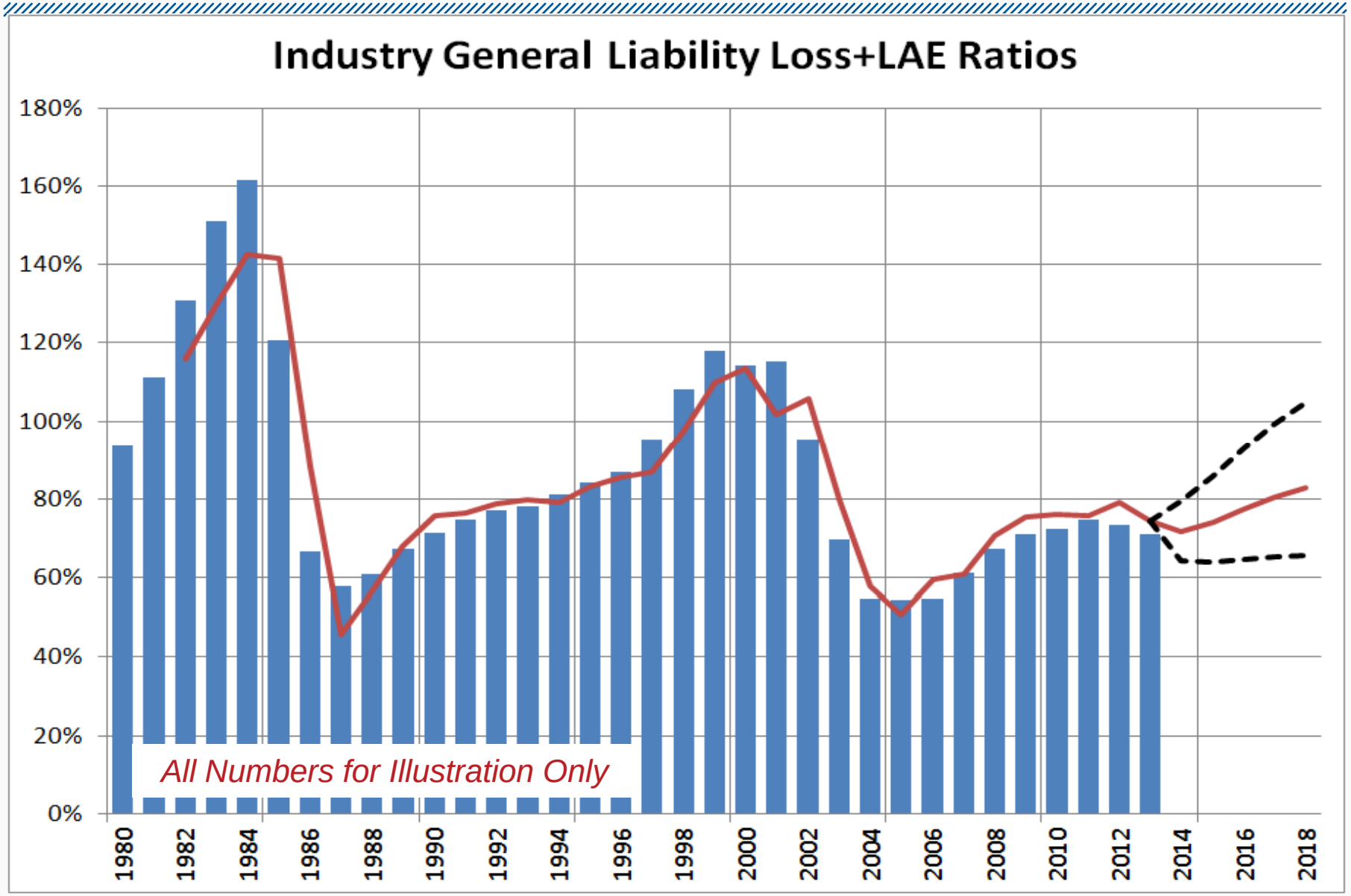


Including Model Uncertainty in Pricing

If we have an estimate of the impact of model uncertainty, in terms of a standard deviation or coefficient of variation (CV), it can be included in pricing.

We introduce a secondary “mixing” variable (Y), applied as a multiplier to the loss amount (X). In a collective risk model, secondary variable(s) can be applied to either frequency, severity, or both.

The increase in CV after mixing is given by a simple formula:

$$CV_{X \cdot Y} = \sqrt{CV_X^2 + CV_Y^2 + CV_X^2 \cdot CV_Y^2}$$

Including Model Uncertainty in Pricing

There are some special cases where mixing can be incorporated analytically.

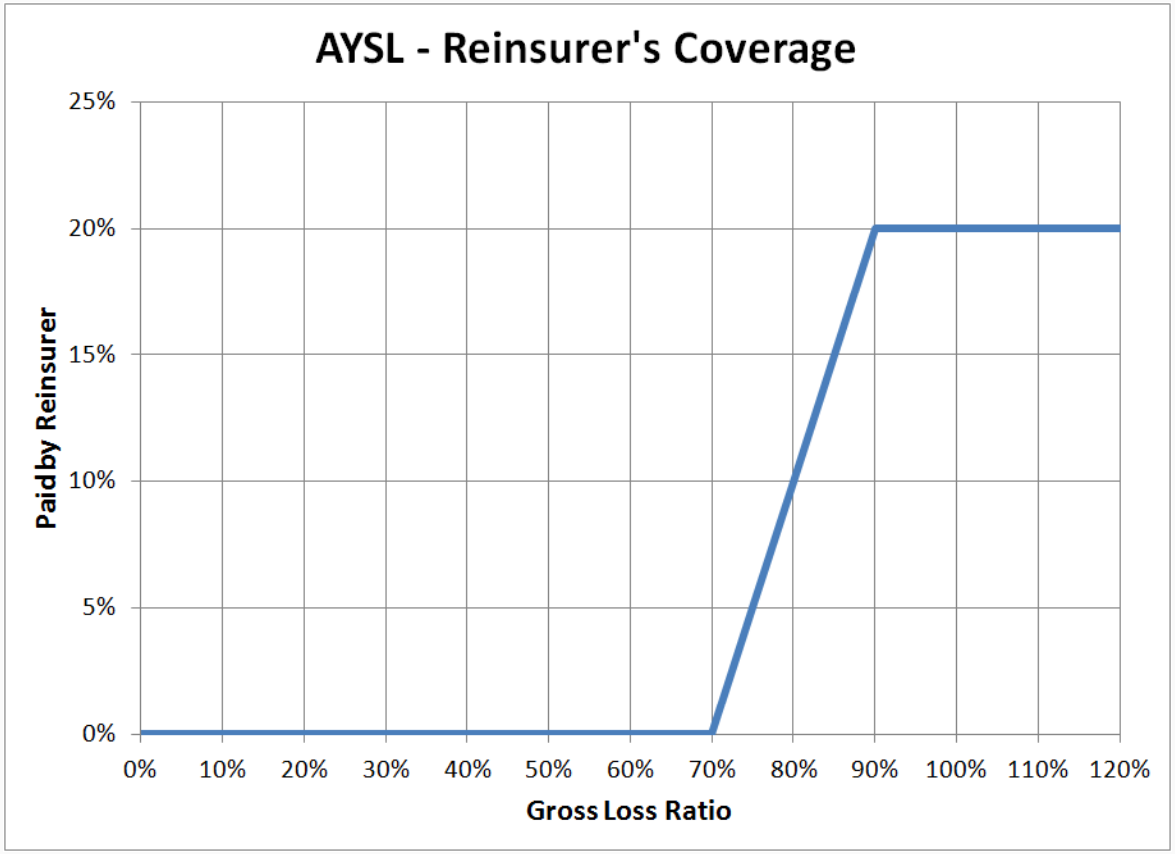
Process Variance	Mean Mixed with	Mixed Distribution
Exponential	Gamma	Pareto
Poisson	Gamma	Negative Binomial
Lognormal	Lognormal	Lognormal

Where an analytic form is overly complicated, method-of-moments or simulation can be used.

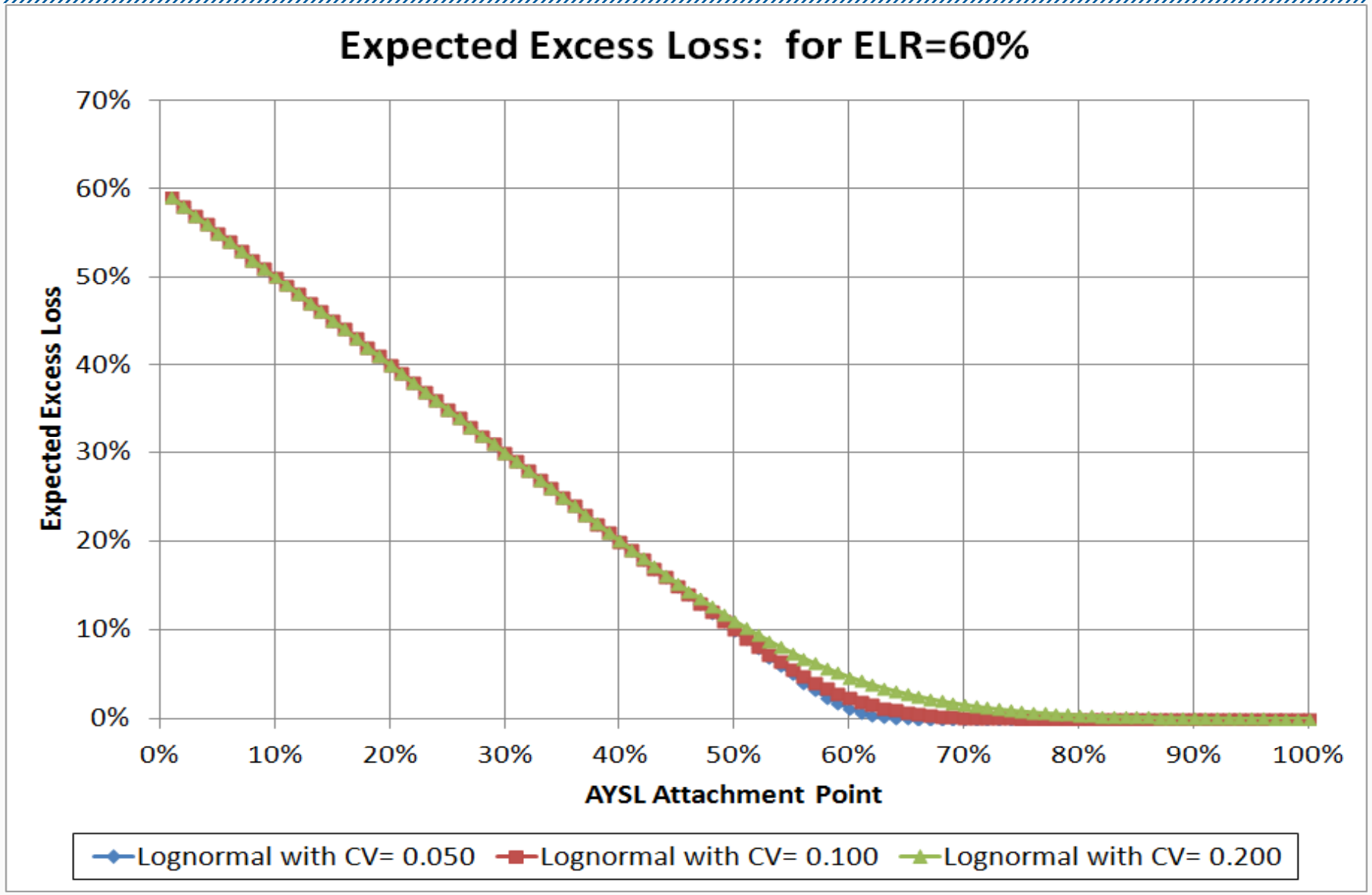
Accident Year Stop Loss (AYSL)

An accident year stop loss (AYSL) is a reinsurance treaty that covers aggregate losses in excess of some attachment point, defined in terms of a loss ratio.

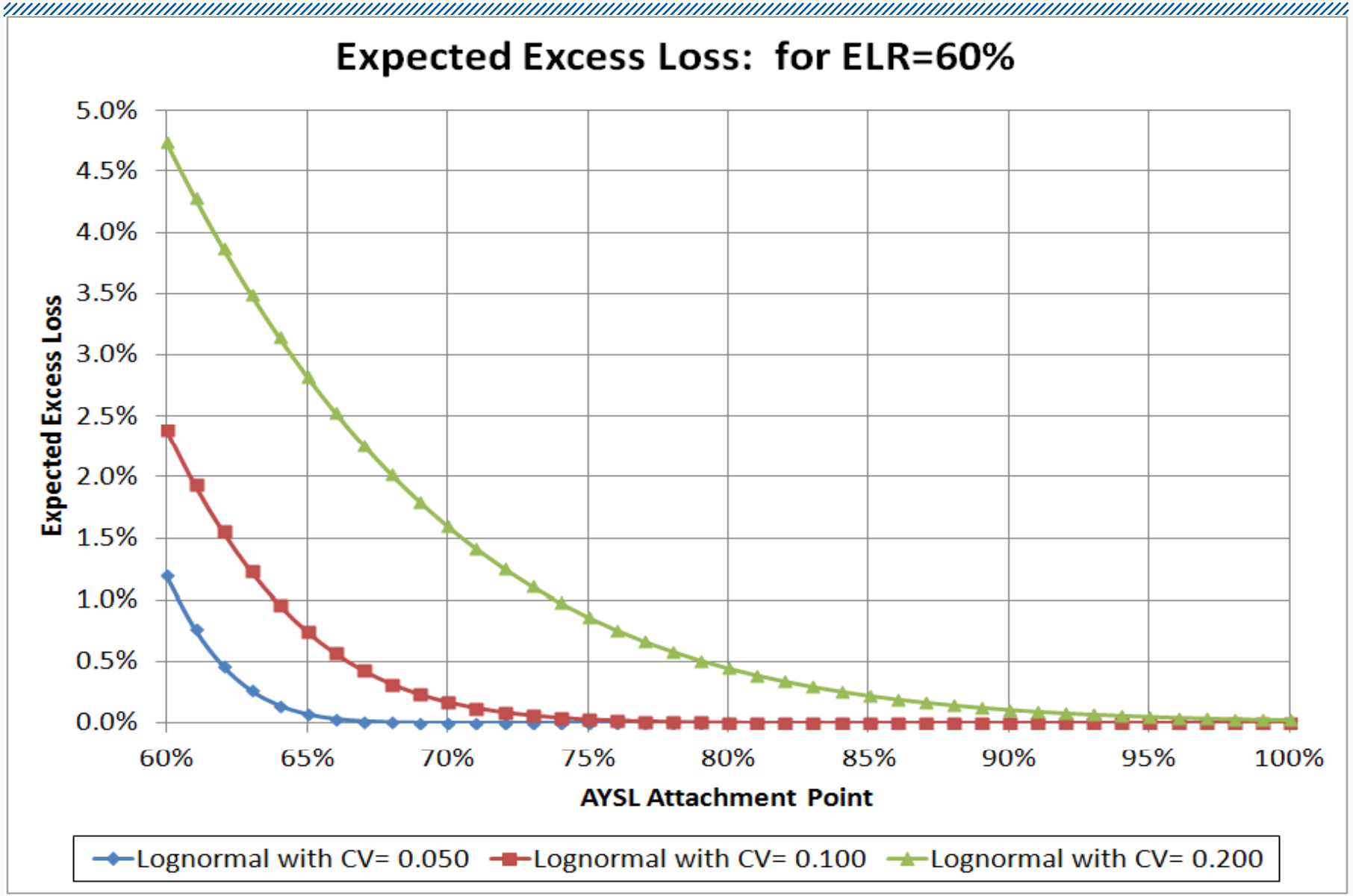
Example: Reinsurer covers losses excess of 70% LR, up to 90%



Accident Year Stop Loss (AYSL)



Accident Year Stop Loss (AYSL)



Stop Loss and Disguised Stop Loss Treaties

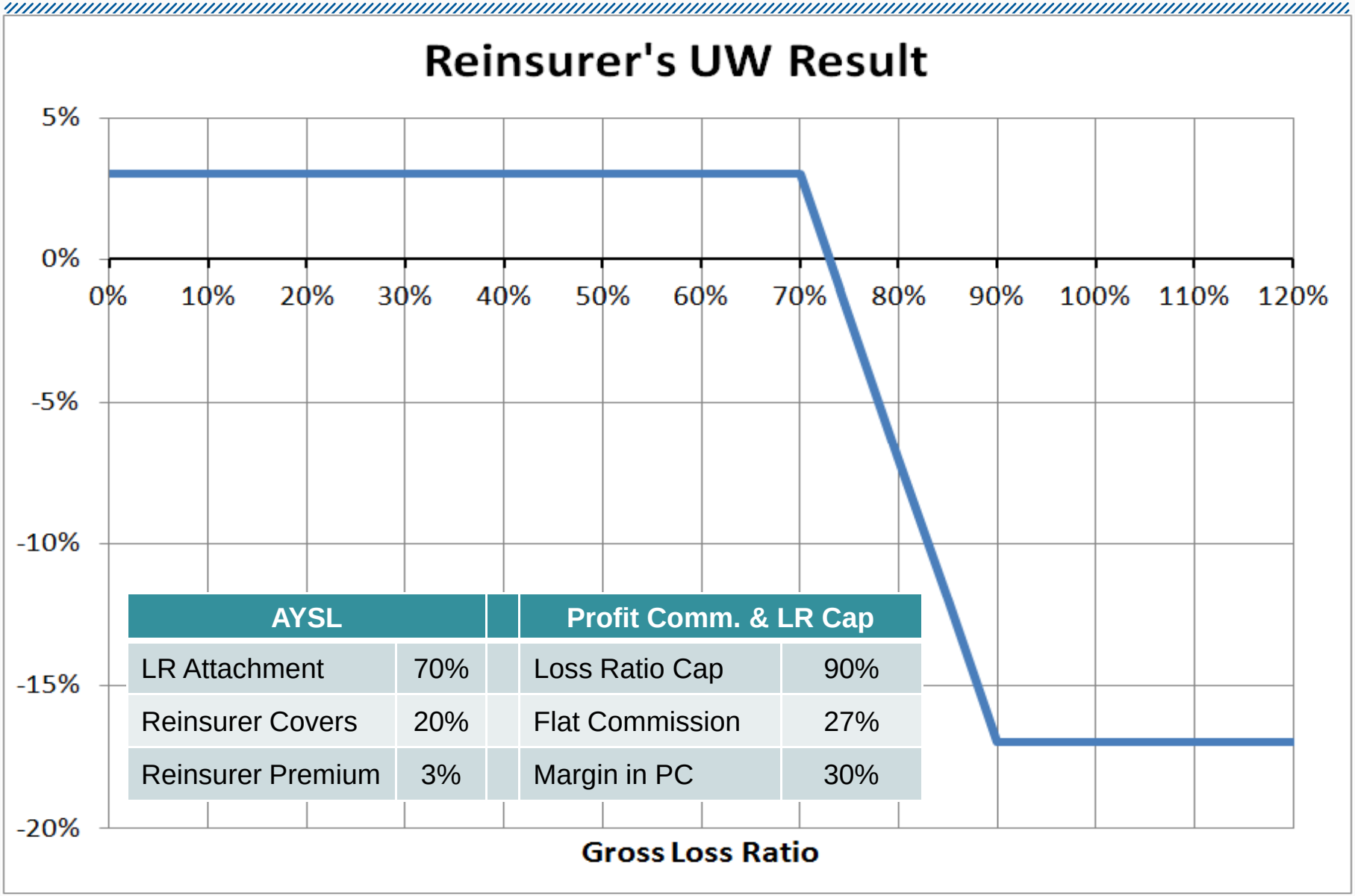
The profitability of the accident year stop loss (AYSL) is dependent upon the variance of the loss distribution.

Other structures are equally sensitive to the loss distribution, but not always recognized as being equivalent to AYSL.

AYSL			Profit Comm. & LR Cap	
LR Attachment	70%		Loss Ratio Cap	90%
Reinsurer Covers	20%		Flat Commission	27%
Reinsurer Premium	3%		Margin in PC	30%

Loss-sensitive treaty structures are sensitive to variance assumptions.

Stop Loss and Disguised Stop Loss Treaties



AYSL			Profit Comm. & LR Cap	
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Model Uncertainty in Per-Occurrence Excess

Per-occurrence excess treaties cover individual losses in excess of some attachment point. These treaties are typically priced using experience and exposure rating models. We will focus on exposure rating.

Exposure rating requires a detailed size-of-loss distribution, used with the ceding company policy limit profile.

The mixed exponential model includes a weighted average exponential distributions.

Table	Mean	Weight
1	1,526	0.350000
2	6,104	0.350000
3	24,414	0.180000
4	97,656	0.072000
5	390,625	0.028800
6	1,562,500	0.011520
7	6,250,000	0.004608
8	25,000,000	0.001843
9	100,000,000	0.001229
For illustration only		1.000000

Model Uncertainty in Per-Occurrence Excess

We can include model uncertainty as a mix applied to each exponential. Uncertainty may be due to severity inflation or tail factor estimation.

This turns the size-of-loss distribution into a weighted average of Paretos.

Table	Mean	Weight	Param CV	Pareto Q
1	1,526	0.350000	0.25	16
2	6,104	0.350000	0.25	16
3	24,414	0.180000	0.25	16
4	97,656	0.072000	0.25	16
5	390,625	0.028800	0.25	16
6	1,562,500	0.011520	0.25	16
7	6,250,000	0.004608	0.25	16
8	25,000,000	0.001843	0.25	16
9	100,000,000	0.001229	0.25	16
For illustration only		1.000000		

	LEV at 1,000,000	LEV at 2,000,000	Exposure 1Mx1M
Mixed Exponential	40,273	52,137	22.76%
Mixed Pareto	39,931	51,720	22.79%

Model Uncertainty in Per-Occurrence Excess

Why is there little impact on per-occurrence excess pricing?

It is because the random variability is much greater than the model uncertainty, and therefore does not greatly change the size-of-loss distribution.

$$CV_X = 1 \quad \text{Random variability}$$

$$CV_Y = .25 \quad \text{Model uncertainty}$$

$$CV_{X.Y} = \sqrt{CV_X^2 + CV_Y^2 + CV_X^2 \cdot CV_Y^2} \approx 1.06$$

HOWEVER: The impact of model uncertainty due to inflation or market cycle is a systematic risk that does not diversify away. It means that model uncertainty is more significant in a portfolio ERM analysis than in pricing for an individual treaty.

Conclusions

- Model uncertainty can be significant.
- Model uncertainty exists even if you think you are not using a model.
- Some components of model uncertainty can be reasonably estimated and incorporated into pricing.
- Some reinsurance products are especially sensitive to model uncertainty and require careful attention:
  - Accident Year Stop Loss (AYSL)
  - Other loss-sensitive features acting like AYSL
  - Multiyear programs

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THANK YOU FOR YOUR ATTENTION

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