Ceded Reserving –
It's Not as Simple as Subtraction
CAS Seminar on Reinsurance

Bruce D. Fell, FCAS, MAAA, CERA, CFA
Deloitte Consulting

Joseph Milicia, FCAS, MAAA
Willis Towers Watson

June 5, 2017

Discussion Outline

- Reinsurance Contract Types and Approaches
- Examples
- Potential Pitfalls
- Reserve Ranges
- Other Issues

WARNING

1. **ACCOUNTANTS, AUDITORS AND OTHER PROFESSIONALS WHO LIKE NUMBERS TO ALWAYS ADD UP MAY NOT AGREE WITH SOME OR ALL OF THE ENCLOSED MESSAGES**
2. **SOME OF THE ENCLOSED APPROACHES MAY CREATE ADVERSE OR FAVORABLE DEVELOPMENT WHICH COULD GENERATE A LOSS OF CREDIBILITY FOR THE ACTUARY**
3. **USE ALL METHODS WITH A SIGNIFICANT DOSE OF REALITY**

Reinsurance contract types

- Working layer excess of loss
  - Plain vanilla coverage
  - Aggregate limits, corridors, annual aggregate deductibles
- Quota share
  - Straightforward percentage
  - Contains caps or corridors
- Aggregate excess of loss
- Loss portfolio transfer
- High layer excess of loss
  - Low frequency/high severity
- Catastrophe excess of loss

4

Approaches used for various reinsurance contract types

- Working layer excess of loss
  - Plain vanilla contracts
    - Estimate gross ultimate loss and net ultimate loss and subtract to estimate ceded ultimate loss
    - Estimate ceded losses directly reflecting the attachment point and limits
    - Apply reinsurance program to gross estimates
    - Gross up the net analysis
  - Contract contains deductibles, aggregate limits, etc.
    - Estimate ceded losses directly reflecting the specific contract features.
    - Simulation method or direct consideration of the full distribution of losses in the layer to properly reflect reinsurance terms
  - With any method the consistency of the estimates should be checked
    - Ceded development patterns will typically be slower while net development patterns will typically be faster than the gross patterns
    - Review gross, ceded and net ultimates as well as unpaid liability estimates
    - Ranges should typically be narrower for net than gross

5

Approaches used for various reinsurance contract types

- Working Layer excess of loss

Method	Pros	Cons
Estimating Gross and Net	• Gross and net data typically more credible than ceded data • Gross and net reserves are what appear on the Actuarial Opinion • Development patterns are typically more mature for gross and net	• Important to consider that loss development and IELRs need to be different • If little or no ceded activity, then data will not reveal required differences • Varying limits and retentions need to be reflected in the net analysis
Estimating Ceded Directly	• Relies on actual ceded history	• If ceded data is sparse then development and IELRs must be derived • Varying limits and retentions require different assumptions for each year • Typically development patterns are less mature for ceded data
Apply reinsurance terms to gross	• Benefit from credibility of gross data • Common reinsurance terms makes it relatively easy	• May be difficult to develop assumptions that accurately reflect reinsurance terms
Gross up net	• Benefit from credibility of net data • Industry benchmarks can assist with expected excess losses	• Varying limits and retentions need to be reflected in the net analysis • Reliance on industry data or other data sources

6

Approaches used for various reinsurance contract types

- Quota Share
  - Straightforward
    - Estimate gross ultimate loss, then apply quota share percentage to estimate ceded ultimate loss
    - Estimating net ultimate losses and subtracting from gross estimates should generate identical results
  - Contract contains loss corridors, caps, etc.
    - Estimate ceded losses directly to specifically reflect portions of quota share with reinsurance protection versus portions retained net
    - Must consider full distribution of gross losses in order to reflect the true ceded losses

7

Approaches used for various reinsurance contract types

- Quota Share

Method	Pros	Cons
Estimating Gross and Multiplying QS Percentage	• Simplest form of reinsurance so this yields correct answers if you do the math correctly	• Varying quota share percentages could complicate analysis especially if they change within a reserving time period (e.g. quarterly)
Estimating Gross and Net	• Should yield same results as above • Can possibly make varying percentages easier to deal with	• Changes in quota share percentages could be masked by this approach

8

Approaches used for various reinsurance contract types

- Aggregate Excess of Loss/Adverse Development Cover
  - Estimate ceded ultimate losses directly based on gross ultimate losses reflecting the full distribution of gross losses
  - Just because gross losses are below the attachment point does not mean that there is not a ceded liability
- Loss Portfolio Transfer
  - Estimate ceded ultimate losses directly based on gross ultimate losses reflecting the full distribution of gross losses
  - Just because gross losses are below the contract limit does not mean that all of the gross liability can be ceded
  - A net liability can still exist when gross ultimate losses are below the contract limit

9

Approaches used for various reinsurance contract types

- High layer excess of loss
  - Very likely that historical ceded losses to the layer are not credible
    - Traditional approaches to Gross and Net unlikely to be reasonable as Net is likely to be too similar to Gross (Ceded liabilities are likely to be understated)
    - Similarly for Ceded, cannot rely upon traditional techniques due to inadequate data
  - Even if all claims are below attachment, ceded is not necessarily zero
  - Must estimate ceded losses directly
  - Utilize Frequency/Severity approaches
    - Pro: Will be more accurate than traditional actuarial approaches
    - Con: Requires more "advanced" assumptions and may be more difficult to communicate to non-actuarial audiences
    - Need to select a credible excess layer upon which you can do analysis
  - Same consistency tests are relevant for high excess contracts as they are for working layer excess

10

Approaches used for various reinsurance contract types

- Catastrophe Excess of Loss
  - Similar to the high layer excess of loss, traditional methods are unlikely to produce credible estimates
  - Estimate Ceded directly
  - First consideration needs to be layers and perils covered
    - If high excess and only covers named storms, has there been a named storm that has any possibility of piercing the layer? If not, earned reserves are zero
  - After an event has occurred, various approaches may be utilized depending on available data:
    - Cat model run on actual portfolio
    - Industry loss estimates and market share approach
    - Ground-up claims development
  - A-Priori estimates should utilize exposure based approaches
    - Leverage results of cat modeling
    - Ensure that you consider seasonality of covered perils

11

Examples: Basic Assumptions

- Gross unpaid liabilities result from identical accident years
- Premium = $1.5 million/year
- Ultimate losses = $1 million/year

- Loss ratio = 66.7%
- Expected direct/gross unpaid liabilities = $2 million

- Loss development patterns as follows:

	12	24	36	48	60	72	84
Paid %	29.0%	40.0%	62.5%	80.0%	91.0%	97.5%	100.0%
Reported %	50.0%	75.0%	90.0%	95.0%	100.0%	100.0%	100.0%

12

Gross paid loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	290,000	400,000	625,000	800,000	910,000	975,000	1,000,000
2010	290,000	400,000	625,000	800,000	910,000	975,000	
2011	290,000	400,000	625,000	800,000	910,000		
2012	290,000	400,000	625,000	800,000			
2013	290,000	400,000	625,000				
2014	290,000	400,000					
2015	290,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	1.379	1.563	1.280	1.138	1.071	1.026	
2010	1.379	1.563	1.280	1.138	1.071		
2011	1.379	1.563	1.280	1.138			
2012	1.379	1.563	1.280				
2013	1.379	1.563					
2014	1.379						
Incremental	1.379	1.563	1.280	1.138	1.071	1.026	1.000
Cumulative	3.448	2.500	1.600	1.250	1.099	1.026	1.000

13

Gross reported loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	500,000	750,000	900,000	950,000	1,000,000	1,000,000	1,000,000
2010	500,000	750,000	900,000	950,000	1,000,000	1,000,000	
2011	500,000	750,000	900,000	950,000	1,000,000		
2012	500,000	750,000	900,000	950,000			
2013	500,000	750,000	900,000				
2014	500,000	750,000					
2015	500,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	1.500	1.200	1.056	1.053	1.000	1.000	
2010	1.500	1.200	1.056	1.053	1.000		
2011	1.500	1.200	1.056	1.053			
2012	1.500	1.200	1.056				
2013	1.500	1.200					
2014	1.500						
Incremental	1.500	1.200	1.056	1.053	1.000	1.000	1.000
Cumulative	2.000	1.333	1.111	1.053	1.000	1.000	1.000

14

Gross estimate of ultimate losses

Acc Yr	Paid Losses	Reported Losses	Ultimate Losses	Case Reserves	IBNR	Total Unpaid Liabilities
2009	1,000,000	1,000,000	1,000,000	0	0	0
2010	975,000	1,000,000	1,000,000	25,000	0	25,000
2011	910,000	1,000,000	1,000,000	90,000	0	90,000
2012	800,000	950,000	1,000,000	150,000	50,000	200,000
2013	625,000	900,000	1,000,000	275,000	100,000	375,000
2014	400,000	750,000	1,000,000	350,000	250,000	600,000
2015	290,000	500,000	1,000,000	210,000	500,000	710,000
Total	**5,000,000**	**6,100,000**	**7,000,000**	**1,100,000**	**900,000**	**2,000,000**

15

Example #1: Working Layer Excess of Loss

- Further, assume that the company purchases reinsurance excess of $50,000 so that there is a frequency of claims ceded to the reinsurer
- In this case, either approach could work:
  - Gross less net
  - Ceded analysis directly
- It is important to make sure that the analysis is consistent

16

Working layer excess ceded paid loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	25,000	100,000	150,000	200,000	250,000	275,000	300,000
2010	25,000	100,000	150,000	200,000	250,000	275,000	
2011	25,000	100,000	150,000	200,000	250,000		
2012	25,000	100,000	150,000	200,000			
2013	25,000	100,000	150,000				
2014	25,000	100,000					
2015	25,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	4.000	1.500	1.333	1.250	1.100	1.091	
2010	4.000	1.500	1.333	1.250	1.100		
2011	4.000	1.500	1.333	1.250			
2012	4.000	1.500	1.333				
2013	4.000	1.500					
2014	4.000						
Incremental	4.000	1.500	1.333	1.250	1.100	1.091	1.000
Cumulative	12.000	3.000	2.000	1.500	1.200	1.091	1.000

17

Working layer excess ceded reported loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	50,000	175,000	225,000	275,000	300,000	300,000	300,000
2010	50,000	175,000	225,000	275,000	300,000	300,000	
2011	50,000	175,000	225,000	275,000	300,000		
2012	50,000	175,000	225,000	275,000			
2013	50,000	175,000	225,000				
2014	50,000	175,000					
2015	50,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	3.500	1.286	1.222	1.091	1.000	1.000	
2010	3.500	1.286	1.222	1.091	1.000		
2011	3.500	1.286	1.222	1.091			
2012	3.500	1.286	1.222				
2013	3.500	1.286					
2014	3.500						
Incremental	3.500	1.286	1.222	1.091	1.000	1.000	1.000
Cumulative	6.000	1.714	1.333	1.091	1.000	1.000	1.000

18

Working layer excess ceded estimate of ultimate losses

Acc Yr	Paid Losses	Reported Losses	Ultimate Losses	Case Reserves	IBNR	Total Unpaid Liabilities
2009	300,000	300,000	300,000	0	0	0
2010	275,000	300,000	300,000	25,000	0	25,000
2011	250,000	300,000	300,000	50,000	0	50,000
2012	200,000	275,000	300,000	75,000	25,000	100,000
2013	150,000	225,000	300,000	75,000	75,000	150,000
2014	100,000	175,000	300,000	75,000	125,000	200,000
2015	25,000	50,000	300,000	25,000	250,000	275,000
Total	**1,300,000**	**1,625,000**	**2,100,000**	**325,000**	**475,000**	**800,000**

19

Net of working layer excess paid loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	265,000	300,000	475,000	600,000	660,000	700,000	700,000
2010	265,000	300,000	475,000	600,000	660,000	700,000	
2011	265,000	300,000	475,000	600,000	660,000		
2012	265,000	300,000	475,000	600,000			
2013	265,000	300,000	475,000				
2014	265,000	300,000					
2015	265,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	1.132	1.583	1.263	1.100	1.061	1.000	
2010	1.132	1.583	1.263	1.100	1.061		
2011	1.132	1.583	1.263	1.100			
2012	1.132	1.583	1.263				
2013	1.132	1.583					
2014	1.132						
Incremental	1.132	1.583	1.263	1.100	1.061	1.000	1.000
Cumulative	2.642	2.333	1.474	1.167	1.061	1.000	1.000

20

Net of working layer excess reported loss triangle

Acc Yr	12	24	36	48	60	72	84
2009	450,000	575,000	675,000	675,000	700,000	700,000	700,000
2010	450,000	575,000	675,000	675,000	700,000	700,000	
2011	450,000	575,000	675,000	675,000	700,000		
2012	450,000	575,000	675,000	675,000			
2013	450,000	575,000	675,000				
2014	450,000	575,000					
2015	450,000						
Acc Yr	**24-Dec**	**24-36**	**36-48**	**48-60**	**60-72**	**72-84**	**84-Ult**
2009	1.278	1.174	1.000	1.037	1.000	1.000	
2010	1.278	1.174	1.000	1.037	1.000		
2011	1.278	1.174	1.000	1.037			
2012	1.278	1.174	1.000				
2013	1.278	1.174					
2014	1.278						
Incremental	1.278	1.174	1.000	1.037	1.000	1.000	1.000
Cumulative	1.556	1.217	1.037	1.037	1.000	1.000	1.000

21

Net of working layer excess estimate of ultimate losses

Acc Yr	Paid Losses	Reported Losses	Ultimate Losses	Case Reserves	IBNR	Total Unpaid Liabilities
2009	700,000	700,000	700,000	0	0	0
2010	700,000	700,000	700,000	0	0	0
2011	660,000	700,000	700,000	40,000	0	40,000
2012	600,000	675,000	700,000	75,000	25,000	100,000
2013	475,000	675,000	700,000	200,000	25,000	225,000
2014	300,000	575,000	700,000	275,000	125,000	400,000
2015	265,000	450,000	700,000	185,000	250,000	435,000
Total	**3,700,000**	**4,475,000**	**4,900,000**	**775,000**	**425,000**	**1,200,000**

22

Example #1: Working layer excess of loss

- Development factors definitely vary for the three segments

	Paid LDFs			Reported LDFs		
Age	Gross	Ceded	Net	Gross	Ceded	Net
12	3.448	12.000	2.642	2.000	6.000	1.556
24	2.500	3.000	2.333	1.333	1.714	1.217
36	1.600	2.000	1.474	1.111	1.333	1.037
48	1.250	1.500	1.167	1.053	1.091	1.037
60	1.099	1.200	1.061	1.000	1.000	1.000
72	1.026	1.091	1.000	1.000	1.000	1.000
84	1.000	1.000	1.000	1.000	1.000	1.000

Acc Yr	Gross Reserves	Ceded Reserves	Net Reserves
2009	0	0	0
2010	25,000	25,000	0
2011	90,000	50,000	40,000
2012	200,000	100,000	100,000
2013	375,000	150,000	225,000
2014	600,000	200,000	400,000
2015	710,000	275,000	435,000
Total	**2,000,000**	**800,000**	**1,200,000**

- If you were using Bornhuetter-Ferguson methods you would also want to ensure that initial expected losses are consistent

23

Example #1: Working layer excess of loss

- What if this is the only data you have looking only at gross and net?

Gross				
Acc Yr	12	24	36	48
2012	500,000	750,000	900,000	950,000
2013	500,000	750,000	900,000	
2014	500,000	750,000		
2015	500,000			
Acc Yr	24-Dec	24-36	36-48	48-60
2012	1.500	1.200	1.056	
2013	1.500	1.200		
2014	1.500			
				Tail?
Incremental	1.500	1.200	1.056	

Net				
Acc Yr	12	24	36	48
2012	450,000	575,000	675,000	675,000
2013	450,000	575,000	675,000	
2014	450,000	575,000		
2015	450,000			
Acc Yr	24-Dec	24-36	36-48	48-60
2012	1.278	1.174	1.000	
2013	1.278	1.174		
2014	1.278			
				Tail?
Incremental	1.278	1.174	1.000	

- Should the tail factors be the same?
- What about looking at ceded data?

24

Example #1: Working layer excess of loss

- What if this is the only data you have looking only at gross and net?

Gross				
Acc Yr	12	24	36	48
2012	500,000	750,000	900,000	950,000
2013	500,000	750,000	900,000	
2014	500,000	750,000		
2015	500,000			
Acc Yr	24-Dec	24-36	36-48	48-60
2012	1.500	1.200	1.056	
2013	1.500	1.200		
2014	1.500			
				Tail?
Incremental	1.500	1.200	1.056	

Net				
Acc Yr	12	24	36	48
2012	450,000	575,000	675,000	675,000
2013	450,000	575,000	675,000	
2014	450,000	575,000		
2015	450,000			
Acc Yr	24-Dec	24-36	36-48	48-60
2012	1.278	1.174	1.000	
2013	1.278	1.174		
2014	1.278			
				Tail?
Incremental	1.278	1.174	1.000	

Ceded				
Acc Yr	12	24	36	48
2012	50,000	175,000	225,000	275,000
2013	50,000	175,000	225,000	
2014	50,000	175,000		
2015	50,000			
Acc Yr	24-Dec	24-36	36-48	48-60
2012	3.500	1.286	1.222	
2013	3.500	1.286		
2014	3.500			
				Tail?
Incremental	3.500	1.286	1.222	

- Should the tail factors be the same?
- What about looking at ceded data?
- Make sure the tails hang together!
- What if cessions were less frequent?

25

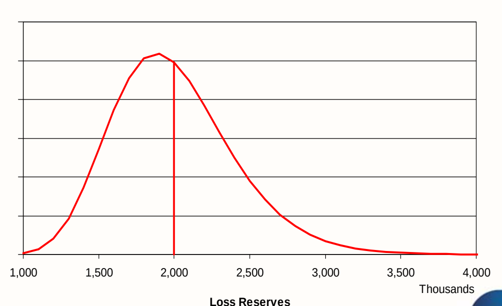
Examples: More Basic Assumptions

- Gross unpaid liabilities result from identical accident years
- Premium = $1.5 million/year
- Ultimate losses = $1 million/year
- **AY loss distribution = LogN(μ = 13.784, σ = 0.25) [CV=0.254]**
- Loss ratio = 66.7%
- Expected direct/gross unpaid liabilities = $2 million
- **Reserve distribution = LogN (μ = 14.489, σ = 0.20) [CV=0.202]**
- Loss development patterns as follows:

	12	24	36	48	60	72	84
Paid %	29.0%	40.0%	62.5%	80.0%	91.0%	97.5%	100.0%
Reported %	50.0%	75.0%	90.0%	95.0%	100.0%	100.0%	100.0%

26

Gross unpaid loss liabilities distribution



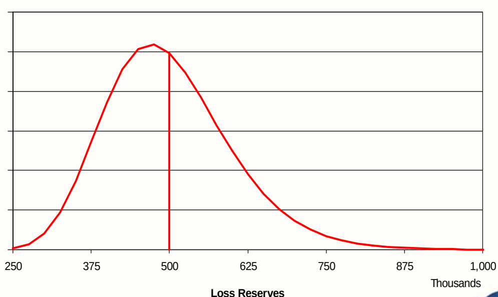
27

Example #2: Quota share

- Assume 25% quota share of business
- Premium = $375,000/year
- Ultimate losses = $250,000/year
- Loss ratio = 66.7%
- Expected ceded unpaid liabilities = $500,000
- Unpaid liabilities dist. = Lognormal (μ = 13.102, σ = 0.20)
- Appropriate methods for cedant:
  - Apply quota share percentage to gross losses
  - No need to separately use loss development or B-F
  - Easy and straightforward cession
- Appropriate methods for reinsurer
  - Typically one contract in portfolio of similar quota shares
  - Similar straightforward loss development and/or BF methods

28

Ceded loss reserve distribution



29

Example #3: Quota Share with corridor

- Assume 25% quota share of business
- Premium = $375,000/year
- Ultimate losses = $250,000/year
- Loss ratio = 66.7%
- Loss ratio corridor between 70% and 75%
  - Cedant retains liability in this 5% corridor
- **Are the expected ceded unpaid liabilities still = $500,000?**
- **Should the reinsurer's unpaid liabilities be less than $500,000?**

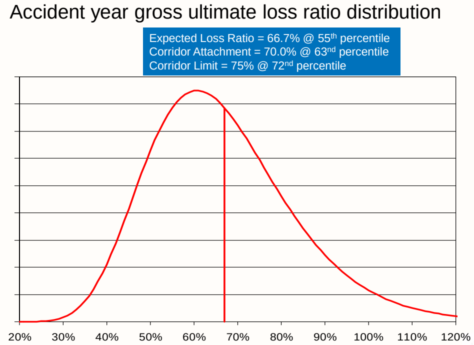
30

Example #3: Quota Share with corridor

- Assume 25% quota share of business
- Premium = $375,000/year
- Ultimate losses = $250,000/year
- Loss ratio = 66.7%
- Loss ratio corridor between 70% and 75%
  - Cedant retains liability in this 5% corridor
- **Are the expected ceded unpaid liabilities still = $500,000?**
  NO, The $500,000 represents the cession of the expected gross reserves instead of the expected ceded reserves. How do we handle this in order to get the correct number?
- **Should the reinsurer's unpaid liabilities be less than $500,000?**
  YES, the reinsurer's liability drops due to the corridor.

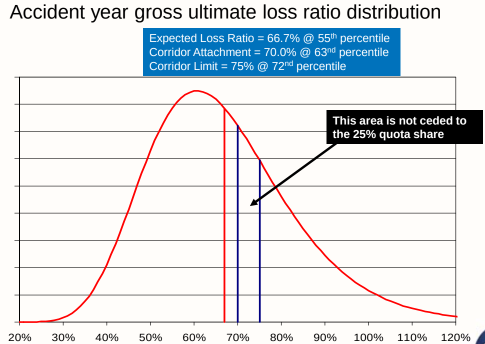
31

Accident year gross ultimate loss ratio distribution



32

Accident year gross ultimate loss ratio distribution



33

Accident year gross and ceded losses

- Gross E(X) = $1,000,000
- Gross Limited Expected Value @ 70% loss ratio = $921,112
- Gross Limited Expected Value @ 75% loss ratio = $945,365

- Quota Share w/o corridor E(X) = 25% x 1,000,000 = $250,000
- Quota Share LEV @70% LR = $230,278
- Quota Share LEV @75% LR = $236,341
- E(X) between 70% and 75% = $6,063
- Quota Share w/corridor E(X) = $250,000 - $6,063 = $243,937

- This also points out the economic cost of the corridor!

34

Impact on loss reserves

- In order to calculate the correct ceded unpaid liabilities, one must consider the variability of the liabilities for each year to determine the appropriate adjustment
- As accident years mature there is less variability in the unpaid liabilities and therefore less chance that the corridor will be reached

- From a reinsurer's perspective, this contract would likely be within a portfolio and therefore the corridor may not be reflected explicitly
- However, for material contracts, the reinsurer would follow a similar approach as described for the cedant

35

Example #4: Adverse development cover

- Assume gross expected unpaid loss liabilities of $2 million
- Adverse development cover is purchased that attaches excess of $2.5 million with a $1 million limit
- How much should the company reflect as a ceded reserve for this contract?

  Zero

  Something greater than zero that reflects the expected value of the losses that could potentially reach the reinsurance

36

Example #4: Adverse development cover

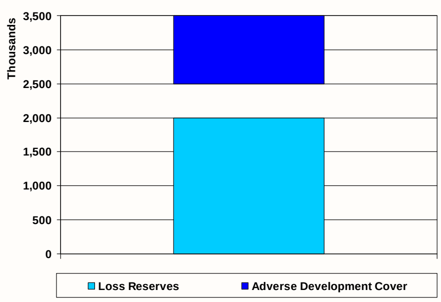
- Assume gross expected unpaid loss liabilities of $2 million
- Adverse development cover is purchased that attaches excess of $2.5 million with a $1 million limit
- How much should the company reflect as a ceded reserve for this contract?

Zero

Something greater than zero that reflects the expected value of the losses that could potentially reach the reinsurance

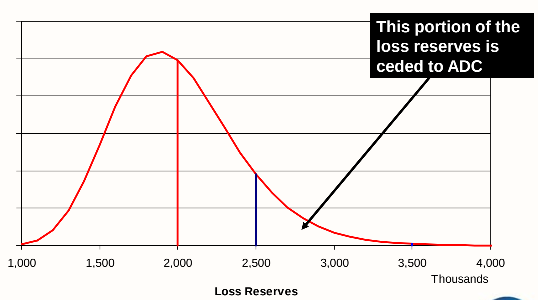
37

Example #4: Adverse development cover



38

Gross unpaid loss liabilities distribution



39

Adverse development cover ceded unpaid liabilities

- Gross E(X) = $2,000,000
- Gross Limited Expected Value @ $2.5m = $1,970,352
- Gross Limited Expected Value @ $3.5m = $1,999,596

- ADC E(X) between $2.5m and $3.5 m = $29,245
- Net E(X) = $1,970,755

- Reinsurer would typically also consider contract pricing
  - Assume reinsurance premium was 20% rate on line = $200,000 reflecting significant risk margin and expenses
  - At the expected estimate above, loss ratio = 14.6%
  - Likely reinsurer would initially reserve at a higher loss ratio to reflect increased risk

40

Example #4A: Adverse development cover

- What if the $2 million of loss reserves develops adversely to $3 million after the coverage is purchased?
- Adverse development cover is purchased that attaches excess of $2.5 million with a $1 million limit
- New reserve distribution = Lognormal (μ = 14.907, σ = 0.12)
- How much should the company reflect as a ceded reserve for this contract?

$500,000

Something greater than $500,000

Something less than $500,000

It depends

41

Example #4A: Adverse development cover

- What if the $2 million of loss reserves develops adversely to $3 million after the coverage is purchased?
- Adverse development cover is purchased that attaches excess of $2.5 million with a $1 million limit
- New reserve distribution = Lognormal (μ = 14.907, σ = 0.12)
- How much should the company reflect as a ceded reserve for this contract?

$500,000

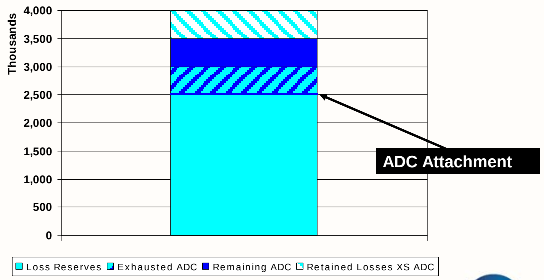
Something greater than $500,000

Something less than $500,000

It depends

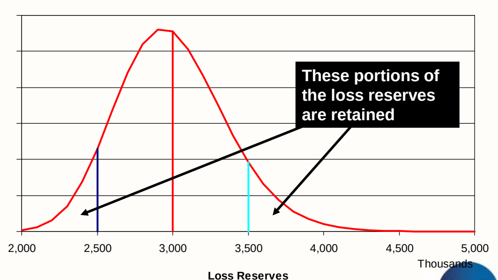
42

Example #4A: Adverse development cover



43

Gross unpaid loss liabilities distribution



44

Adverse development cover ceded unpaid liabilities

- Gross E(X) = $3,000,000
- Gross Limited Expected Value @ $2.5m = $2,490,798
- Gross Limited Expected Value @ $3.5m = $2,981,731

- ADC E(X) between $2.5m and $3.5 m = $490,934
- Net E(X) = $2,509,066

45

Example #4A: Adverse development cover

- What if the $2 million of loss reserves develops adversely to $3 million after the coverage is purchased?
- Adverse development cover is purchased that attaches excess of $2.5 million with a $1 million limit
- New reserve distribution = Lognormal (μ = 14.907, σ = 0.12)
- How much should the company reflect as a ceded reserve for this contract?

$500,000

Something greater than $500,000

Something less than $500,000 – Actual ceded = 490,934

It depends

46

Example #5: Loss Portfolio Transfer

- Assume gross expected loss reserves of $2 million
- Loss portfolio transfer is purchased with a $2.5 million limit
- How much should the company reflect as a ceded reserve for this contract?

$2 million

Something less than $2 million that reflects that the company still retains a potential liability

47

Example #5: Loss Portfolio Transfer

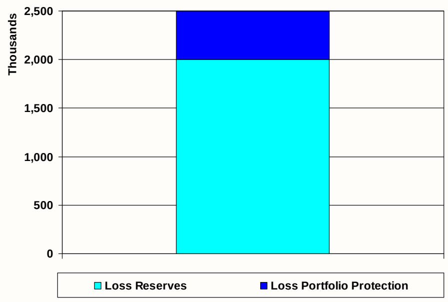
- Assume gross expected loss reserves of $2 million
- Loss portfolio transfer is purchased with a $2.5 million limit
- How much should the company reflect as a ceded reserve for this contract?

$2 million

Something less than $2 million that reflects that the company still retains a potential liability

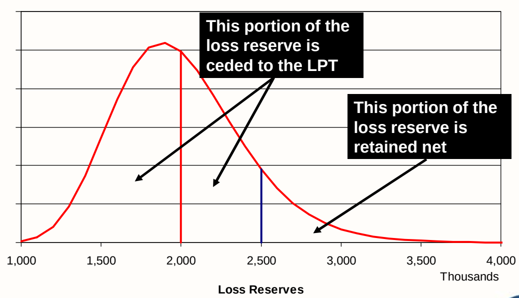
48

Example #5: Loss Portfolio Transfer



49

Gross unpaid loss liabilities distribution



50

Loss portfolio transfer cover ceded and net liabilities

- Gross E(X) = $2,000,000
- Limited Expected Value @ $2.5m = $1,970,352
  - Equivalent to the reserve ceded to the LPT
- Retained net reserves = $29,648
- Reinsurer's liabilities are equivalent to the LEV of $1,970,352, however reinsurer would likely reflect full $2 million or even something higher to reflect chance of adverse development.

51

Example #6: High layer excess of loss

- Losses tend to be low frequency but high severity
- Ceded data is rarely credible and if credible does not display typical loss development
  - Often, reported loss experience is 0 and then pops, reported may then change a bit as the claim matures and new information is uncovered but largest change is when it enters layer and when it settles
  - Paid data, can often be zero until settlement
- Traditional approaches rarely work well. How would you approach?

52

Example #6: High layer excess of loss

- Losses tend to be low frequency but high severity
- Ceded data is rarely credible and if credible does not display typical loss development
  - Often, reported loss experience is 0 and then pops, reported may then change a bit as the claim matures and new information is uncovered but largest change is when it enters layer and when it settles
  - Paid data, can often be zero until settlement
- Traditional approaches rarely work well. How would you approach?

Use frequency/severity approach

53

High layer excess of loss – Frequency/Severity method: Basic steps

- Pick a data limit where credible excess claims data exists
- Estimate the annual number of claims above the data limit
  - 37.5 claims greater than $150,000
- Use size-of-loss curves to project the number of claims above the reinsurance retention
  - 8.6 (of 37.5 claims) greater than $300,000
- Use size-of-loss curves to project average severity of claims in reinsurance layer
  - $246,020 average severity of claims in $700,000 excess of $300,000 layer
- Multiply the frequency and the severity projections to estimate the total ultimate losses
- Incorporate frequency/severity estimate into Bornhuetter-Ferguson method
- Most common distribution used is the Single-parameter Pareto

54

High layer excess of loss – Frequency/Severity method: Why use the single-parameter Pareto distribution?

- Shape of tail
- Ease of calculation (even though it's not built into Excel)
  - survival function S(X) = (Theta / X) ^ Alpha
  - conditional limited expected value is a simple formula (see following pages)
  - simple to incorporate trend
- Easy to parameterize
  - Theta must be set in advance (equal to the data limit)
  - maximum likelihood estimated Alpha parameter is simple to calculate
    - normalize losses greater than the data limit by dividing by the data limit = X
    - take the natural log of the normalized losses = ln(X)
    - MLE Alpha = the number of losses > the data limit / sum[ln(X)]
- Always a good idea to look at the graph of your observations and fitted distribution
  - beware over-weighting to smaller values
  - keep in mind what layer you are interested in

55

Example #6: High layer excess of loss: Estimate frequency above data limit

Accident Year	Detrended Data Limit	Act #> Detrended Data Limit	Claim Count Develpoment Factors	Individual Total Excess Counts (3 x 4)
(1)	(2)	(3)	(4)	(5)
2010	$112,089	38	1.125	42.8
2011	118,814	34	1.282	43.6
2012	125,943	25	1.408	35.2
2013	133,499	31	1.555	48.2
2014	141,509	22	1.927	42.4
2015	150,000	11	2.618	28.8
Total		161		240.9

(2) Assumes 6% trend.

56

Example #6: High layer excess of loss: Estimate frequency above data limit (continued)

Acc. Year	Projected # of claims > Data Limit	Subject Earned Premium	On-Level SEP	Indicated Frequency (2 / 4)	Selected Frequency	Selected # of Excess Claims
(1)	(2)	(3)	(4)	(5)	(6)	(7)
2010	42.8	$50,000	$62,750	0.681		42.8
2011	43.6	50,000	63,550	0.686		43.6
2012	35.2	55,000	63,525	0.554		35.2
2013	48.2	60,000	63,000	0.765		48.2
2014	42.4	55,000	55,000	0.771	0.750	41.3
2015	28.8	50,000	50,000	0.576	0.750	37.5
Total	240.9	$320,000	$357,825	0.673		248.5

57

Example #6: High layer excess of loss: Estimate excess losses using single-parameter Pareto

Accident Year	Layer Limit	XS Retention	Projected # > $150,000	Projected # > Retention	Average Severity in Layer	Ultimate Loss & ALAE in Layer (4 x 5)
(1)	(2)		(3)	(4)	(5)	(6)
2010	$800,000	$200,000	42.8	12.5	$171,963	$2,147,832
2011	800,000	200,000	43.6	14.4	171,963	2,478,598
2012	800,000	200,000	35.2	13.2	171,963	2,265,462
2013	750,000	250,000	48.2	12.7	210,543	2,675,806
2014	750,000	250,000	41.3	12.3	210,543	2,591,561
2015	700,000	300,000	37.5	8.6	246,020	2,115,012
Total			248.5	73.7		$14,274,272

Notes: (4) from pareto size-of-loss curve frequency formula; (3) x [Basic Limit / Attachment] ^ Alpha
(5) from pareto size-of-loss curve severity formula;
[Retention / (Alpha - 1)] * { 1 - [Retention / (Limit + Retention)] ^ (Alpha - 1) }
Assumes Alpha parameter of 2.125, basic limit detrended at 6%.

58

Example #7: Catastrophe excess of loss

- Traditional actuarial techniques don't work for ceded reserves
- Estimation of gross losses typically require special treatment
- Ceded reserves can be determined by applying the reinsurance coverage directly to the gross losses
- Example:
  - Major hurricane occurs in Florida where primary company has 10,000 homeowners claims
  - Calculate gross ultimate losses for all claims related to the event (this may include evaluating catastrophe model results post-event)
  - Apply reinsurance coverage to the aggregated losses
- For reinsurers, they are dependent upon cedants to report losses and need to aggregate across all contracts

59

Example #7: Catastrophe excess of loss

- How do you determine the ceded reserves if no event has occurred?
- How should reinsurers determine their assumed losses when no event occurred?

60

Example #7: Catastrophe excess of loss

- How do you determine the ceded reserves if no event has occurred?

  Accounting rules do not allow for the accrual of reserves when the event has not occurred.

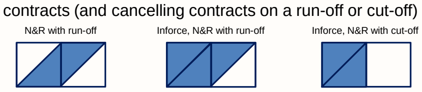
- How should reinsurers determine their assumed losses when no event occurred?

  Similarly, reinsurers can not establish assumed reserves for catastrophes that have not occurred. However, it is possible they may not be aware of all events and therefore may establish some provision.

61

Potential pitfalls

- Industry benchmarks (especially tail development factors)
  - Make sure you are correctly reflecting limits and layers of coverage
- Changes in reinsurance structures over time
- Differences between treaty year and accident year (or mid-year renewals)
- Contracts covering new and renewal business versus inforce contracts (and cancelling contracts on a run-off or cut-off)



- Portfolio in/portfolio out (liabilities are transferred to the next reinsurance contract --- common at Lloyds)
- What pitfalls are we missing?

62

Reserve ranges

- Lows and Highs are not additive if they reflect:
  - True uncertainty of business (e.g. the loss distribution)
  - There is less than perfect positive correlation
- Therefore gross less ceded does not equal net!
- Ranges should reflect differences in:
  - Volume of business
  - Variability of development patterns
  - Variability of different lines of business
- Stochastic techniques can be used to think about ranges

63

Other issues

- Data availability
  - Individual claim data availability
  - Appropriate loss triangles and/or loss development factors
- Ceded loss reserves **should not** reflect the cession of the expected value of the gross loss liabilities → $E(f(x)) \neq f(E(x))$
- Ceded loss reserves **should** reflect the expected value of the ceded loss liabilities
- Some believe the ceded reserves should be “consistent” with the gross liabilities (e.g. a function of the mean) but yet it can overstate or understate the ceded reserves and therefore impact the net reserves
- Would you always expect assumed and ceded liabilities to mirror each other?
- When you sign a loss reserve opinion, what are you “opining” on?

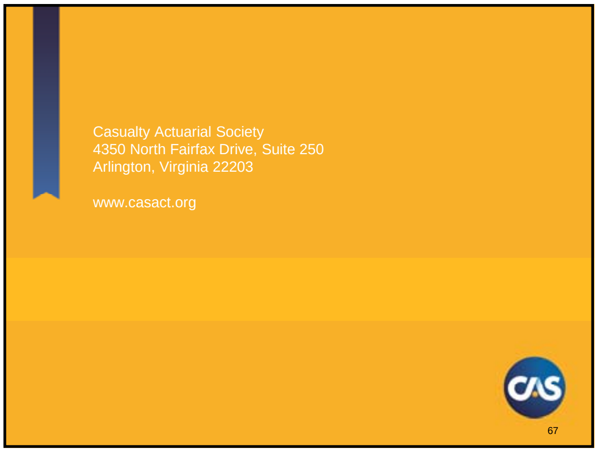
64

QUESTIONS

65

THANK YOU

66


Casualty Actuarial Society
4350 North Fairfax Drive, Suite 250
Arlington, Virginia 22203
www.casact.org
67