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Developing Treaty Experience

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Economic Capital Modeling

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Agenda

The Treaty Experience Development Problem

- Account profitability evaluation and pricing context
- Losses Occurring vs Risks Attaching basis
- Changes in treaty duration
- How to modify LDF??

Understanding Loss Development

- Development of Exposure to Loss vs Claim Development
- Decomposition Model of Loss Development – Convolution
- Two factor development approximation formula
- General Accident period formula
- AMOL approximation

Application to Treaty Development

- Extended or stub period on losses occurring treaty
- Risks Attaching treaty cut-off

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LO Treaty Experience Example

INS Co X LOB A Treaty Experience

Eval date 12/31/2017

Treaty	Basis	Eff date	Expiry Date	Duration (months)	Ult Treaty Prem	Treaty Ceded Loss TD	Treaty LR TD vs Ult Prem	ATU LDF	LDF Method Treaty LR TD vs Ult Prem	A Priori ELR	BF Method Treaty LR TD vs Ult Prem
6	LO	1/1/2017	12/31/2017	24.0	125.0	14.0	11.2%			65.0%	
5	LO	1/1/2016	12/31/2016	18.0	100.0	25.0	25.0%			62.0%	
4	LO	7/1/2015	12/31/2015	33.0	60.0	30.0	50.0%			61.0%	
3	LO	4/1/2013	6/30/2014	39.0	140.0	70.0	50.0%			64.0%	
2	LO	4/1/2011	3/31/2012	24.0	90.0	63.0	70.0%			67.0%	
1	LO	4/1/2010	3/31/2011	1,335.0	95.0	62.0	65.3%			63.0%	
								Total	0.0%	63.7%	0.0%

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RA Treaty Experience Example

INS Co X LOB A Treaty Experience

Eval date 12/31/2017

Treaty	Basis	Eff date	Expiry Date	Exposure Cutoff Date	Duration (months)	Ult Treaty Prem	Treaty Ceded Loss TD	Treaty LR TD vs Ult Prem	ATU LDF	LDF Method Treaty LR TD vs Ult Prem	A Priori ELR	BF Method Treaty LR TD vs Ult Prem
6	RA	7/1/2016	6/30/2017	6/30/2018	12.0	140.0	15.0	10.7%			63.0%	
5	RA	7/1/2015	6/30/2016	6/30/2017	12.0	100.0	33.0	33.0%			62.0%	
4	RA/Cutoff	7/1/2014	6/30/2015	6/30/2015	12.0	60.0	26.0	43.3%			61.0%	
3	RA	7/1/2013	6/30/2014	6/30/2015	12.0	110.0	50.0	45.5%			64.0%	
2	RA	7/1/2012	6/30/2013	6/30/2014	12.0	100.0	72.0	72.0%			68.0%	
1	RA	7/1/2011	6/30/2012	6/30/2013	12.0	95.0	65.0	68.4%			63.0%	
									Total	0.0%	63.6%	0.0%

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Claims Process vs Bucketing and Evaluation
Development of Exposure to Loss vs Claim Development
Convolution Equation
Two Factor Decomposition Approximation Formula
General Accident Period Equation

UNDERSTANDING LOSS DEVELOPMENT

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Claim Development

- Development can be described as a series of paired dates and amounts relating to actions on a claim

Date	Action	Change in Paid	Cumulative Paid	Current Reserve	Claim Development	Cumulative Development
4/3/2011	Trigger event occurs	$0	$0	$0	$0	$0
8/2/2011	Claim reported/ intitial reserve set	$0	$0	$25,000	$25,000	$25,000
7/1/2012	payment/reserve takedown/revision	$47,000	$47,000	$15,000	$37,000	$62,000
4/1/2013	payment/reserve takedown/revision	$13,000	$60,000	$25,000	$23,000	$85,000
9/15/2013	reserve takedown	$0	$60,000	$15,000	-$10,000	$75,000
2/1/2014	Final payment/ reserve set to $0	$18,000	$78,000	$0	$3,000	$78,000

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Bucketing and Evaluation

- Exposure Period Begins
- Claim Occurrence Date
- Claim is Reported
- Claim valuation at end of each cal period

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Total Loss Development

$$T/A = T+A$$

- T= Underlying claim development RV
  - T = Time at which a dollar of loss is paid
  - T is conceptually driven by a series of paired dates and amounts
- A = Bucketing and valuation process
- T and A are independent

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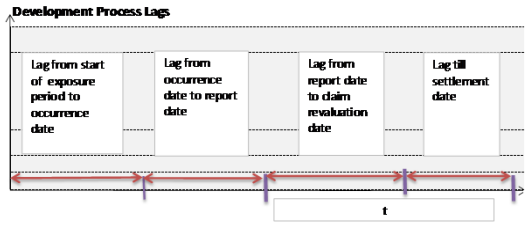
Loss Development Process Lags



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Random Variable Model of Exposures

- A = Exposure random variable
- F_A=CDF of A
- f_A= exposure density

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Conditional ADOL ,AMOL, and Variance

Conditional Average Date of Loss Exposure:
$m_A(t) = E_A[T|T < t]$

Conditional Average Maturity of Loss Exposure:
$r_A(t) = t - m_A(t)$

Condtional Variance of Date of Loss Exposure:
$v_A(t) = E_A[(T - m_A(t))^2|T < t]$

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Exposure Densities

$F_A(s) = Cumulative\ exposure\ \%\ as\ of\ time\ s.$

AY: $f_{AY}(s) = 1\ for\ 0 \le s \le 1$

PY Cutoff: $f_{PYCO}(s) = 2s\ for\ 0 \le s \le 1$

PY : $f_{PY}(s) = s\ for\ 0 \le s \le 1$
$f_{PY}(s) = 2 - s\ for\ 1 \le s \le 2$

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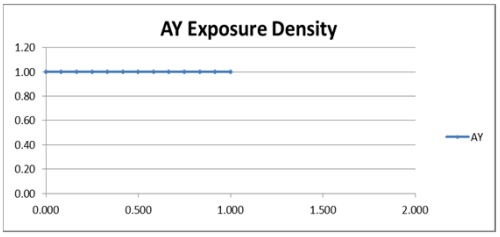
AY Exposure Density Graph



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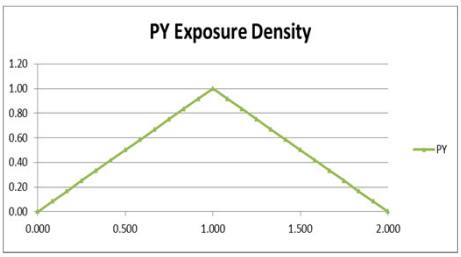
PY Exposure Density Graph



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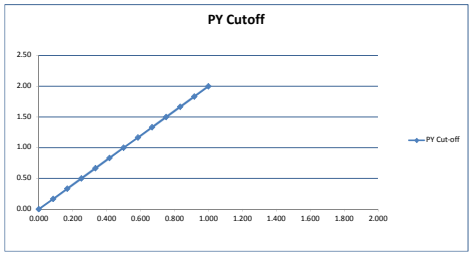
PY Cutoff Exposure Density Graph



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AY Exposure Statistics

Average Date of Loss	$m_A(t) = \begin{Bmatrix} \frac{1}{2}t & for\ t<1 \\ \frac{1}{2} & for\ t\geq 1 \end{Bmatrix}$
Average Maturity of Loss	$r_A(t) = \begin{Bmatrix} \frac{1}{2}t & for\ t<1 \\ t-\frac{1}{2} & for\ t\geq 1 \end{Bmatrix}$
Variance of Loss Exposure Date	$v_A(t) = \begin{Bmatrix} \frac{1}{12}t^2 & for\ t<1 \\ \frac{1}{12} & for\ t\geq 1 \end{Bmatrix}$

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PY Cut-off Exposure Statistics

Average Date of Loss	$m_A(t) = \begin{Bmatrix} \frac{2}{3}t & for\ t<1 \\ \frac{2}{3} & for\ t\geq 1 \end{Bmatrix}$
Average Maturity of Loss	$r_A(t) = \begin{Bmatrix} \frac{1}{3}t & for\ t<1 \\ t-\frac{2}{3} & for\ t\geq 1 \end{Bmatrix}$

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Robbin-Homer Pct of Ult Convolution Formula

$$PCT_{T|A}(t) = F_{A+T}(t) = \int_0^t ds\ f_A(s) * F_T(t-s)$$

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Decomposition Approximation

$$PCT_{T|A}(t) \approx F_A(t) \cdot \{PCT_T(r_A(t))\}$$
$$Error = \frac{1}{2} \cdot F_A(t) \cdot f'_T(r_A(t)) \cdot v_A\ (t)$$

- Splits total development of losses for a given set of exposures into:
  - Development from start of exposure period up to date of claim occurrence
  - Development on claims that have occurred.

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AY Decomposition Formula - Exponential

$F_T(t) = 1 - \exp(-t/\mu)$

$f_T(t) = (1/\mu) \exp(-t/\mu)$

$f'_T(t) = (1/\mu^2) \exp(-t/\mu)$

$$PCT_{T|A}(t) \approx \begin{bmatrix} t \cdot \left\{1 - \exp\left(-\frac{t}{2\mu}\right)\right\} & for\ t < 1 \\ 1 - \exp\left(-\frac{t}{2\mu}\right) & for\ t > 1 \end{bmatrix}$$

$$error = \frac{-1}{24} \cdot \left(\frac{t}{\mu}\right)^2 \exp\left(-\frac{t}{2\mu}\right)$$

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AY Decomposition Formula Example

AY Development Decomposition		
Exponential	*Mean(months)*	*27.0*

Eval Age	Avg Date of Loss	Avg Maturity of Loss	AY % of Exposure ETD	Development CDF at AMOL	AY Approx PCT of Ultimate	ATU LDF	ATA LDF
0	0.0	0.0	0.0%	-			
3	1.5	1.5	25.0%	5.4%	1.4%	74.0185	3.892
6	3.0	3.0	50.0%	10.5%	5.3%	19.0185	2.190
9	4.5	4.5	75.0%	15.4%	11.5%	8.6852	1.731
12	6.0	6.0	100.0%	19.9%	19.9%	5.0185	1.423
15	6.0	9.0	100.0%	28.3%	28.3%	3.5277	1.266
18	6.0	12.0	100.0%	35.9%	35.9%	2.7869	1.188
21	6.0	15.0	100.0%	42.6%	42.6%	2.3461	1.142
24	6.0	18.0	100.0%	48.7%	48.7%	2.0551	1.111
27	6.0	21.0	100.0%	54.1%	54.1%	1.8499	1.089
30	6.0	24.0	100.0%	58.9%	58.9%	1.6981	1.073
33	6.0	27.0	100.0%	63.2%	63.2%	1.5820	1.061
36	6.0	30.0	100.0%	67.1%	67.1%	1.4907	1.052

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AY PCT of Ult via LEVs

$$PCT_{T|AY}(t) = \begin{Bmatrix} t - LEV(t) & for\ t < 1 \\ 1 - (LEV(t) - LEV(t-1)) & for\ t > 1 \end{Bmatrix}$$

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AY PCT ULT – Exponential Example

Development Distribution		
Exponential	*Mean(months)*	27.00

(1)	(2) LEV Calc	(3) LEV Calc	(4)	(5) ((2)-(3))/12	(6) 100%-(4)	(7) 100%-(4)	(8)
Generating AY LDF via LEV Formula							
Age	LEV(t)	LEV(t-12))	t-LEV(t)	LEV(t)-LEV(t-12)	PCT ULT	ATU LDF	Approx ATU LDF
0	0.000		0.000		0.00%		
3	2.839		0.161		1.34%	74.691	74.019
6	5.380		0.620		5.17%	19.358	19.019
9	7.654		1.346		11.22%	8.913	8.685
12	9.688	0.000	2.312	0.807	19.27%	5.191	5.019
15	11.509	2.839	3.491	0.722	27.76%	3.603	3.528
18	13.138	5.380	4.862	0.646	35.35%	2.829	2.787
21	14.596	7.654	6.404	0.578	42.15%	2.372	2.346
24	15.900	9.688	8.100	0.518	48.23%	2.073	2.055

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Losses Occurring Treaty Program with Extended and Stub Periods
Risks Attaching Treaty Program with Cutoff

TREATY DEVELOPMENT EXAMPLES

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Losses Occurring Treaty Example

INS Co X LOB A Treaty Experience												
Eval date			12/31/2017									
Treaty	Basis	Eff date	Expiry Date	Duration (months)	Ult Treaty Prem	Treaty Ceded Loss TD	Treaty LR TD vs Ult Prem	ATU LDF	LDF Method Treaty LR TD vs Ult Prem	A Priori ELR	BF Method Treaty LR TD vs Ult Prem
6	LO	1/1/2017	12/31/2017	12.0	125.0	14.0	11.2%	5.019	56.2%	65.0%	63.2%
5	LO	1/1/2016	12/31/2016	12.0	100.0	25.0	25.0%	2.055	51.4%	62.0%	56.8%
4	LO	7/1/2015	12/31/2015	6.0	60.0	30.0	50.0%	1.582	79.1%	61.0%	72.4%
3	LO	4/1/2013	6/30/2014	15.0	140.0	70.0	50.0%	1.195	59.7%	64.0%	60.4%
2	LO	4/1/2011	3/31/2012	12.0	90.0	63.0	70.0%	1.066	74.6%	67.0%	74.2%
1	LO	4/1/2010	3/31/2011	12.0	95.0	62.0	65.3%	1.042	68.0%	63.0%	67.8%
								Total	63.0%	63.9%	64.8%

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Losses Occurring Treaty LDF

INS Co X LOB A Treaty - LDF by Treaty

Eval date 12/31/2017 *Exponential* *Mean(months)* *27.0*

Treaty	Basis	Eval Age (months)	Avg Date of Loss	Avg Loss Occ Date Lag	Average Maturity of Loss	Development CDF at AMOL	Treaty % Exposure ETD	Treaty PCT ULT Approx	Treaty ATU LDF Approx
6	LO	12.0	7/1/2017	6.00	6.00	19.9%	100%	19.9%	5.019
5	LO	24.0	7/1/2016	6.00	18.00	48.7%	100%	48.7%	2.055
4	LO	30.0	10/1/2015	3.00	27.00	63.2%	100%	63.2%	1.582
3	LO	57.0	12/1/2013	7.50	49.00	83.7%	100%	83.7%	1.195
2	LO	81.0	10/1/2011	6.00	75.00	93.8%	100%	93.8%	1.066
1	LO	93.0	10/1/2010	6.00	87.00	96.0%	100%	96.0%	1.042

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Risks Attaching Treaty Example

INS Co X LOB A Treaty Experience

Eval date 12/31/2017

Treaty	Basis	Eff date	Expiry Date	Exposure Cutoff Date	Duration (months)	Eval Age (months)	Ult Treaty Prem	Treaty Ceded Loss TD	Treaty LR TD vs Ult Prem	ATU LDF	LDF Method Treaty LR TD vs Ult Prem
6	RA	7/1/2016	6/30/2017	6/30/2018	12.0	18.0	140.0	15.0	10.7%	4.917	52.7%
5	RA	7/1/2015	6/30/2016	6/30/2017	12.0	30.0	100.0	33.0	33.0%	2.055	67.8%
4	RA/Cutoff	7/1/2014	6/30/2015	6/30/2015	12.0	42.0	60.0	26.0	43.3%	1.396	60.5%
3	RA	7/1/2013	6/30/2014	6/30/2015	12.0	54.0	110.0	50.0	45.5%	1.268	57.6%
2	RA	7/1/2012	6/30/2013	6/30/2014	12.0	66.0	100.0	72.0	72.0%	1.157	83.3%
1	RA	7/1/2011	6/30/2012	6/30/2013	12.0	78.0	95.0	65.0	68.4%	1.095	74.9%
										Total	65.4%

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Risks Attaching Treaty LDF

INS Co X LOB A Treaty - LDF by Treaty

Eval date 12/31/2017 *Exponential* *Mean(months)* *27.0*

Treaty	Basis	Eval Age (months)	Avg Date of Loss	Avg Loss Occ Date Lag	Average Maturity of Loss	Development CDF at AMOL	Treaty % Exposure Etd	Treaty PCT ULT Approx	Treaty ATU LDF Approx
6	RA	18.0	6/1/2017	10.86	7.14	23.2%	87.5%	20.3%	4.917
5	RA	30.0	1/1/2016	12.00	18.00	48.7%	100.0%	48.7%	2.055
4	RA/Cutoff	42.0	1/1/2015	8.00	34.00	71.6%	100.0%	71.6%	1.396
3	RA	54.0	1/1/2014	12.00	42.00	78.9%	100.0%	78.9%	1.268
2	RA	66.0	1/1/2013	12.00	54.00	86.5%	100.0%	86.5%	1.157
1	RA	78.0	1/1/2012	12.00	66.00	91.3%	100.0%	91.3%	1.095

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Conclusions and Questions

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