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Intermediate Experience Rating: Not All Models are Wrong

CAS Seminar on Reinsurance
June 1-2, 2015

David R. Clark

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Agenda

1. Experience Rating as a GLM
   - Selection of Weights
   - Testing assumptions
2. XOL Treaties and Policy Limit “Drift”
   - Testing for PL Drift
   - Adjusting for PL Drift
3. Property – Experience Rating for Occurrence Layers
4. Proportional Treaties on Umbrella and Trend on “Missing” losses

Preliminaries

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Accident Year	Onlevel Premium	LDF	Trended Loss	Loss Rate (estimator)
2005				
2006				
2007				
2008				
2009				
2010				
2011				
2012				
2013				
				?

$$ELR_i = \frac{E[Loss_i \cdot LDF_i]}{Premium_i}$$

What is the “right” way to use this information to estimate a single ELR?

Experience Rating as a GLM

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Each year in the experience period gives us one estimate of the ELR.

$$ELR_i = \frac{E[Loss_i \cdot LDF_i]}{Premium_i}$$

We can rearrange these terms into a linear model (equivalently GLM with identity link function). Letting Y = losses reported-to-date.

$$E[Loss_i] = \left(\frac{Premium_i}{LDF_i}\right) \cdot ELR$$

$$E[Y] = X \cdot \beta$$

$$Var(Y) = \phi \cdot E[Y]^p$$

Experience Rating as a GLM

Several examples re-casting this as GLM, with identity link and alternative variance structures. Each variance function leads to a different estimator.

GLM	Variance	Best Estimator
Overdispersed Poisson	$Var(Y) = \phi \cdot E[Y]^1$	$\widehat{ELR} = \frac{\sum Loss_i}{\sum Prem_i / LDF_i}$
Overdispersed Poisson	$Var(Y) = \phi \cdot \left(\frac{1}{LDF_i}\right) \cdot E[Y]^1$	$\widehat{ELR} = \frac{\sum Loss_i \cdot LDF_i}{\sum Prem_i}$
Gamma	$Var(Y) = \phi \cdot E[Y]^2$	$\widehat{ELR} = \frac{1}{n} \sum \frac{Loss_i \cdot LDF_i}{Prem_i}$

Y = Trended losses reported-to-date

Experience Rating as a GLM

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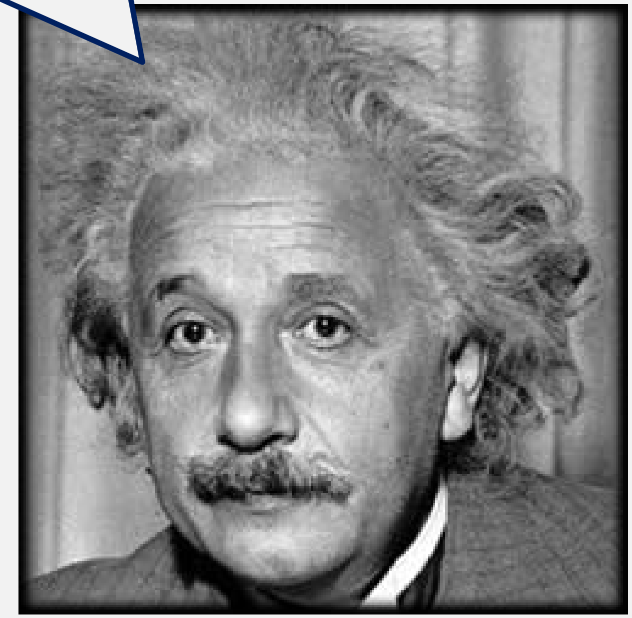
What does this get us?

- Guidance for best weighting scheme
  - Including how to give weight to recent partially-earned year
- Standardized residuals to test model assumptions (is there a remaining trend or cycle?)
- Criteria for deciding to include additional information
- Ability to perform multiple experience ratings in a single model that shares some parameters

Testing Experience Rating

No amount of experimentation can ever prove me right;
a single experiment can prove me wrong.



Testing Experience Rating

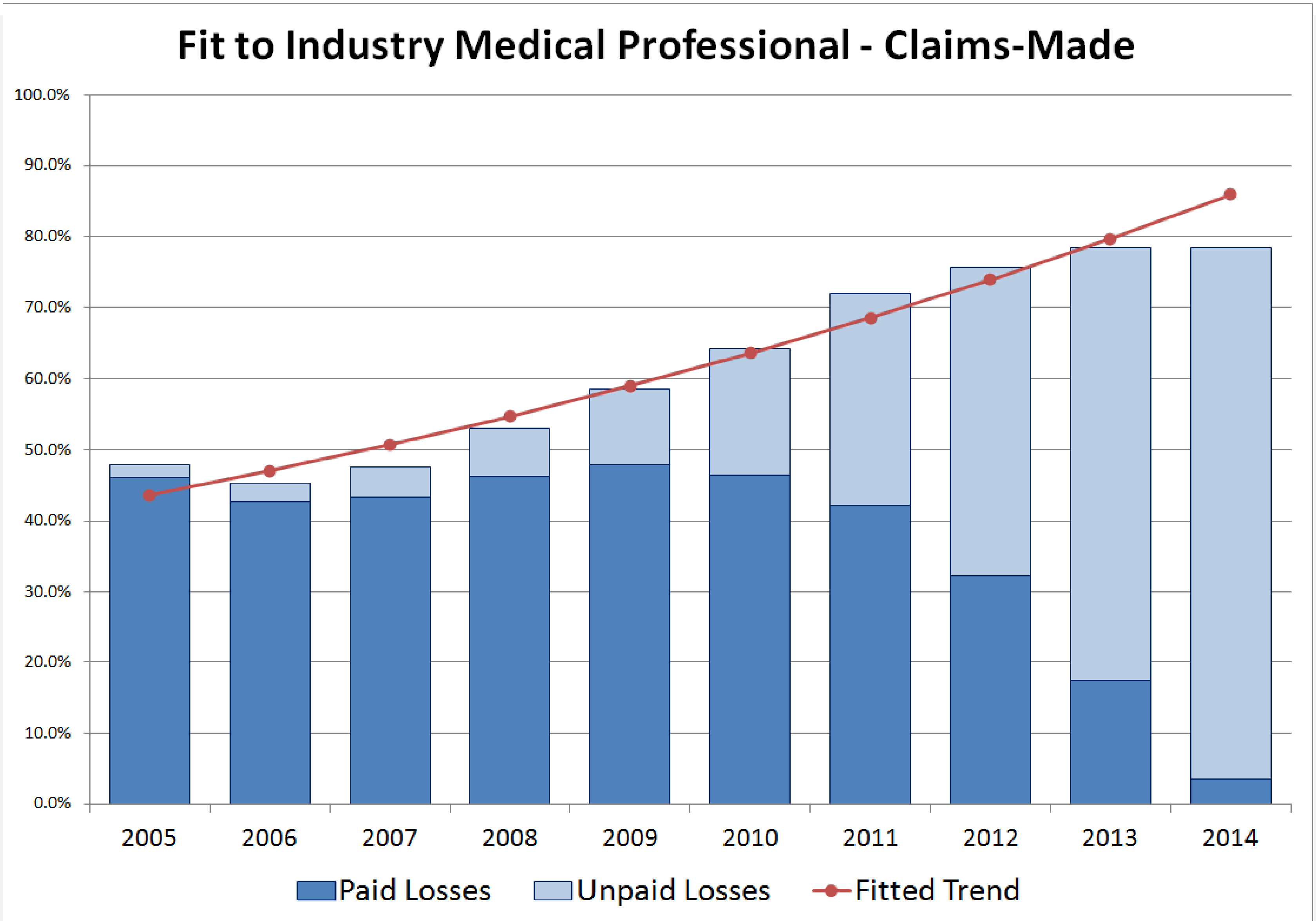
Another quote:

> “A model for data, no matter how elegant or correctly derived, must be discarded or revised if it does not fit the data or when new or better data are found and it fails to fit them.“
>
> - Paul Velleman
>
> “Truth, Damn Truth, and Statistics" in the Journal of Statistical Education, 2008.

Standard part of the experience rating model is the assumption that, after adjustments to current level, the ***ELR*** is constant over time.

Does this assumption “fit” the data?

Testing Experience Rating



Testing Experience Rating

Each year in the experience period gives us one estimate of the ELR.

$$ELR_i = \frac{E\left[Loss_i \cdot LDF_i \cdot (1+t)^i\right]}{Premium_i}$$

We can again rearrange these terms into a GLM with "log-link" and then test the significance of the trend.

$$E[Loss_i] = \left(\frac{Premium_i}{LDF_i}\right) \cdot ELR \cdot (1+t)^i$$

$$E[Y] = w_i \cdot exp(\beta_0 + i \cdot \beta_1)$$

$$Var(Y) = \phi \cdot E[Y]^p$$

Testing Experience Rating

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We can estimate trend via GLM, even if some years have zero losses.

Pricing factors (frequency, severity trends, benefit changes, rate changes, etc) are interpreted as *explanatory variables*. We want to test how well they explain movements in the data.

If the "residual trend" on the historical experience is zero (or not significantly different than zero), then we conclude the pricing factors were effective.

For a single treaty, a residual trend may not be significant. We can run multiple GLMs simultaneously to test if there is a residual trend to the portfolio.

Testing Experience Rating

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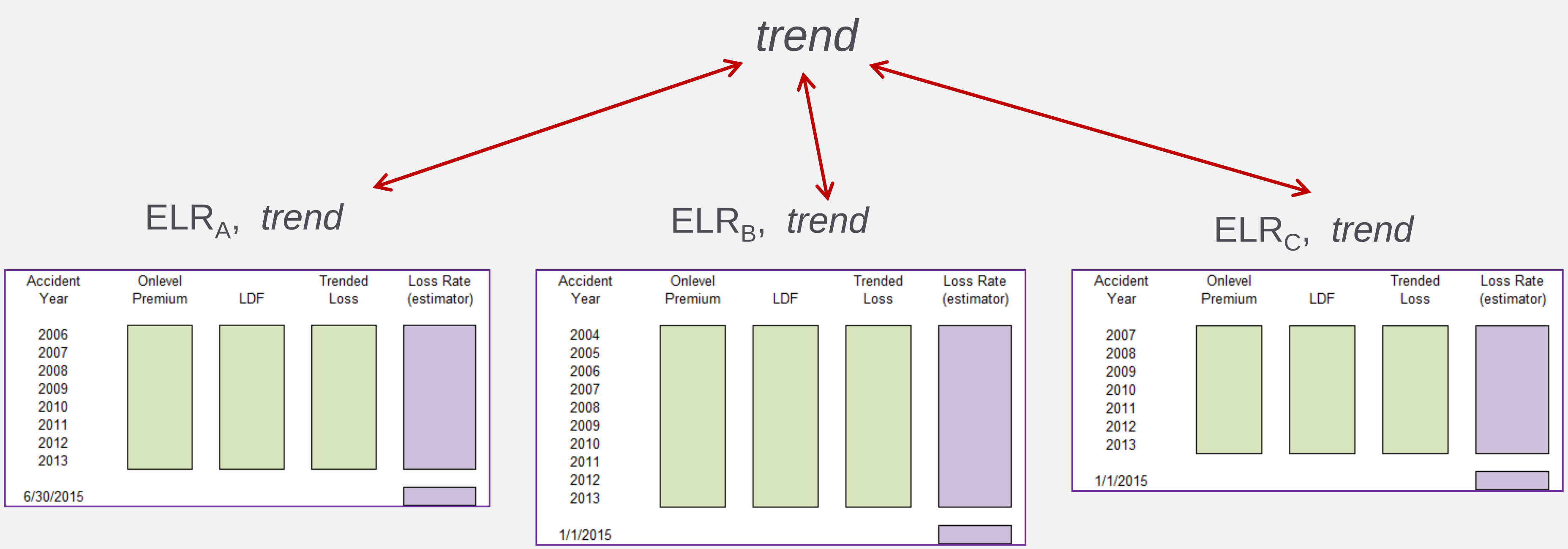
If we observe a residual trend that is consistent across accounts, it may indicate a systemic problem in the market segment.

Simultaneous GLMs (or alternatively, mixed models) can help detect systemic effects.



Examples of Changes in Experience:

Three Problems… One Solution!

Policy Limit “Drift”

The problem: when pricing Casualty nonproportional (aka XOL) treaties, there may be more policies that expose the layers in some years than in other years.

If the policy limits or hazard classes are different in the historical period than in the prospective period, then the experience rating will be biased.

To address this, we need information on the policy limit distribution.

- Testing for Policy Limit Drift
- Adjusting for Policy Limit Drift (Mata & Verheyen)

Testing for Policy Limit Drift

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Given a listing of large losses, with their policy limits, we have a **sample** of the historical policy limits profile. This sample profile can be compared with the prospective (or in-force) profile used as input for the exposure-rating.

			Policy Limits			
Limits Sold:	500,000	750,000	1,000,000	2,000,000	5,000,000	Total for all PL
Premium	5,000,000	25,000,000	50,000,000	10,000,000	10,000,000	100,000,000
ELR	62.5%	62.5%	62.5%	62.5%	62.5%	62.5%
Expected Loss	3,125,000	15,625,000	31,250,000	6,250,000	6,250,000	62,500,000
Threshold:	250,000					
Count > Thresh:	3.1	14.0	26.0	4.5	3.6	51.2
Normalized	12.3	54.7	101.6	17.5	13.9	200.0
Std Dev	3.4	6.3	7.1	4.0	3.6	
Historical Counts	31.0	58.0	95.0	13.0	3.0	200.0

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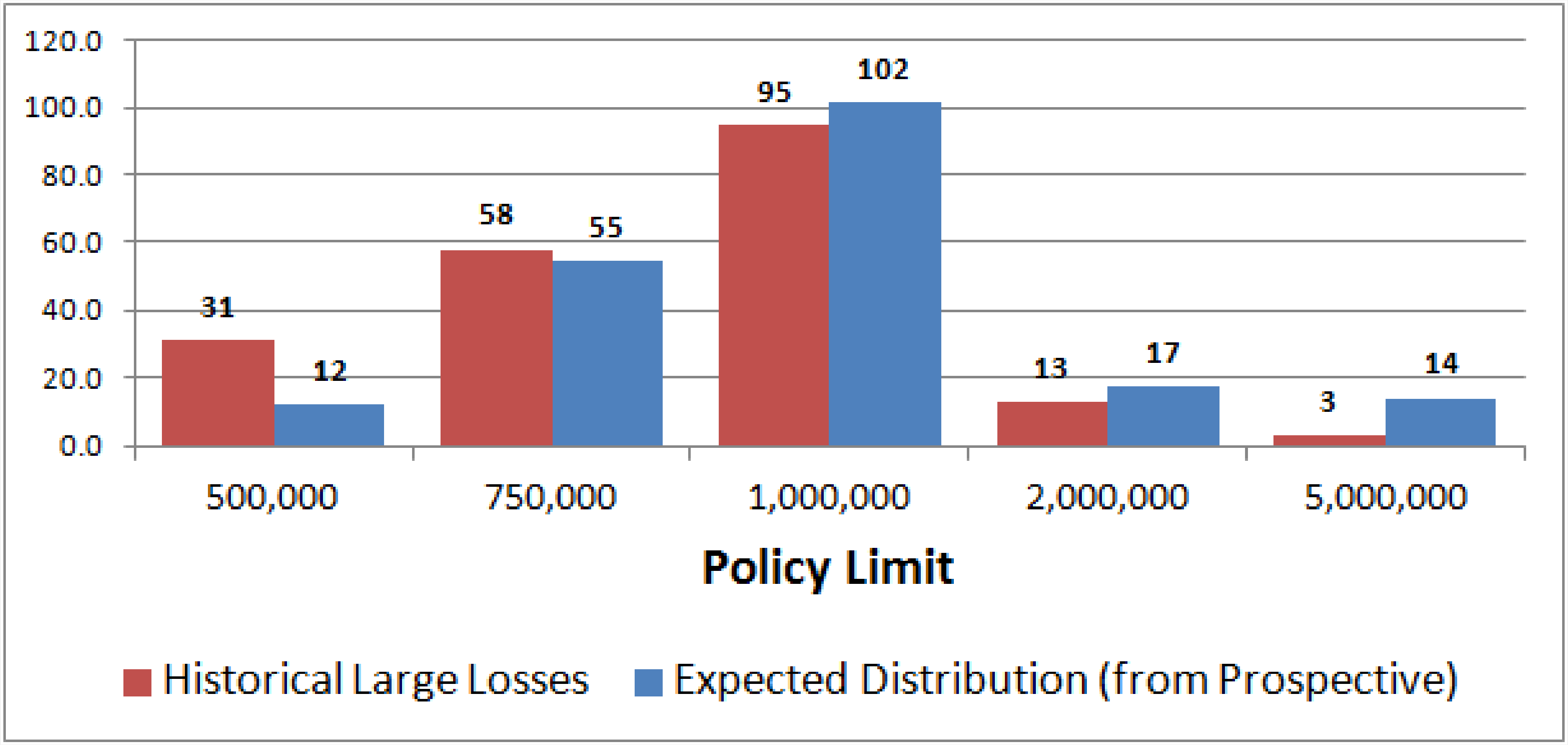
Testing for Policy Limit Drift

Does this represent a drift in policy limits?



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Adjusting for Policy Limit Drift (Mata/Verheyen)

The exposure rate is estimated using the risk profile and size-of-loss severity distribution for the prospective period.

$$ExposRate = \frac{\sum_{RiskProfile} Prem_i \cdot \frac{\left(LEV(L+R) - LEV(R)\right)}{LEV(PL_i)}}{\sum_{RiskProfile} Prem_i}$$

Alternatively, we can estimate the exposure rate for each historical period, by changing the risk profile (but always using the prospective size-of-loss distribution).

$$ExposRate_{2012}$$
$$ExposRate_{2013}$$
$$ExposRate_{2014}$$
$$\dots$$
$$ExposRate_{2015}$$

Adjusting for Policy Limit Drift (Mata/Verheyen)

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Mata & Verheyen suggest using these exposure rates to create an index applied to losses.

$$Index_{2012} = \frac{ExposRate_{2015}}{ExposRate_{2012}}$$

$$Index_{2013} = \frac{ExposRate_{2015}}{ExposRate_{2013}}$$

$$\ldots$$

I recommend a small change to this and instead use the inverse of this index, to be applied to premium.

This removes the possibility of the index “blowing up” (divide-by-zero error) if one of the historical periods does not expose the treaty layer.

Adjusting for Policy Limit Drift (Mata/Verheyen)

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For a given client, we have a profile showing premium by policy limit (and by risk class) for each historical period.

Policy Limit	2007	2008	2009	2010	2011	2012	2013	2014	2015
500,000	7,460,000	6,220,000	5,180,000	4,320,000	3,600,000	3,000,000	2,500,000	2,250,000	2,000,000
1,000,000	70,000,000	70,000,000	70,000,000	70,000,000	70,000,000	70,000,000	72,500,000	75,000,000	75,000,000
2,000,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,500,000	2,000,000	1,000,000
5,000,000	890,000	1,190,000	1,580,000	2,110,000	2,810,000	3,750,000	5,000,000	6,000,000	6,000,000
10,000,000	1,790,000	2,380,000	3,170,000	4,220,000	5,630,000	7,500,000	10,000,000	10,000,000	12,500,000
25,000,000	2,000,000	2,000,000	5,000,000	5,000,000	5,000,000	10,000,000	10,000,000	10,000,000	10,000,000
500 xs 500	12.25%	12.37%	12.17%	12.19%	12.16%	11.67%	11.60%	11.65%	11.56%
4M xs 1M	1.49%	1.74%	2.51%	2.90%	3.39%	4.59%	5.17%	5.23%	5.53%
5M xs 5M	0.50%	0.60%	1.02%	1.17%	1.36%	2.00%	2.21%	2.15%	2.44%
15M xs 10M	0.39%	0.39%	0.93%	0.92%	0.91%	1.68%	1.59%	1.55%	1.53%

All numbers for illustration purposes only.

Adjusting for Policy Limit Drift (Mata/Verheyen)

We bring in the exposure rates by historical period and use them as an index on the historical onleveled premium.

Year	Onlevel Premium	LDF	Prem / LDF	Exposure Rate	Index for Mix	Indexed Premium	Trended Layered Losses	Rate to Subject Premium
	[A]	[B]	[C]=[A]/[B]	[D]	[E]*	[F]=[C]x[E]	[G]	[G] / [F]
2007	84,640,000	1.134	74,638,448	1.49%	0.269	20,092,317	1,893,234	9.42%
2008	84,290,000	1.173	71,858,483	1.74%	0.314	22,569,478	2,000,000	8.86%
2009	87,430,000	1.228	71,197,068	2.51%	0.453	32,276,367	701,581	2.17%
2010	88,150,000	1.309	67,341,482	2.90%	0.524	35,280,521	0	0.00%
2011	89,540,000	1.434	62,440,725	3.39%	0.613	38,296,279	434,018	1.13%
2012	96,750,000	1.649	58,671,922	4.59%	0.831	48,729,189	1,563,271	3.21%
2013	102,500,000	2.093	48,972,766	5.17%	0.935	45,811,943	4,000,000	8.73%
2014	105,250,000	3.559	29,572,914	5.23%	0.945	27,956,734	0	0.00%
Total	738,550,000		484,693,809			271,012,828	10,592,104	3.91%
2015	106,500,000			5.53%	1.000	106,500,000	4,162,383	3.91%

* $[E]_t = [D]_{2015} / [D]_t$

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Adjusting for Policy Limit Drift (Mata/Verheyen)

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Alternatively, this can be considered as estimating an average experience/exposure relativity.

Year	Onlevel Premium	LDF	Prem / LDF	Exposure Rate	Layered Premium	Trended Layered Losses	Relative to Layered Premium
	[A]	[B]	[C]=[A]/[B]	[D]	[E]=[C]x[D]	[F]	[F] / [E]
2007	84,640,000	1.134	74,638,448	1.49%	1,111,442	1,893,234	1.703
2008	84,290,000	1.173	71,858,483	1.74%	1,248,470	2,000,000	1.602
2009	87,430,000	1.228	71,197,068	2.51%	1,785,424	701,581	0.393
2010	88,150,000	1.309	67,341,482	2.90%	1,951,604	0	0.000
2011	89,540,000	1.434	62,440,725	3.39%	2,118,426	434,018	0.205
2012	96,750,000	1.649	58,671,922	4.59%	2,695,541	1,563,271	0.580
2013	102,500,000	2.093	48,972,766	5.17%	2,534,168	4,000,000	1.578
2014	105,250,000	3.559	29,572,914	5.23%	1,546,476	0	0.000
Total	738,550,000		484,693,809		14,991,551	10,592,104	0.707
2015	106,500,000			5.53%	5,891,235	4,162,383	0.707

Same

All numbers for illustration purposes only.

Property – Experience Rating for Occurrence Layers

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The problem: some events are considered "attritional catastrophes" for which historical experience is relevant. These include:

- Tornado
- Hail
- Winter storm

The historical loss event may need to be adjusted for demographic changes, and mix of business, as well as for loss inflation.

We can do this by running catastrophe models for these attritional events using historical exposure profiles. The AAL's on [inflation-trended] TIVs can be used in the same way that Policy Limit drift was included for casualty lines.

(see also Ira Robbin's 2009 paper on a similar calculation for rate change)

Property – Experience Rating for Occurrence Layers

The adjustment for the exposure growth uses Annual Aggregate Loss (AAL) from the catastrophe model – but follows same format as PL Drift calculation.

Year	Onlevel Premium	LDF	Prem / LDF	Cat Model Exposure AAL / TIV	Index for Exposure Changes	Indexed Premium	Inflated Layered Cat Losses	Rate to Subject Premium
	[A]	[B]	[C]=[A]/[B]	[D]	[E]*	[F]=[C]x[E]	[G]	[G] / [F]
2007	60,000,000	1.000	60,000,000	0.07140	0.636	38,147,818	0	0.00%
2008	65,000,000	1.000	65,000,000	0.07620	0.679	44,105,076	15,260,000	34.60%
2009	70,000,000	1.001	69,930,070	0.08150	0.726	50,750,674	1,510,000	2.98%
2010	75,000,000	1.003	74,775,673	0.08700	0.775	57,929,506	5,200,000	8.98%
2011	80,000,000	1.017	78,662,734	0.09240	0.823	64,723,389	14,560,000	22.50%
2012	85,000,000	1.076	78,996,283	0.09780	0.871	68,796,406	24,710,000	35.92%
2013	90,000,000	1.371	65,645,514	0.10300	0.917	60,209,154	3,250,000	5.40%
2014	95,000,000	3.770	25,198,939	0.10780	0.960	24,189,186	0	0.00%
Total	620,000,000		518,209,212			408,851,210	64,490,000	15.77%
2015	100,000,000			0.11230	1.000	100,000,000	15,773,464	15.77%

* $[E]_t = [D]_{2015} / [D]_t$

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Proportional Treaties on Umbrella – “Missing Trend”

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The problem: when pricing proportional (aka Quota-Share) treaties covering excess or umbrella policies, there are some losses that are below the layer covered by the ceding company’s policy.

These losses would have pierced into the ceding company’s attachment point on an inflated basis. They are missing in the historical data.

This “leveraged trend” can be included in one of two ways:

- Trend the aggregated losses from the ceding company using a leveraged trend factor.
- Trend the individual losses from the ceding company, including the attachment point on known losses, and then include an additional load for the “missing” losses.

Proportional Treaties on Umbrella – "Missing Trend"

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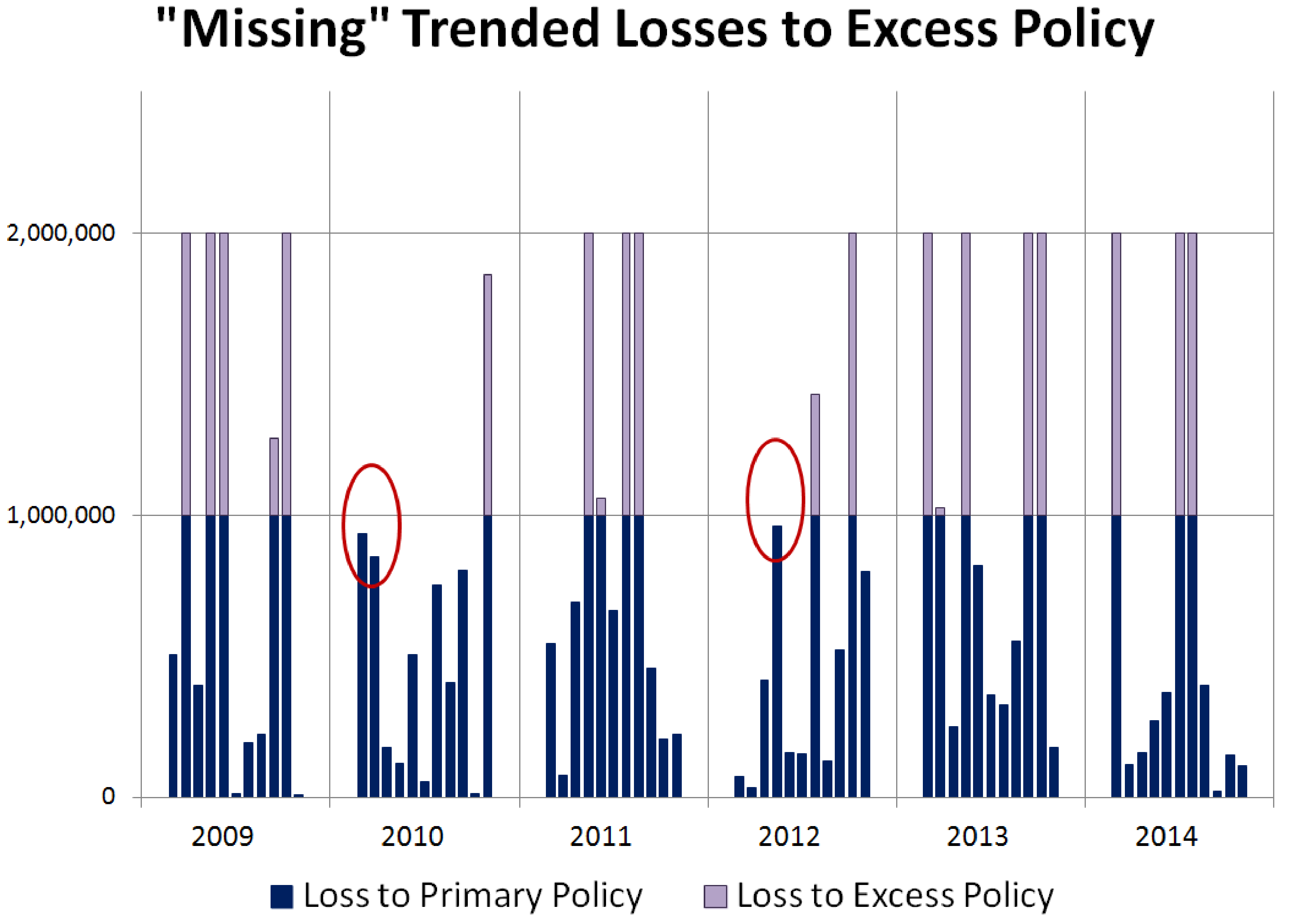

Proportional Treaties on Umbrella – "Missing Trend"

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The expected value of the "missing" losses that would trend into the layer for each year can be estimated from the size-of-loss distribution.

$$\int_{R}^{R\cdot(1+t)^i} (x-R)\,dF$$

$$= LEV\left(R\cdot(1+t)^i\right) - LEV(R)$$

$$- R\cdot\left[(1+t)^i - 1\right]\cdot\left\{1 - F\left(R\cdot(1+t)^i\right)\right\}$$

We can calculate exposure rates with and without this "missing" component to approximate the amount needed.

Proportional Treaties on Umbrella – "Missing Trend"

This formula produces a factor for each historical period that can be applied in exactly the same format as the other calculations given by Mata & Verheyen.

$$\frac{\text{Exposure Rate excl. "missing" losses}}{\text{Full Exposure Rate}} =$$

$$\frac{LEV_{L+R} - LEV_{R\cdot(1+t)^i} + R \cdot \left[(1+t)^i - 1\right] \cdot \left[1 - F\left(R \cdot (1+t)^i\right)\right]}{LEV_{L+R} - LEV_R}$$

Proportional Treaties on Umbrella – “Missing Trend”

The factor for “missing” trend will be different for each historical period.

It can be applied as an adjustment to premium, just as in the other examples

Year	Umbrella Limit		Umbrella Retention	Trend at 6.00%	Onlevel Premium	LDF	Prem / LDF	Adjustment	Adj Premium	% Missing
					[A]	[B]	[C]=[A]/[B]	[D]	[E]=[C]x[D]	
2005	4,000,000	xs	1,000,000	1.791	10,000,000	1.083	9,233,610	0.8643	7,980,602	-13.57%
2006	4,000,000	xs	1,000,000	1.689	10,000,000	1.105	9,049,774	0.8867	8,024,266	-11.33%
2007	4,000,000	xs	1,000,000	1.594	10,000,000	1.134	8,818,342	0.9077	8,004,145	-9.23%
2008	4,000,000	xs	1,000,000	1.504	10,000,000	1.173	8,525,149	0.9271	7,903,535	-7.29%
2009	4,000,000	xs	1,000,000	1.419	10,000,000	1.228	8,143,322	0.9447	7,693,231	-5.53%
2010	4,000,000	xs	1,000,000	1.338	10,000,000	1.309	7,639,419	0.9604	7,336,807	-3.96%
2011	4,000,000	xs	1,000,000	1.262	10,000,000	1.434	6,973,501	0.9738	6,791,001	-2.62%
2012	4,000,000	xs	1,000,000	1.191	10,000,000	1.649	6,064,281	0.9848	5,972,101	-1.52%
2013	4,000,000	xs	1,000,000	1.124	10,000,000	2.093	4,777,831	0.9930	4,744,492	-0.70%
2014	4,000,000	xs	1,000,000	1.060	10,000,000	3.559	2,809,778	0.9982	2,804,714	-0.18%
				Grand Total:	100,000,000		72,035,008	0.9336	67,254,895	-6.64%

All numbers for illustration purposes only.

Conclusions

All of this is a lot of work... is the juice worth the squeeze?

- Materiality of change to the expected loss?
- Improvement in standard error of expected loss estimate?
- Improved understanding of client business? (information asymmetry)

Conclusions

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Usefulness of thinking of experience rating as Predictive Model:

- Answer questions about the “best” weighting method
- Systematic way of determining if model assumptions are correct
- Systematic way of integrating additional information
- Systematic way of determining value of additional information

To fully benefit we need risk profiles for each historical period.

This should become a standard for reinsurance submissions.

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Thank you very much
for your attention

David R. Clark

daveclark@munichreamerica.com

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