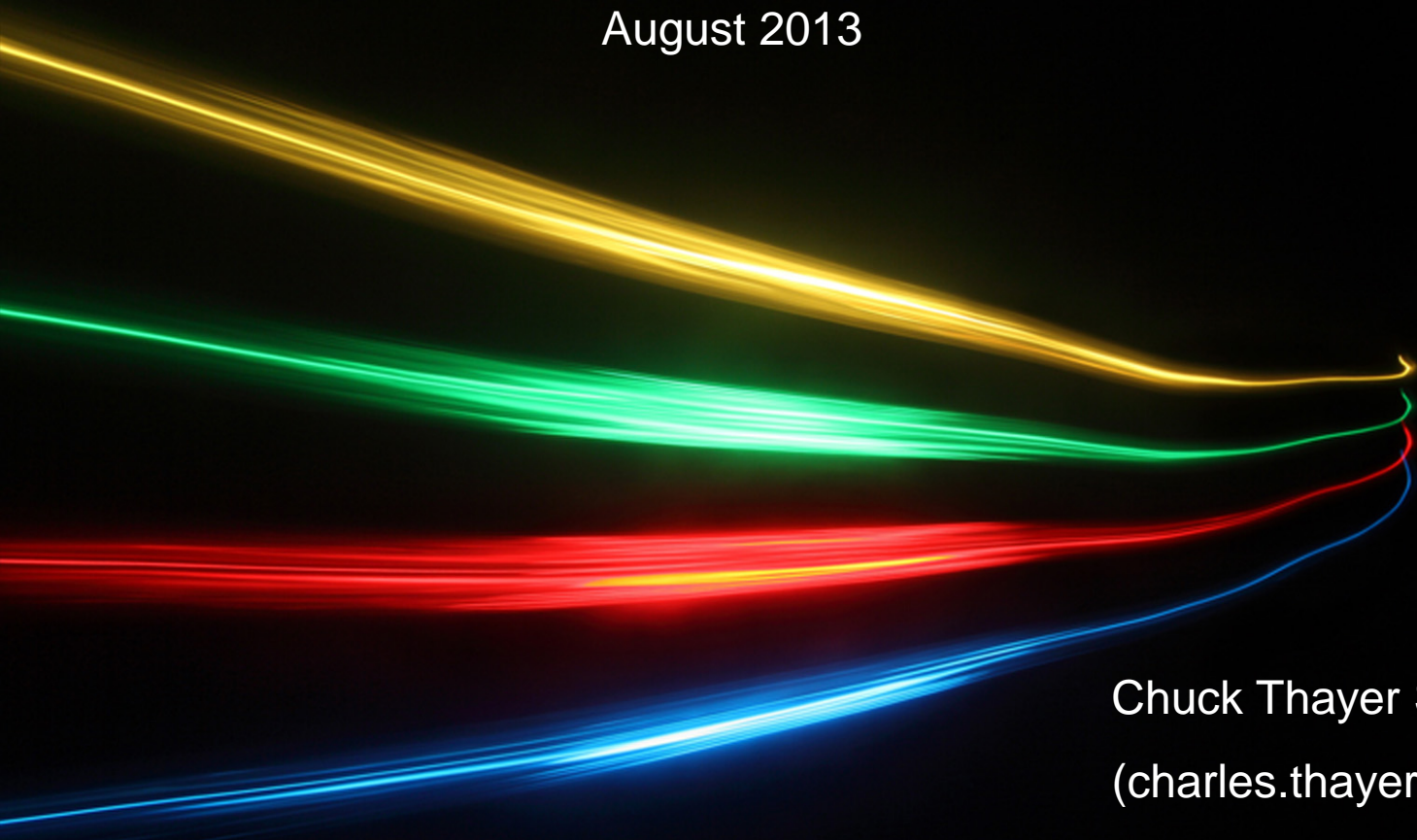


# LOSS SENSITIVE TREATY FEATURES

CAS Boot Camp on Reinsurance  
Pricing Techniques

August 2013



Chuck Thayer SVP, Willis Re  
([charles.thayer@willis.com](mailto:charles.thayer@willis.com))

# Goals for today

- Define the term: loss sensitive feature
- Purpose: Resolve differing views on pricing and risk
- How to value loss sensitive features: QS, XOL
- Describe basic tools for simulation
- Question time

# THE BASICS

Willis Re

MANAGING EXTREMES

Define “loss sensitive features”

Features by treaty type

Zooming in : profit commissions



# Definition

- Loss sensitive features in a reinsurance contract:
  - Adjust treaty behavior based on loss experience, to bridge loss pick gaps in pricing negotiations
  - Can affect ceded premium, losses, or expenses
  - Can be combined to create incentives to manage the quality of the subject book of business
- Premiums or commissions start at “provisional” level
  - Index up or down in response to loss activity
- Loss terms may involve sharing losses among parties
  - Degree and type of loss sharing affects final cost

# Why loss sensitive terms?

- Treaty pricing aim: Leave everyone “equally unhappy”
- Reinsurer loss picks can seem high to ceding insurers
  - Reinsurers conservative due to lack of information
  - Insurer is confident about underwriting expertise
- Loss sensitive terms make a treaty placement viable
  - Adjust initial premium/commissions retrospectively
  - Limit exposure in exchange for rate concession
- Any concessions are conditional on good experience
  - Loss sensitive terms can settle “bets” on loss picks
  - Each side needs to know the value of these bets

# Types of loss sensitive terms?

- Ceded Premium “concessions” made possible by terms that adjust premiums retrospectively as losses come in
  - Reinstatement provisions, or additional premiums
  - Swing-rated contracts
- Ceded Commission: Sliding scale ceding commission and profit commissions even things up via commission adj’s
- Ceded Loss features directly affect exposure to treaty, cutting premiums by reducing dollar-trading
  - Annual aggregate deductibles (AAD)
  - Loss ratio corridors and caps
  - 2<sup>nd</sup> or 3<sup>rd</sup> event covers can have specific triggers

# Features used by treaty type

- Pro rata / QS treaties
  - Profit commission
  - Sliding scale
  - Loss corridor (% , \$)
  - Aggregate cap (% , \$)
  - Event cap
- Excess of Loss (XOL)
  - Profit commission
  - Reinstatements
  - Swing rating plans
  - No Claims Bonus
  - AADs
  - Annual Agg Limits (\$)
  - Loss Ratio Cap (%)
  - *Experience funds (out of scope ☹)*

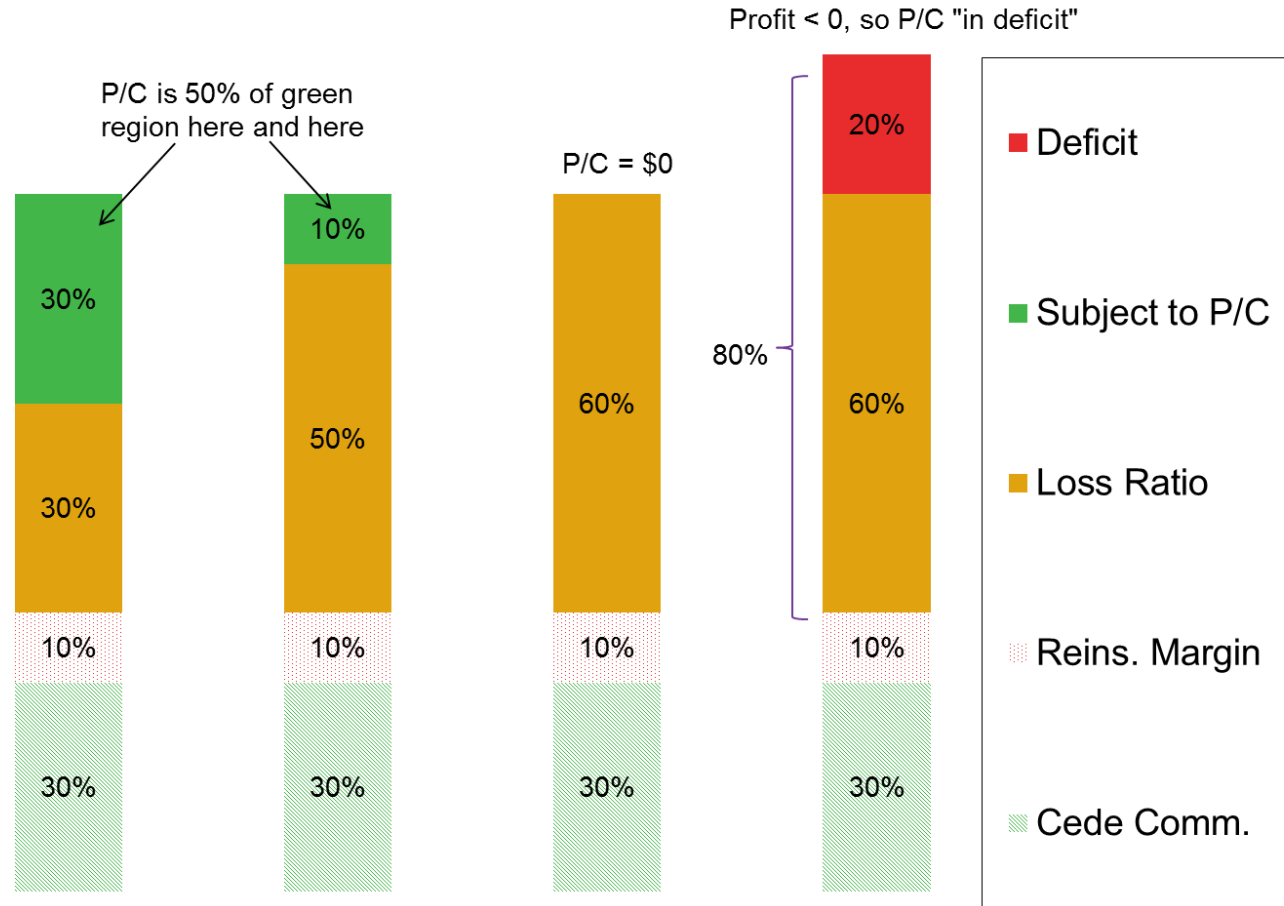
# Profit commission

- Used in Quota Share or XOL to reward good results, so it can be used to settle pricing disputes
  - Idea: Cedent gets defined share of treaty “profit”
- Profit formulas vary, but typically
  - Profit = Premium – Loss – Commission – Margin
  - “Margin” is a provision for reinsurer expenses
- PCs often given using shorthand: “50 after 10” (%)
  - With flat ceding commission of 30%, we have
  - Profit (%) =  $(1 - 30\% \text{ CC} - 10\% \text{ RM} - \text{LR}\%)$
  - So PC % =  $50\% \times \max(0, 60\% - \text{Loss Ratio } \%)$



# Profit commission "illustrations"

- "50 after 10"
- 30% "cede"
- 10% "margin"
- Loss Ratios:
  - 30%,
  - 50%,
  - 60%,
  - 80%.
- Last case 80% LR lands in "deficit"



# Profit commission – value?

- Question: What's the **expected** cost of the PC?
- Suppose the treaty ELR is 60%, where PC is zero
  - Does this imply that expected cost of PC is zero?
- Uh, no. Expected Cost(PC)  $\neq$  PC at Expected LR
- Why? Don't cite Jensen's Inequality. Show us and tell us.
  - 60% is the **expected** LR, not the only possible LR
  - There's a probability distribution around the ELR
  - Some possible values trigger payments on PC
- Numerical illustrations, pictures and animations help you drive your point home. Use these tools to communicate.

# Profit commission – oversimplified example

- Profit Commissions are a one-way street. They pay in good times, but don't surcharge in bad years.
- California property QS with EQ exposure (all/nothing)
  - Non-Cat ELR = 40% (certain)
  - PC is 50 after 10, net of 30% ceding commission
  - Cat (EQ) ELR = 30%, based on at most 1 EQ/yr
    - (LR | No EQ) = 0%, Pr[No EQ] = 90%
    - (LR | EQ) = 300%, Pr[EQ] = 10%
- Results: What's the value of PC with and without EQ?
- Expected cost PC:  $10\% \times \text{PC}(\text{EQ}) + 90\% \times \text{PC}(\text{No EQ})$

# Profit commission – oversimplified example answers

Answers: **Did you get these?**

- With 50 after 10, and a 30% Cede
  - PC | No EQ =  $0.5 \times (1 - 30\% - 10\% - 40\% - 0\%)$ 
    - Value is 10%. Right?
  - PC | EQ =  $0.5 \times (1 - 30\% - 10\% - 40\% - 3\%)$ 
    - Profit is quite negative, so PC = 0.
- Expected cost PC:  $10\% \times \text{PC}(\text{EQ}) + 90\% \times \text{PC}(\text{No EQ})$ 
  - $90\% \times 10\% + 10\% \times 0\% = 9\%$  of Ceded Premium

# VALUATION PRINCIPLES

Willis Re

MANAGING EXTREMES

General workflow

Loss sensitive features on pro rata treaties

Loss sensitive features on XOL treaties

Comments on multi-year terms



# General workflow: Cost / benefit of loss sensitive features

- Create an Aggregate Loss Distribution
  - Think of it as a discrete list of possible Loss Ratio outcomes with assigned probabilities
- You can fit adjusted historical premium/loss data to curve
  - Aggregate loss ratio distribution (e.g. Lognormal)
  - Fit Frequency/Severity distributions and simulate
    - Detailed curve-fitting is **out of scope** 😞
- Apply loss sensitive terms at each table row or scenario
- Find probability-weighted average cost (benefit) of the loss sensitive features in the contract

# Profit commission: “50 after 10” revisited: 30% Cede, 60% ELR

	Prob	LR	Cede	Cost of PC at LR	UW Ratio
1	4.0%	25.0%	30.0%	17.5%	72.5%
2	10.0%	35.0%	30.0%	12.5%	77.5%
3	20.0%	40.0%	30.0%	10.0%	80.0%
4	25.0%	50.0%	30.0%	5.0%	85.0%
5	20.0%	60.0%	30.0%	<b>0.0%</b>	90.0%
6	15.0%	70.0%	30.0%	0.0%	100.0%
7	2.0%	80.0%	30.0%	0.0%	110.0%
8	2.0%	145.0%	30.0%	0.0%	175.0%
9	1.0%	350.0%	30.0%	0.0%	380.0%
10	1.0%	450.0%	30.0%	0.0%	480.0%
Total	100.0%	60.0%	30.0%	<b>5.2%</b>	95.2%

- Once again, we can see that Expected Cost of PC is not equal to the PC Cost evaluated at the Expected Loss Ratio.

# Huh? Why doesn't this work?

- Key point: Loss distribution determines the answer
  - Distribution assigns probability to each LR value
  - Loss ratio determines PC (\$) for scenario or value
- With skewed distributions seen in reinsurance, you may often pay the cedent under a PC arrangement
  - Esp. true for XOL, but you always need lots of favorable scenarios to balance extreme scenarios
  - Favorable scenarios are the ones that trigger PC
- Loss distribution drives all of your pricing. Take care.
  - Loss sensitive feature cost estimates and overall treaty pricing assumptions must line up (not easy)



# Profit commission: Vary the loss distribution assumption

	Prob	LR	Cede	Cost of PC at LR	UW Ratio
1	0.0%	25.0%	30.0%	17.5%	72.5%
2	1.0%	35.0%	30.0%	12.5%	77.5%
3	15.0%	40.0%	30.0%	10.0%	80.0%
4	25.0%	50.0%	30.0%	5.0%	85.0%
5	30.0%	60.0%	30.0%	<b>0.0%</b>	90.0%
6	20.0%	70.0%	30.0%	0.0%	100.0%
7	6.0%	80.0%	30.0%	0.0%	110.0%
8	3.0%	145.0%	30.0%	0.0%	175.0%
9	0.0%	350.0%	30.0%	0.0%	380.0%
10	0.0%	450.0%	30.0%	0.0%	480.0%
Total	100.0%	60.0%	30.0%	<b>2.9%</b>	92.9%

- What if your loss distribution is more like this?

# Profit commission: Vary the loss distribution assumption

	Prob	LR	Cede	Cost of PC at LR	UW Ratio
1	0.0%	25.0%	30.0%	17.5%	72.5%
2	0.0%	35.0%	30.0%	12.5%	77.5%
3	0.0%	40.0%	30.0%	10.0%	80.0%
4	33.3%	50.0%	30.0%	5.0%	85.0%
5	33.3%	60.0%	30.0%	<b>0.0%</b>	90.0%
6	33.3%	70.0%	30.0%	0.0%	100.0%
7	0.0%	80.0%	30.0%	0.0%	110.0%
8	0.0%	145.0%	30.0%	0.0%	175.0%
9	0.0%	350.0%	30.0%	0.0%	380.0%
10	0.0%	450.0%	30.0%	0.0%	480.0%
Total	100.0%	60.0%	30.0%	<b>1.7%</b>	91.7%

- Or maybe like this?

# Loss sensitive features for QS (proportional) treaties

- Pro Rata: Quota Share, Surplus Share treaties
  - Profit commission (seen this already)
  - Sliding scale commission
  - Loss corridor
  - Loss ratio cap
- Event caps can be written into QS contract
  - Usually applies when QS is underneath XOL
  - Use net aggregate loss distribution after XOL, including mass point created at XOL retention.
  - So this is really an XOL topic, not purely QS

# Sliding scale commissions

- Applies when parties disagree on the loss ratio pick
  - Provisional Cede paid at start pegged to implied ELR (say, “30 at a 60”), then “slides” with LR
  - Adjusts **up** as the LR goes **down**, up to a Max
  - Adjusts **down** as the LR goes **up**, down to a Min
- In good years, slide increases cedent net profit by lowering net expense, so net leverage unaffected.
- In bad years, reinsurer gets some margin protection from rebated commission.
- “*Put your money where your mouth is*” provision.

# Sliding scale example

- Suppose sliding scale terms are given by this table:

	Cede	@	LR	Cede + LR	Margin
Min	15%	@	75%	90%	10%
Prov	20%	@	65%	85%	15%
Max	25%	@	60%	85%	15%

- Provisional Cede “20 at a 65” goes up/down with LR
- If the Loss Ratio turns out to be:
  - < 65%: Slides up 1:1 for each 1% LR drop to 25%
  - > 65%: Drops by ½:1 each add'l +1% LR to 15%
- Question: So... If ELR = 60%, is the Expected Ceding Comm. equal to Ceding Comm. at Expected LR?

# Value a sliding scale commission

	Prob	LR	Cede	UW Ratio
1	4.0%	25.0%	25.0%	50.0%
2	10.0%	35.0%	25.0%	60.0%
3	20.0%	40.0%	25.0%	65.0%
4	25.0%	50.0%	25.0%	75.0%
5	20.0%	60.0%	<b>25.0%</b>	85.0%
6	15.0%	70.0%	17.5%	87.5%
7	2.0%	80.0%	15.0%	95.0%
8	2.0%	145.0%	15.0%	160.0%
9	1.0%	350.0%	15.0%	365.0%
10	1.0%	450.0%	15.0%	465.0%
Total	100.0%	60.0%	<b>23.3%</b>	83.3%

- Again,  $E[\text{Commission}] \neq \text{Commission @ Expected LR}$

# Loss ratio corridor

- Provision assigns all (or part) of losses in a given LR range (“corridor”) to be retained by ceding company
  - Roughly speaking: “I bleed, you bleed” approach
  - Not as common as slides
- Example: Cedent keeps 100% of losses when LR is 75% to 85% – “10 point corridor attaching at 75%”
  - Subject LR = 75%: Ceded LR = Subject LR = 75%
  - Subject LR = 80%: Ceded LR = 75%
  - Subject LR = 85%: Ceded LR = <wait> 75%
  - Subject LR = 100%: Ceded LR = ???
- Note: Corridor does not have to be 100% retained

# Loss ratio cap

- Provision assigns a maximum Ceded LR for treaty
  - Once you hit the aggregate cap, the party's over
- Example: QS with 200% loss ratio cap
  - Ceded LR before cap = 150%: Ceded LR = 150%
  - Ceded LR before cap = 300%: Ceded LR = 200%
- Useful on start-ups: Limit / Premium can be volatile
  - New Umbrella program offers \$10M policy limits, but only writes \$3M in premium in first year of operation
  - Can be the only way to get the treaty placed.
  - While the cap may be set high, at least downside is limited
- Note: See your auditor for an opinion on risk transfer.



# Breathe

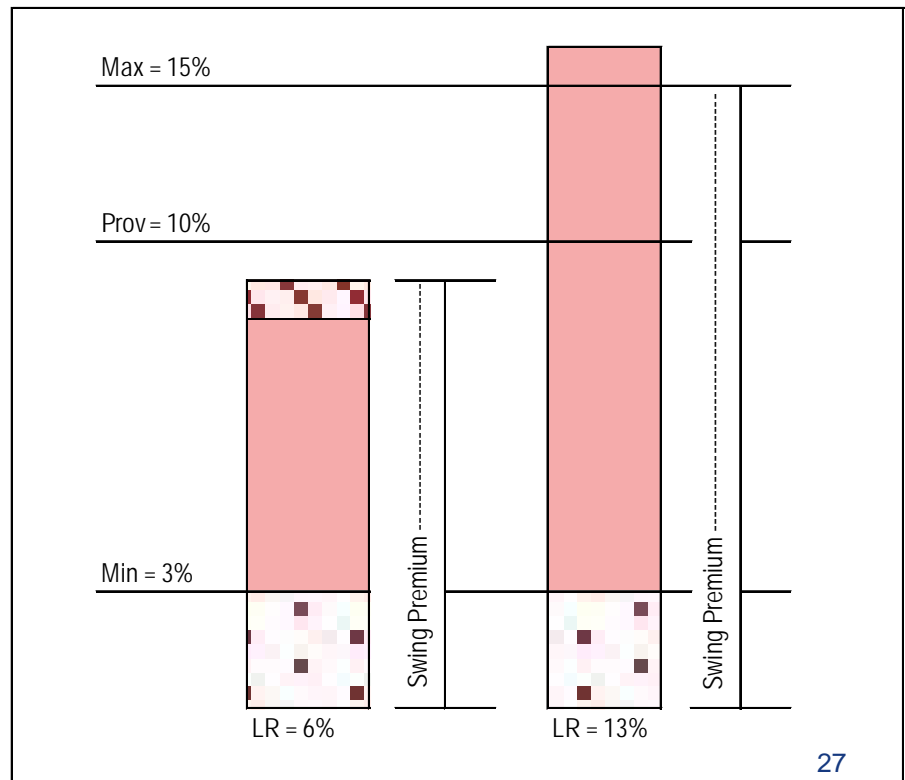
# Loss sensitive features on XOLs

- Excess of loss (XOL) treaties
  - Profit commission (seen this before)
  - Swing rates
  - Reinstatements and Aggregate Limits
  - Annual Aggregate Deductibles
  - No Claims Bonuses (if anywhere, Cat XOLs)
  - Loss ratio cap (seen this before)

# Swing rating provisions

- Swing rates set a provisional Ceded Premium, then dials it up/down with later adjustments based on ceded losses
  - Terms can vary. Read the contract to see how it works.
- Typical Swing (Rates ~ % SPI)
  - Provisional Rate = 10%;  
Minimum Rate/Margin = 3%;  
Maximum Rate = 15%
  - “Losses Loaded” at = 1.1
- Ceded Rate = Minimum Rate + Ceded Loss % x 1.1,  
subject to Max Rate of 15%
- Question: What Ceded Loss % puts you at the Maximum Rate?

Answer



# Swing rating example

Swing Rated Contract				
Min / Margin = 3%, Losses Loaded at 1.1, Max = 15%, Provisional = 10%				
	Prob	Burn	Final Rate	LR
1	48.5%	0.0%	3.0%	
2	20.0%	5.0%	8.5%	
3	19.5%	7.5%	11.3%	
4	7.0%	25.0%	15.0%	
5	5.0%	35.0%	15.0%	
<b>Total</b>	<b>100.0%</b>	<b>6.0%</b>	<b>7.1%</b>	<b>83.4%</b>
Burn = Ceded Loss to SPI				

# Limited reinstatement provisions

- Many XOL treaties have reinstatement provisions that cap the number of times you can tap the treaty's risk limit.
  - Reinstatements can be free or paid
  - Paid reinstatements are based on the initial premium, as in 1<sup>st</sup> @ 50%, 2<sup>nd</sup> @ 75%, etc.
  - Catastrophe treaties often have “1 @ 100%”
    - One full reinstatement of the limit for the full premium
- Limited reinstatements imply an annual aggregate limit.
- Treaty Aggregate Limit = Risk Limit x (1 + # reinstatements)

# Limited reinstatement provisions

- Example: \$1M x \$1M layer with one reinstatement
  - As the first \$1M limit, a second limit becomes available
  - Treaty Aggregate Limit =  $\$1M \times (1 + 1) = \$2M$
  - Reinstatements can be free or paid – Read the contract
  - “Free” is a euphemism for “Prepaid”
- Many Property Cat XOLs have limits that are exhausted in the aggregate. Reinstatements are “pro rata as to Amount”
  - Pay next reinstatement premium proportionally as you use the current limit. On final limit, premium is paid up.
- Summary: Reinstatement premium is an additional premium that reinsurers receive depending on loss experience

# Limited reinstatement example

\$1M x \$1M Layer						
1 reinstatement paid at 100% - Pro rata as to amount, 100% as to time						
Upfront Ceded Premium = \$200,000						
	Year 1			Year 2		
	Ground up Loss	Ceded Loss	Reinst. Prem	Ground up Loss	Ceded Loss	Reinst. Prem
1	2,000	1,000	200	1,500	500	100
2	2,000	1,000	-	1,500	500	100
3	2,000	-	-	2,000	1,000	-
Total	6,000	2,000	200	5,000	2,000	200

# Valuing a limited reinstatement provision

\$1M x \$1M Layer							
1 reinstatement paid at 100% - Pro rata as to amount, 100% as to time							
Upfront Ceded Premium = \$300,000							
	Prob	Loss to Layer	Losses after limitation	Upfront Premium	Reinst. Premium	Total Prem	LR
1	75.0%	-	-	300	-	300	
2	15.0%	1,000	1,000	300	300	600	
3	5.0%	2,000	2,000	300	300	600	
4	3.0%	3,000	2,000	300	300	600	
5	2.0%	4,000	2,000	300	300	600	
<b>Total</b>	<b>100.0%</b>	<b>420</b>	<b>350</b>	<b>300</b>	<b>75</b>	<b>375</b>	<b>93%</b>



# Annual Aggregate Deductible

- Annual Agg. Deductible (AAD): Added barrier of retained **in-layer** losses that would otherwise go to the treaty
  - AAD eliminates the **first** losses to hit the layer
  - Similar to loss corridor, but AAD always hits first
- Example: XOL cover: \$500 x \$500 XOL and AAD of \$750
  - Total Loss to Layer = \$500?
    - Cedent retains entire \$500. Ceded loss = \$0
  - Total Loss to Layer = \$1M?
    - Cedent retains \$750, Reinsurer pays \$250
- Question: If we impose a \$500 AAD, should the actuary reduce her expected layer losses of \$1M by \$500?

# Uh... No! (But you knew that, right?)

\$1M x \$1M Layer				
AAD = \$500,000				
	Prob	Loss to Layer	After AAD	AAD Savings
1	48.5%	-	-	-
2	20.0%	1,000	500	500
3	19.5%	2,000	1,500	500
4	7.0%	3,000	2,500	500
5	5.0%	4,000	3,500	500
Total	100.0%	1,000	743	258

As with any of these examples, a different loss distribution would result in different estimated savings.

# No Claims Bonus

- A No Claims Bonus provision can be added to an excess of loss contract – it's exactly what it sounds like
- QS contracts usually attach at first dollar of loss
  - A no claims bonus doesn't make much sense
- Very binary: If there are no losses, cedent can receive a small % of premium back
- If there is a small layer loss, we have a conundrum:
  - Take the NCB rebate, and **commute** the treaty.
  - Wait and see how the layer loss develops.
- Not typical feature in Casualty, but it could be useful in Property Catastrophe XOLs that are well off the ground.

# Using loss sensitive features in multi-year blocks

- In all structures presented thus far, each year's results stands on its own.
- Example: XOL with a PC over consecutive years.
  - Year 1 is light (PC pays in full). Year 2 has big losses.
  - Nice for cedent. Reinsurer is hammered from both ends.
- To smooth results and get better rates, loss sensitive terms can apply to total treaty experience across multiple years.
  - E.g., multi-year PC or slide, “2 full limits over 3 years”
- This is called rating on a **Multi Year Block**
- Modeling a multiyear block requires more care in setting your loss distribution. A lot can happen in 3 dice rolls.

# Deficit / credit carryforward (especially for sliding scales)

- Sliding scale commissions can also get out of balance, but can be tamed over time with a deficit/credit carryforward
- If a low LR triggers the max commission, any spillover can roll into next year's slide calculation as a **credit** carryforward.
- Likewise, the excess portion of a high LR can roll into next year as a **deficit** carryforward.
- Typical sliding scale format is given at right.
- Read the contract to know how to handle deficit or credit carryovers in an actual treaty.

	Cede	@	LR
Min	15%	@	75%
Prov	20%	@	65%
Max	25%	@	60%

# VALUATION NUTS AND BOLTS

Willis Re

MANAGING EXTREMES

Aggregate loss distributions and valuation

Fooling yourself: Process and parameter uncertainty

What if there's Cat or large loss exposure?

Frequency/Severity modeling

How do I get started in simulating loss distributions?



# Determining an aggregate loss distribution (3 methods)

- Discrete distribution with hand-picked LR points and judgmentally selected probabilities – when you can't fit.
  - Easy to explain to underwriters, buyers, brokers
  - Care is needed to include enough extreme values
- Fit parametric probability distribution to on-level LRs
  - Can work reasonably well for QS on Gross
  - Actuaries like lognormal: easy, somewhat skewed
- Fit frequency/severity: Simulation / convolution
  - Useful for XOL, Cat-exposure, QS with event cap
  - Lognormal can't do loss-free years, and it's too "light"

# Judgmentally selected aggregate loss distribution

	Prob	LR	Cede	UW Ratio
1	4.0%	25.0%	25.0%	50.0%
2	10.0%	35.0%	25.0%	60.0%
3	20.0%	40.0%	25.0%	65.0%
4	25.0%	50.0%	25.0%	75.0%
5	20.0%	60.0%	<b>25.0%</b>	85.0%
6	15.0%	70.0%	17.5%	87.5%
7	2.0%	80.0%	15.0%	95.0%
8	2.0%	145.0%	15.0%	160.0%
9	1.0%	350.0%	15.0%	365.0%
10	1.0%	450.0%	15.0%	465.0%
Total	100.0%	60.0%	<b>23.3%</b>	83.3%

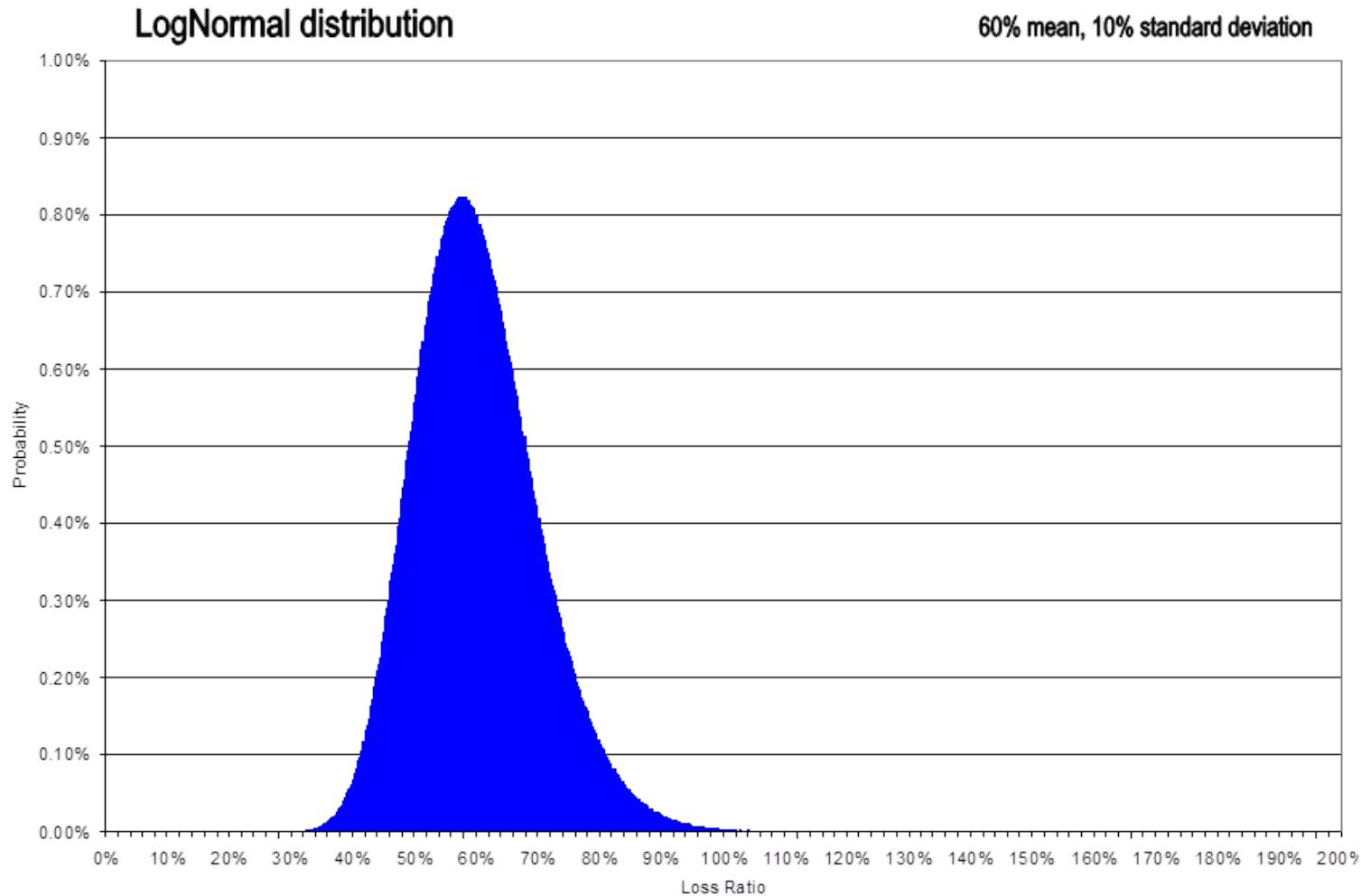


# Lognormal distribution: ELR = 60%, SD = 10%

Ad: To get a better fit to historical experience, try a **shifted Lognormal**.

Cumul Prob	Increm Prob	LR
10.0%	10.0%	48%
20.0%	10.0%	52%
30.0%	10.0%	54%
40.0%	10.0%	57%
50.0%	10.0%	59%
60.0%	10.0%	62%
70.0%	10.0%	64%
80.0%	10.0%	68%
90.0%	10.0%	73%
95.0%	5.0%	78%
99.0%	4.0%	87%
99.6%	0.6%	92%
99.9%	0.3%	98%
Total	100%	60%

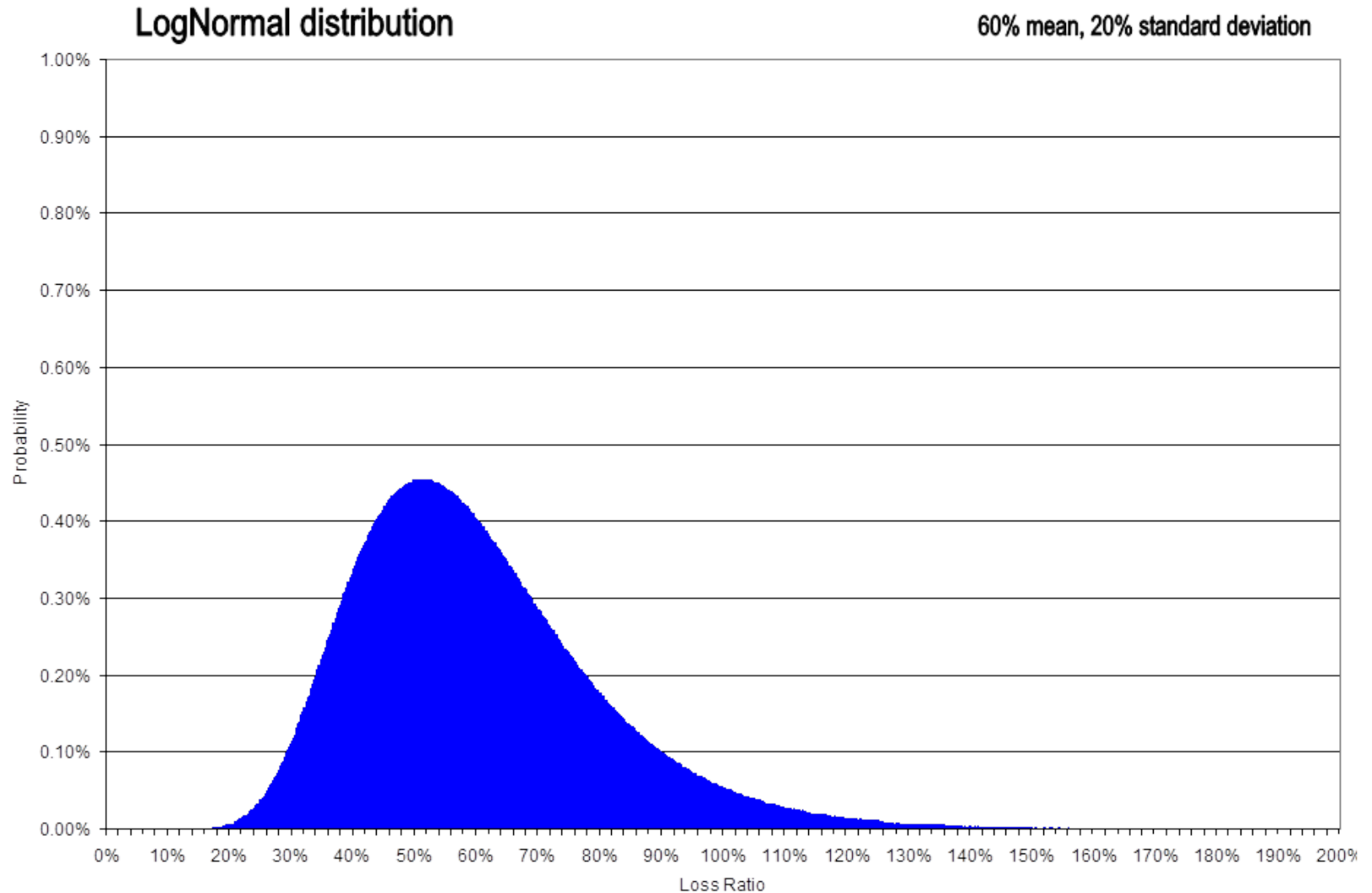
Lognormal distribution: ELR = 60%, SD = 10%



# Lognormal distribution: ELR = 60%, SD = 20%

Cumul Prob	Increm Prob	LR
10.0%	10.0%	38%
20.0%	10.0%	43%
30.0%	10.0%	48%
40.0%	10.0%	52%
50.0%	10.0%	57%
60.0%	10.0%	62%
70.0%	10.0%	68%
80.0%	10.0%	75%
90.0%	10.0%	86%
95.0%	5.0%	97%
99.0%	4.0%	122%
99.6%	0.6%	135%
99.9%	0.3%	157%
Total	100%	60%

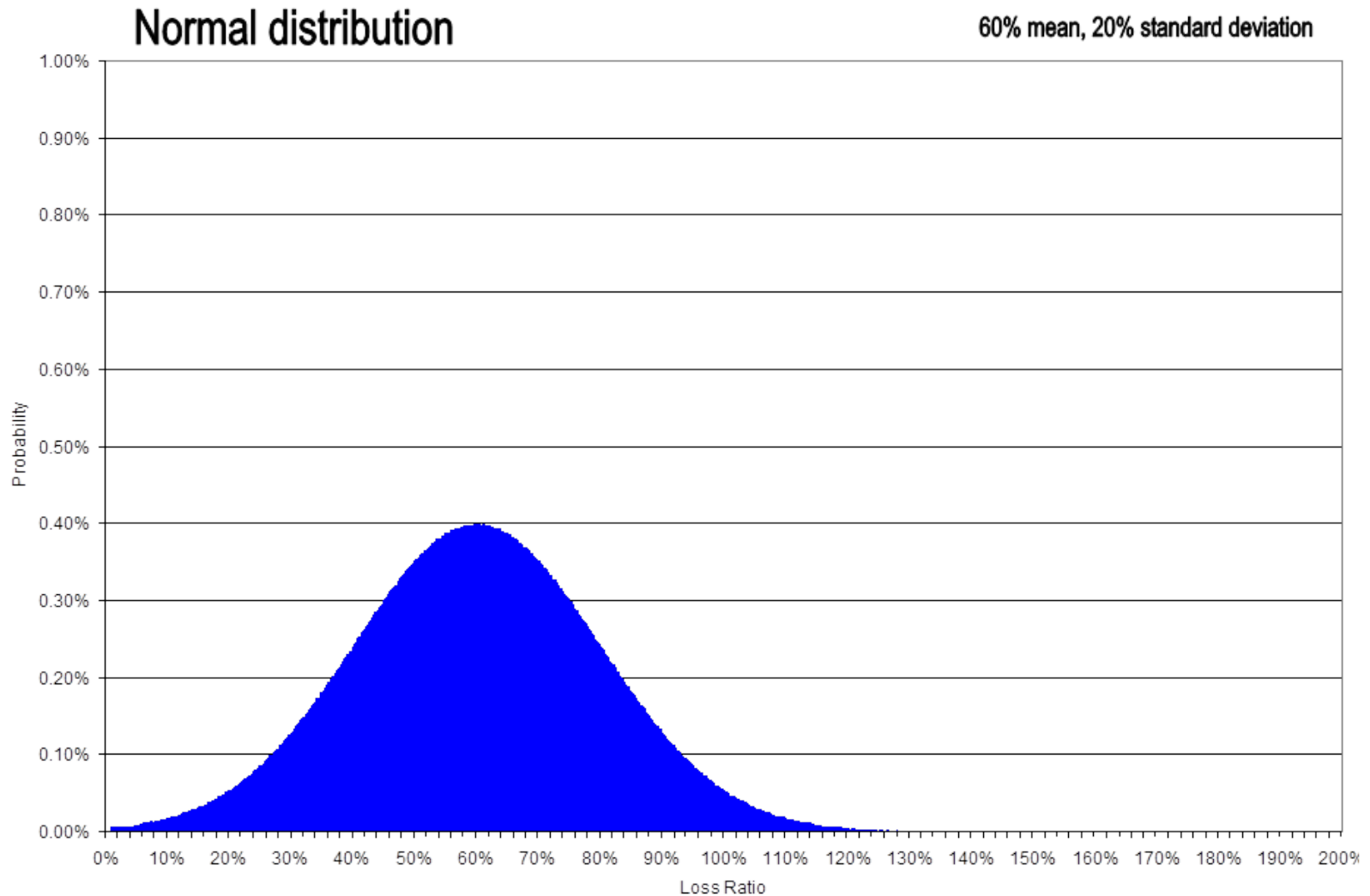
# Lognormal distribution: ELR = 60%, SD = 20%



# Normal distribution: ELR = 60%, SD = 20%

Willis Re

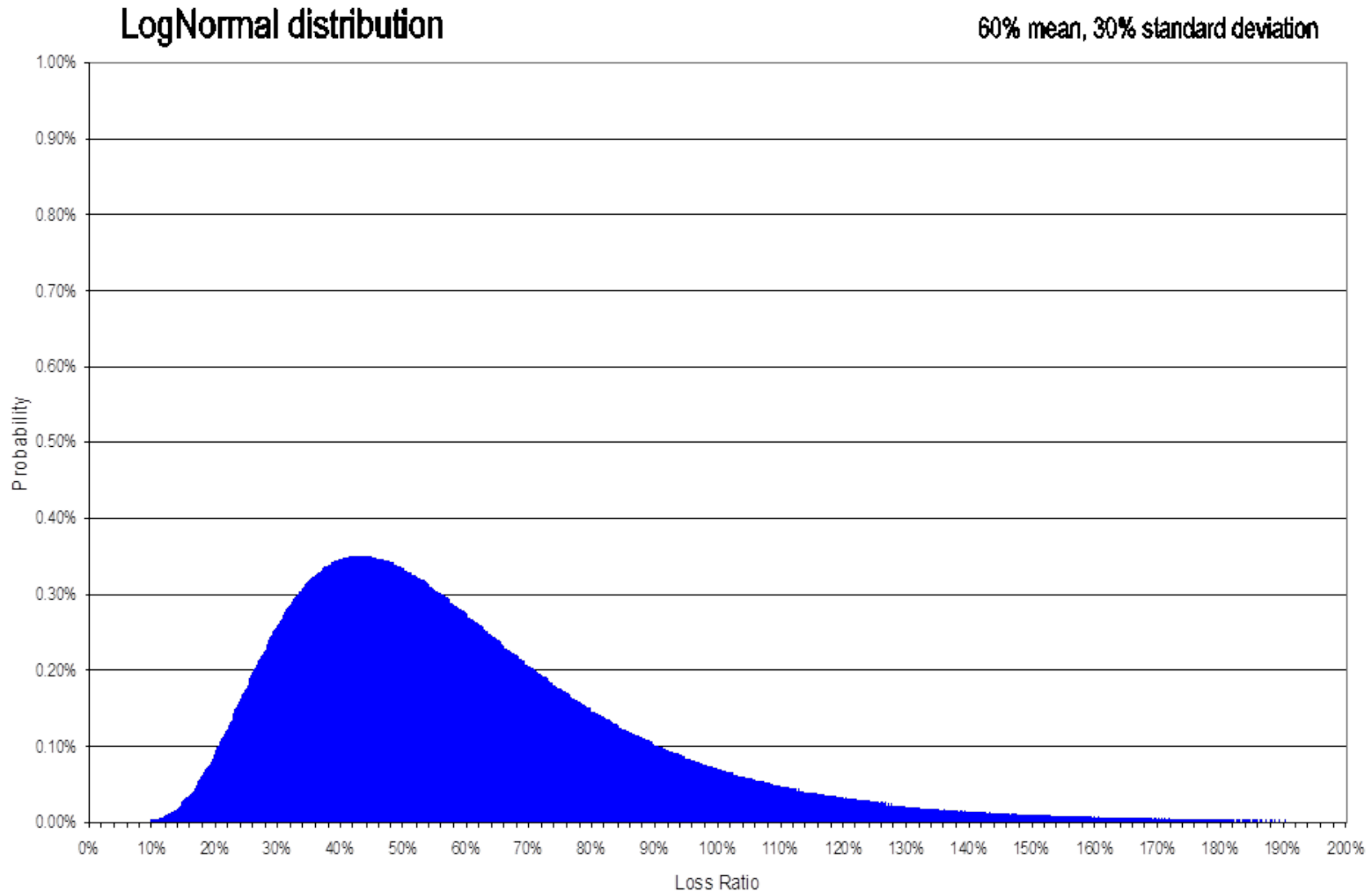
MANAGING EXTREMES



# Lognormal distribution: ELR = 60%, SD = 30%

Cumul Prob	Increm Prob	LR
10.0%	10.0%	29%
20.0%	10.0%	36%
30.0%	10.0%	42%
40.0%	10.0%	47%
50.0%	10.0%	54%
60.0%	10.0%	61%
70.0%	10.0%	69%
80.0%	10.0%	80%
90.0%	10.0%	99%
95.0%	5.0%	115%
99.0%	4.0%	160%
99.6%	0.6%	184%
99.9%	0.3%	218%
Total	100%	60%

# Lognormal distribution: ELR = 60%, SD = 30%



# Is my loss distribution reasonable?

- Reality check: compare to historical results
  - On-level LRs are focus, but check untrended ultimate LRs for patterns you may have missed
  - Do results show volatility beyond rate actions, underwriting measures, rate mods, trends, etc.?
- Do on-level LRs reflect enough downside potential?
  - Cats, shock loss, unexpected frequency jumps
  - Are results in experience period really predictive?
- Does your distribution fly with underwriters, buyers?
- In some cases, you may have to throw out your fitted curve and pick one judgmentally with your group.



# What about this process risk and parameter uncertainty?

- Process risk is the random fluctuation of results around the expected value just due to the random nature of insurance
  - Not every year is going to be the same!
  - Even if we had a Groundhog Day world, there are many possible ways for the next period to play out
- Parameter uncertainty is the fluctuation in results because our fitted parameters used in our loss distribution are never going to be perfect.
  - Even with the right model, we don't have enough observations to give precise parameter estimates
  - We could be wrong about the model. Be humble.

# Parameter uncertainty: Don't stop being an actuary when modeling

- Parameter uncertainty comes in through many doors.
  - Trend, rate changes, loss development assumptions – reasonable and representative?
  - For this book, will the future be like the past?
    - Changes in mix? Changes in claims handling?
    - Change in management or philosophy?
    - Is the book growing? Shrinking? Stable?
- Fitted CVs are generally on the low side for modeling
  - 5 - 10 years of Loss Ratios can't cover the full range of even reasonably-expected possibilities
  - Anything with Cat exposure really needs scrutiny

# Addressing parameter uncertainty with a simple prior distribution

- When the mean is “fuzzy”, don’t stop at just one value for the Expected LR, try several ELRs. Here’s how...
- Assign probability weights to the new ELRs so that they all weight back to your original ELR (say, 60%).
  - Let  $ELR \sim [50\%, 60\%, 70\%]$ , and each has 1/3 chance of being the true mean, and do a 2-stage simulation
  - For each step, randomly select the conditional mean (i.e., 50%, 60%, or 70%), then set the aggregate loss as a Lognormal with this mean and your selected CV
  - Note that the CV covers your “process variance”
- Other “priors” may be better/worse, but you get the idea

# Creating distributions when there is Cat exposure

- If your treaty covers Cat-exposed business, you need to try to model non-Cat and Cat risks separately
  - Non-Cat “attritional” loss  $\sim$  Lognormal  $LR(\mu, \sigma)$
  - Cat losses are much more skewed, and “binary”
- Event-based Cat models fit nicely into simulation
  - Combines Cat, Non-Cat and other risks easily
  - Scenarios let you illustrate loss sensitive features
  - Lets you easily separate effects of Cat vs. Non-Cat
- Lognormal model for combined risk is a dead end
  - Hard to calibrate and explain. Easy to screw up.

# Modeling frequency and severity separately

- While a lognormal aggregate loss distribution is relatively easy to use, it is not usually appropriate for XOL treaties
  - Does not reflect “hit or miss” nature of XOL contracts
  - Ignores the possibility of loss-free years
  - Too light-tailed to account for extreme scenarios
  - Understates the potential of losses MUCH greater than the expected loss
- Modeling Frequency and Severity separately is more common for XOL
  - Usually large losses are simulated individually
  - “Attritional” losses modeled in bulk as LR model

# Common frequency distributions

- Poisson is an easy-to-use distribution to model claim counts
  - Poisson distribution assumes the mean ( $\lambda$ , constant) and variance of the claim count distribution are equal
  - Discrete distribution for # claims = 0, 1, 2, 3, etc...
- Negative Binomial: Poisson with parameter uncertainty
  - Think of a Poisson with  $\Lambda \sim$  Gamma random variable
  - Mean is still  $\lambda$ , but variance =  $\lambda(1 + \lambda c) > \lambda$  ( $c > 0$ ).
  - Preferred because it fits a wider variety of situations
  - The extra variability of the Negative Binomial is more in line with historical experience
- Delaporte distribution: Call me when you get this far.

# Common severity distributions

- Lognormal
- Mixed Exponential (currently used by ISO)
- Pareto
- Truncated Pareto – was used by ISO before moving to the Mixed Exponential
- CAVEAT: If you are fitting a severity distribution to actual claims, don't forget about loss development! (Maybe use ISO curves instead of building your own)

# How do I simulate losses?

- Simulation software: Almost too many choices
  - Excel can do simulations by itself, or with VBA
  - @ Risk, Crystal Ball, MATLAB, R, Python/NumPy
  - Vendor products: Risk Explorer, ADVISE, DIVA
  - Some broker products: MetaRisk, Remetrica
  - Numerical Methods: Use FFT or Heckman-Meyers
- You can use a Lognormal or Gamma for layer losses
  - Parameters would imply implicit frequency/severity
  - It is not that hard to do simulation, once you know some probability concepts and interpretation principles



# Concluding remarks

- There are many loss sensitive features available to help break logjams in reinsurance pricing disputes
- It's up to the actuary to value the requested features and explain the results to underwriters and buyers
- Depending on the loss distribution, your loss sensitive feature's expected cost or savings can vary **greatly**
- A little sensitivity testing on a range of distributions will keep you out of trouble.
- Use lots of illustrations to show how these work.
  - We have computers now. Try an animation in your show & tell to help everyone “see” the risks.

# Loss Sensitive Treaty Features

## Legal Disclaimer

- This analysis has been prepared by Willis Limited and/or Willis Re Inc (“Willis Re”) on condition that it shall be treated as strictly confidential and shall not be communicated in whole, in part, or in summary to any third party without written consent from Willis Re.
- Willis Re has relied upon data from public and/or other sources when preparing this analysis. No attempt has been made to verify independently the accuracy of this data. Willis Re does not represent or otherwise guarantee the accuracy or completeness of such data nor assume responsibility for the result of any error or omission in the data or other materials gathered from any source in the preparation of this analysis. Willis Re, its parent companies, sister companies, subsidiaries and affiliates (hereinafter “Willis”) shall have no liability in connection with any results, including, without limitation, those arising from based upon or in connection with errors, omissions, inaccuracies, or inadequacies associated with the data or arising from, based upon or in connection with any methodologies used or applied by Willis Re in producing this analysis or any results contained herein. Willis expressly disclaims any and all liability arising from, based upon or in connection with this analysis. Willis assumes no duty in contract, tort or otherwise to any party arising from, based upon or in connection with this analysis, and no party should expect Willis to owe it any such duty.
- There are many uncertainties inherent in this analysis including, but not limited to, issues such as limitations in the available data, reliance on client data and outside data sources, the underlying volatility of loss and other random processes, uncertainties that characterize the application of professional judgment in estimates and assumptions, etc. Ultimate losses, liabilities and claims depend upon future contingent events, including but not limited to unanticipated changes in inflation, laws, and regulations. As a result of these uncertainties, the actual outcomes could vary significantly from Willis Re’s estimates in either direction. Willis makes no representation about and does not guarantee the outcome, results, success, or profitability of any insurance or reinsurance program or venture, whether or not the analyses or conclusions contained herein apply to such program or venture.
- Willis does not recommend making decisions based solely on the information contained in this analysis. Rather, this analysis should be viewed as a supplement to other information, including specific business practice, claims experience, and financial situation. Independent professional advisors should be consulted with respect to the issues and conclusions presented herein and their possible application. Willis makes no representation or warranty as to the accuracy or completeness of this document and its contents.
- This analysis is not intended to be a complete actuarial communication, and as such is not intended to be relied upon. A complete communication can be provided upon request. Willis Re actuaries are available to answer questions about this analysis.
- Willis does not provide legal, accounting, or tax advice. This analysis does not constitute, is not intended to provide, and should not be construed as such advice. Qualified advisers should be consulted in these areas.
- Willis makes no representation, does not guarantee and assumes no liability for the accuracy or completeness of, or any results obtained by application of, this analysis and conclusions provided herein.
- Where data is supplied by way of CD or other electronic format, Willis accepts no liability for any loss or damage caused to the Recipient directly or indirectly through use of any such CD or other electronic format, even where caused by negligence. Without limitation, Willis shall not be liable for: loss or corruption of data, damage to any computer or communications system, indirect or consequential losses. The Recipient should take proper precautions to prevent loss or damage – including the use of a virus checker.
- This limitation of liability does not apply to losses or damage caused by death, personal injury, dishonesty or any other liability which cannot be excluded by law.
- This analysis is not intended to be a complete Financial Analysis communication. A complete communication can be provided upon request. Willis Re analysts are available to answer questions about this analysis.
- Willis does not guarantee any specific financial result or outcome, level of profitability, valuation, or rating agency outcome with respect to A.M. Best or any other agency. Willis specifically disclaims any and all liability for any and all damages of any amount or any type, including without limitation, lost profits, unrealized profits, compensatory damages based on any legal theory, punitive, multiple or statutory damages or fines of any type, based upon, arising from, in connection with or in any manner related to the services provided hereunder.
- Acceptance of this document shall be deemed agreement to the above.