A Systematic Approach to Event Modelling / Clash Pricing

6 June 2016

Neil Hyatt
Michael Cane
Dee Shen

Lockton Re….broking *done differently*

LOCKTON®

Introduction

"Better to remain silent and be thought a fool than to speak out and remove all doubt"

We didn't listen…..

Overview

- Losses are usually modelled on an individual risk basis using a frequency-severity approach
- Unfortunately this approach doesn't allow Clash treaties to be modelled, and generates tails that are too thin for Capital modelling
- The following discusses how to modify the usual simulation methodology to simulate events

  This enables us to:
  - Correlate losses within an event
  - Model risk and clash treaties on a coherent basis
  - Price Clash treaties
  - Generate thicker tails to get a more "realistic" view of capital requirements and probability of risk XL horizontal failure


Methodology

Data

We assumed that the premium for the excess book would be presented in a typical Limit/Attachment Profile:

Limit		Attachment		
		A_1	A_2	A_3
	L_1	x_{11}	x_{12}	x_{13}
	L_2	x_{21}	x_{22}	x_{23}
	L_3	x_{31}	x_{32}	x_{33}

“Equivalent” FGU premium

We are going to assume that losses are correlated fgu, so we first estimate the equivalent fgu premium in each cell by assuming that premium is distributed pro-rata to expected loss.

Defining the Limited Expected Value (LEV) in the usual way:

$$LEV(u) = \int_0^u xf(x)dx + u(1 - F(u))$$

where F(x) & f(x) are the cumulative and probability density function respectively.

Then the equivalent fgu premium y_{ij} in each cell is

$$y_{ij} = \frac{x_{ij} * LEV(L_i + A_j)}{LEV(L_i + A_j) - LEV(A_j)}$$

Expected number of fgu losses

Given the assumed loss ratio LR, and a cell frequency of λ_{ij} we have:

$$\lambda_{ij} * LEV\left(L_i + A_j\right) = y_{ij} * LR$$

Re-writing this gives:

$$\lambda_{ij} = \frac{LR * x_{ij}}{LEV\left(L_i + A_j\right) - LEV\left(A_j\right)}$$

And the number of losses per cell as:

Limit		Attachment		
		A_1	A_2	A_3
	L_1	λ_{11}	λ_{12}	λ_{13}
	L_2	λ_{21}	λ_{22}	λ_{23}
	L_3	λ_{31}	λ_{32}	λ_{33}

LOCKTON

Cell Conditional Distribution

If we assume that the frequencies have Poisson distributions and defining

$\Lambda = \sum_i \sum_j \lambda_{ij}$, &

$c_{ij} = {}^{\lambda_{ij}}/_{\Lambda}$

Then:

- The expected number of fgu losses for the book is Λ, and
- The conditional probability for a loss being from a particular limit and attachment cell is c_{ij}.

Event Distribution

If we assume that the distribution for the number of losses coming from an event is:

Number of Losses (k)	Probability
1	p_1
2	p_2
3	p_3
..	..
..	..
n	p_n

Then the expected number of losses given an event has occurred is:

$$N = \sum_k k\, p_k$$

And the expected number of events is:

$$E = \Lambda / N.$$

Simulation

Thus the simulation process is:

- Sample from the distribution for the total number of events, E, to determine how many events have occurred
- For each event , sample off the Number of losses distribution to determine how many losses have occurred, n
- Generate an n x n normal / t copula to correlate n correlated normal samples. The resulting sample being inverted to obtain $y_m \in (0,1)$
- For each random number y_m, sample an fgu loss from $F^{-1}(y_m)$ and then use the appropriate conditional c_{ij}^{k} distribution to determine the particular excess(j) and limit(i) points for the loss sampled, and so the net loss to the insurer

LOCKTON

Simulation

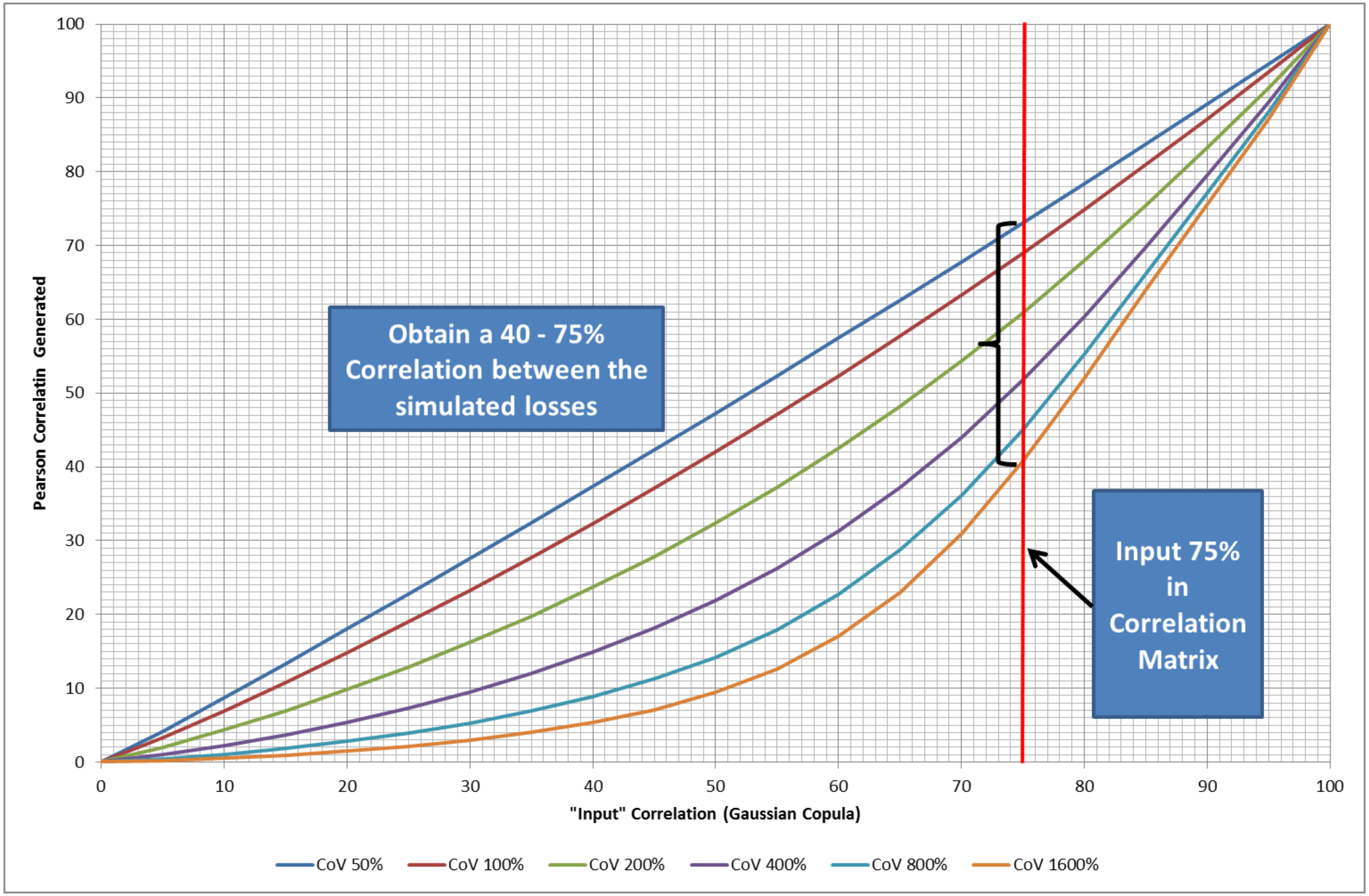
- Apply any Risk XL terms to the individual losses to determine these recoveries
- Aggregate claims (capped at max contribution) for each loss, and when all losses from a particular event have been sampled, apply the Clash Excess of loss terms
- Repeat for all Events
- Repeat for each Simulation

Aside: Correlation

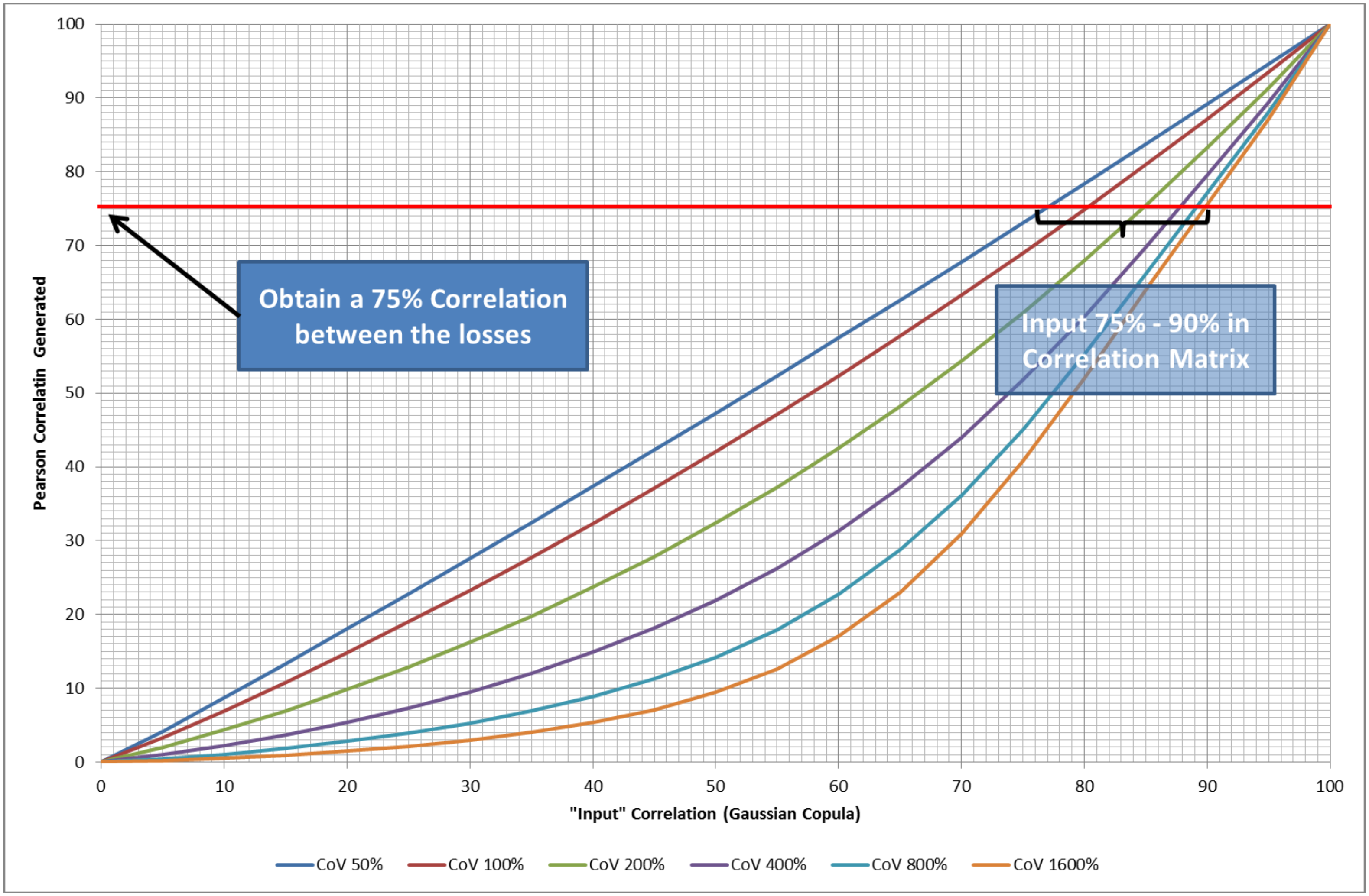
It should be noted that some of the loss distributions modelled are extremely right skew.

At the risk of stating the obvious, a gaussian copula will generate Normal losses with the required correlation. Using a gaussian copula to correlate losses from a right skew distribution will generate lower Pearson correlations than those embedded in the correlation matrix:

Aside: Correlation



Aside: Correlation

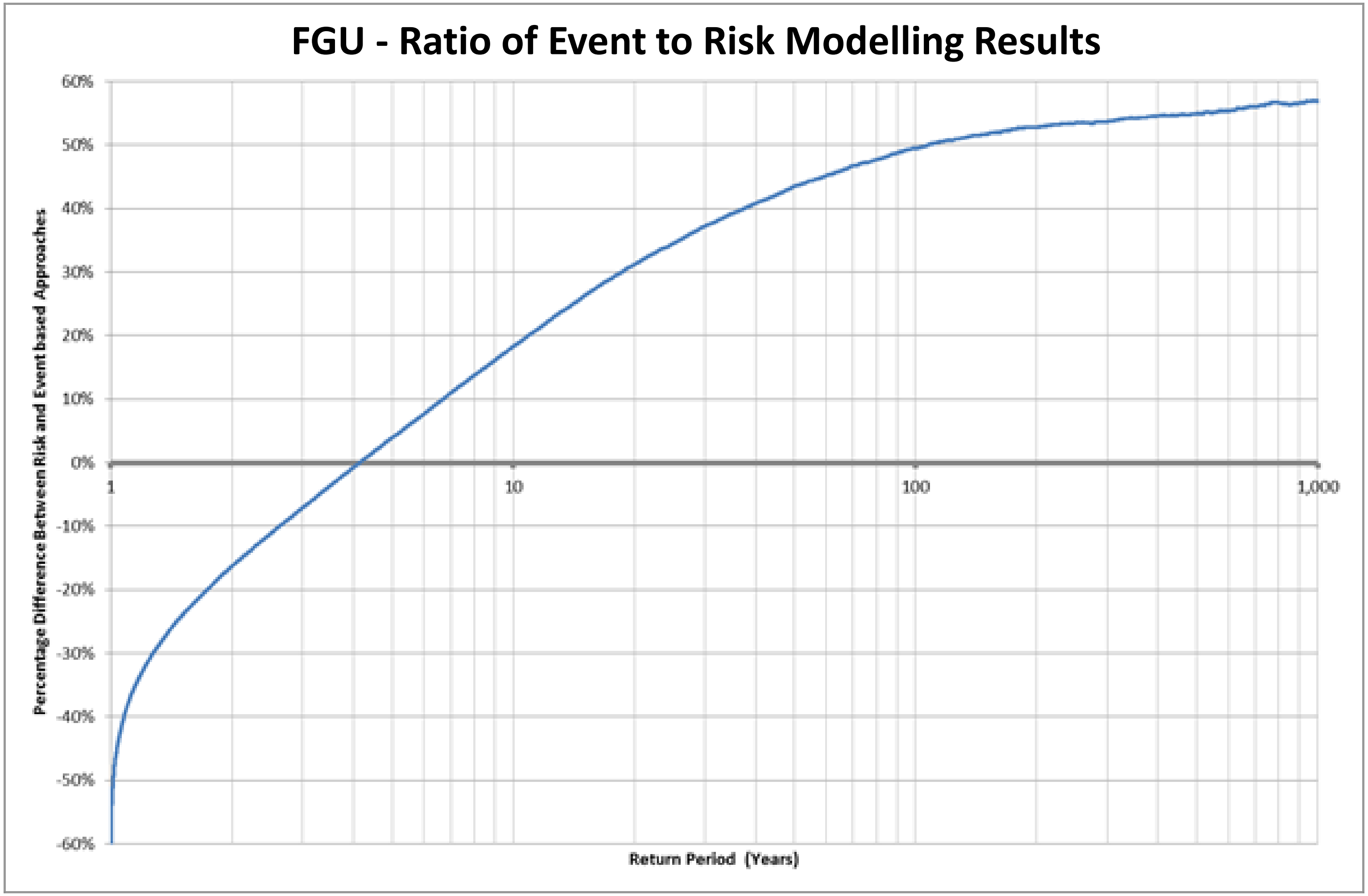


…so when running simulations we uplift the correlation entries.

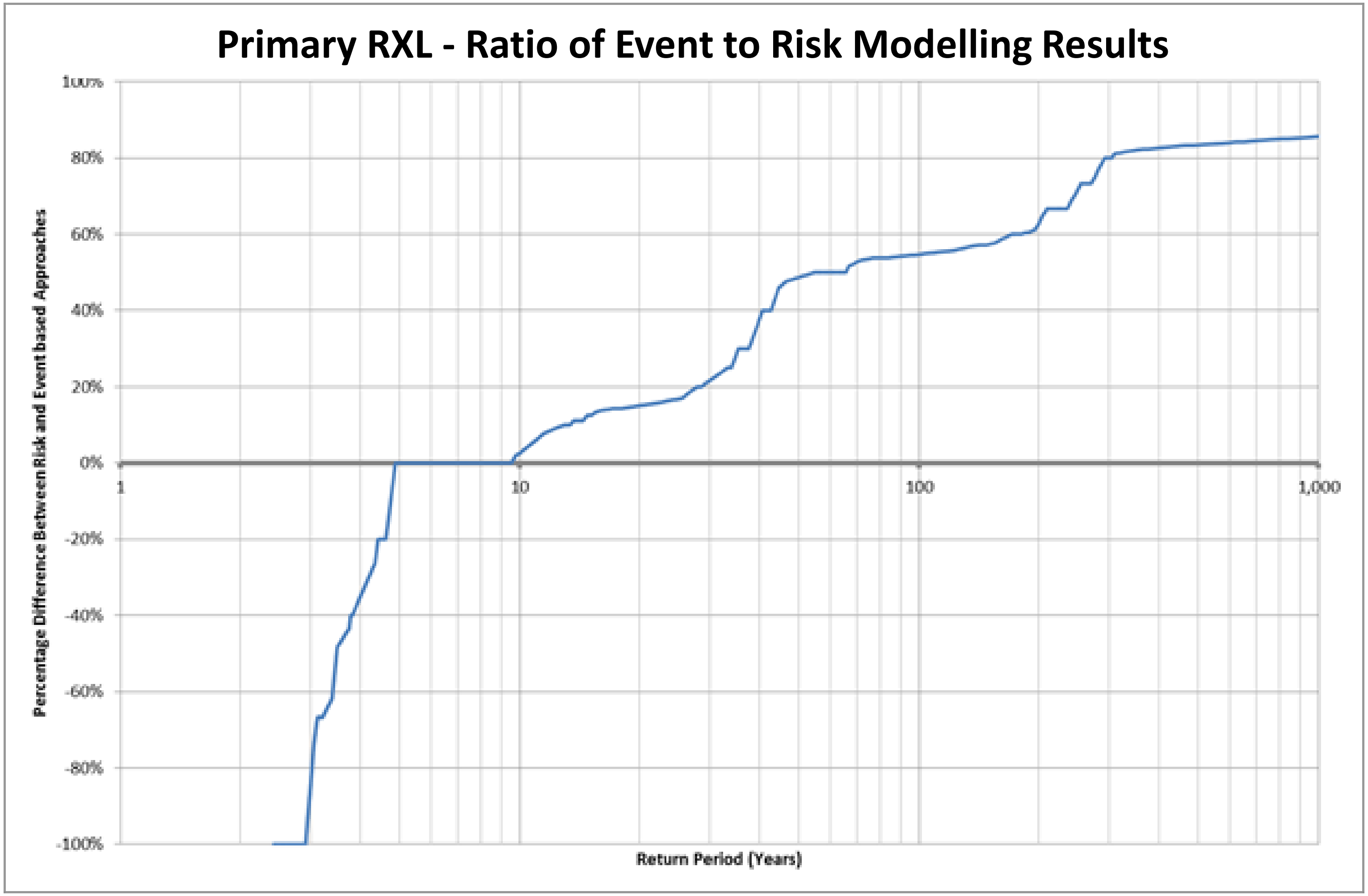
LOCKTON

Results

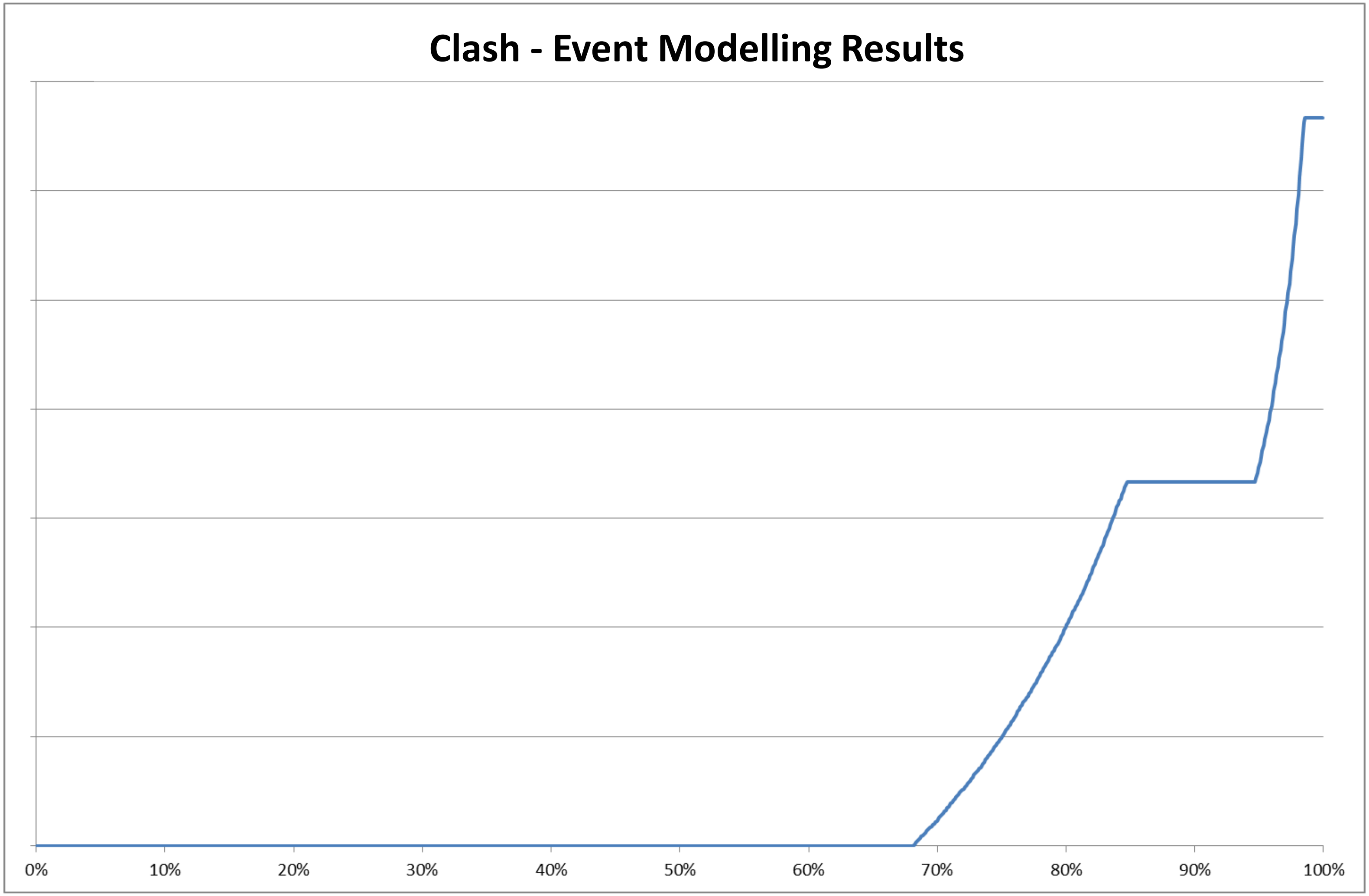
Impact of Event vs Risk Modelling



Impact of Event vs Risk Modelling



Impact of Event vs Risk Modelling



Issues

Problems

- Lack of data available to estimate parameters; calibrate to market price
  - Can "box" the solution to determine what you believe a credible solution looks like
  - Can cross check your assumptions against those implied by the market price
- Markedly slower than a pure risk XL run as you are simulating losses fgu as opposed to excess of an attachment point
  - Much less of an issue today with modern PC's
- Assumes same severity distribution for the individual losses that arise as part of a systemic event verses a "non-systemic" losses
  - This may not be appropriate for all lines, e.g. large FI D&O and E&O losses for tech IPO "laddering" were materially larger than "typical" losses
- Clash reinsurance varies materially in types of events covered; the "events" considered in determining the distribution in number of losses may not fully match the event definition that will trigger clash reinsurance

Enhancements

1) Vary exposure by underwriting year (implicitly assuming exposure flat)

2) Vary loss ratio by Attachment / Limit / Year

3) Explicitly model the different loss processes