Credibility in Loss Development:
Nonproportional Treaty Pricing Application

CAS Seminar on Reinsurance; June 4-5 2018

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Munich RE

Agenda

1) Review of Reinsurance Submission for Skipper Insurance Company
2) Credibility Theory for Loss Development
3) Final Pricing
   a) Experience Rating
   b) Credibility Blending Experience and Exposure Rates
   c) Aggregate Distribution Creation
   d) Calculating Final Price

Submission from Skipper Insurance Company

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Preliminaries: Check for Stability and Policy Limit Drift

Year	Onlevel Premium	Policy Limit Profile 300,000	1,000,000	5,000,000	Allocation of Premium to Layer 400 x 100	500 x 500
2008	na	10.0%	85.0%	5.0%		
2009	18,432,700	9.5%	85.0%	5.5%	26.2%	11.6%
2010	17,258,900	9.0%	85.0%	6.0%	26.2%	11.6%
2011	17,916,600	8.0%	85.0%	7.0%	26.2%	11.7%
2012	18,544,100	7.5%	85.0%	7.5%	26.2%	11.8%
2013	18,470,700	7.0%	85.0%	8.0%	26.2%	11.8%
2014	19,199,500	6.5%	85.0%	8.5%	26.1%	11.9%
2015	19,157,800	5.5%	85.0%	9.5%	26.1%	12.0%
2016	19,374,100	5.0%	85.0%	10.0%	26.1%	12.0%
Future	20,000,000	5.0%	85.0%	10.0%	26.1%	12.0%

All numbers for illustration only

Mata & Verheyen “An Improved Method for Experience Rating Reinsurance Treaties using Exposure Rating Techniques” (2005)
http://www.casact.org/pubs/forum/05spforum/05spf171.pdf

Submission from Skipper Insurance Company

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Reported (paid+case) Development Triangles

400K x 100K

Incurred $ Indemnity+Alae (Prorata) Triangle

	12	24	36	48	60	72	84	96
AY 2009	14,700	462,500	1,082,700	1,675,200	2,156,100	2,458,500	3,347,000	4,296,200
AY 2010	196,900	1,033,300	1,758,900	2,517,000	3,455,800	3,891,300	4,423,300	
AY 2011	275,800	946,400	1,738,400	1,956,200	2,077,100	2,383,000		
AY 2012	215,700	527,800	1,192,300	2,126,000	2,009,200			
AY 2013	332,100	1,447,500	2,562,800	3,170,400				
AY 2014	284,800	1,141,400	1,758,600					
AY 2015	132,800	262,100						
AY 2016	20,100							

Number of Losses: 89

Age-to-Age (ATA) Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96
AY 2009	31.463	2.341	1.547	1.287	1.140	1.361	1.284
AY 2010	5.248	1.702	1.431	1.373	1.126	1.137	
AY 2011	3.431	1.837	1.125	1.062	1.147		
AY 2012	2.447	2.259	1.783	0.945			
AY 2013	4.359	1.771	1.237				
AY 2014	4.008	1.541					
AY 2015	1.974						
Avg	4.007	1.816	1.373	1.172	1.136	1.224	1.284

500K x 500K

Incurred $ Indemnity+Alae (Prorata) Triangle

	12	24	36	48	60	72	84	96
AY 2009	-	322,700	537,600	431,700	450,900	468,000	468,000	468,000
AY 2010	-	27,200	27,200	-	185,700	371,400	371,400	
AY 2011	183,300	422,700	419,500	603,500	604,200	361,700		
AY 2012	-	-	315,300	605,100	531,900			
AY 2013	-	60,600	463,600	678,500				
AY 2014	-	65,500	482,900					
AY 2015	-	-						
AY 2016	-							

Number of Losses: 10.5

Age-to-Age (ATA) Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96
AY 2009	inf	1.666	0.803	1.044	1.038	1.000	1.000
AY 2010	inf	1.000	0.000	inf	2.000	1.000	
AY 2011	2.306	0.992	1.439	1.001	0.599		
AY 2012	inf	inf	1.919	0.879			
AY 2013	inf	7.650	1.464				
AY 2014	inf	7.373					
AY 2015	inf						
Avg	4.903	2.499	1.315	1.081	0.968	1.000	1.000

Credibility Theory: Creating a Prior Distribution

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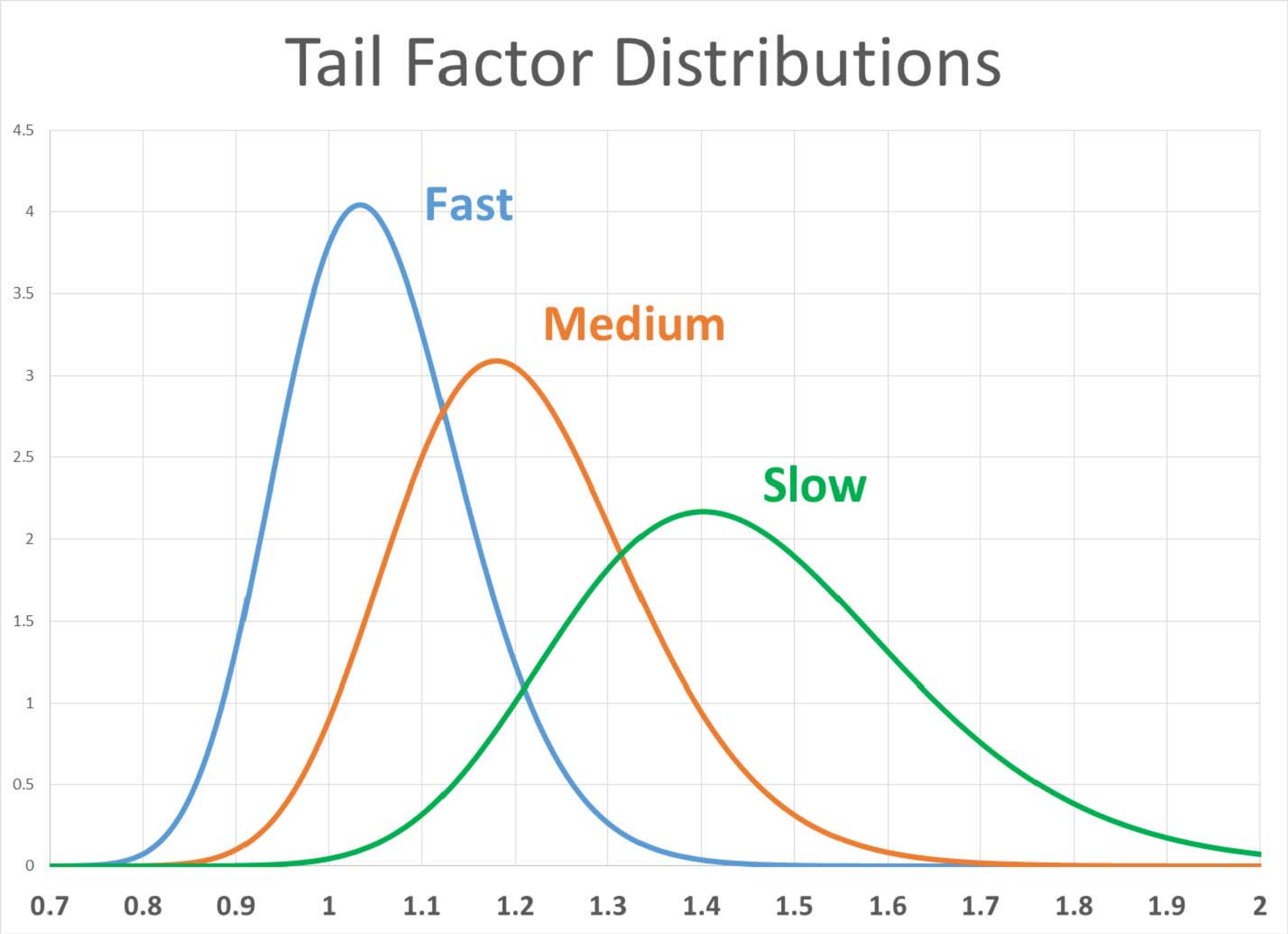
The benchmark patterns should vary for all relevant risk characteristics:

- Line of Business and Subline
- Coverage Trigger (claims-made vs. occurrence)
- Class of Business and Hazard (e.g., are pharmaceutical risks included?)
- Policy and Attachment Point distributions – especially if excess business is included
    - Excess over company’s own underlying (“supported”) or others’ (“unsupported”)
- Claims Handling by insurance company or TPA
- Use of “signal” reserves

Credibility Theory: Creating a Prior Distribution



We can estimate the distribution of tail factors (LDF at age 96 months) as a finite mixture of three categories.

But this still means the tail factor is somewhere between 1.050 and 1.500.

Ideally, these Fast/Medium/Slow categories would correspond to specific risk characteristics.

All numbers for illustration only

Credibility Theory: Creating a Prior Distribution

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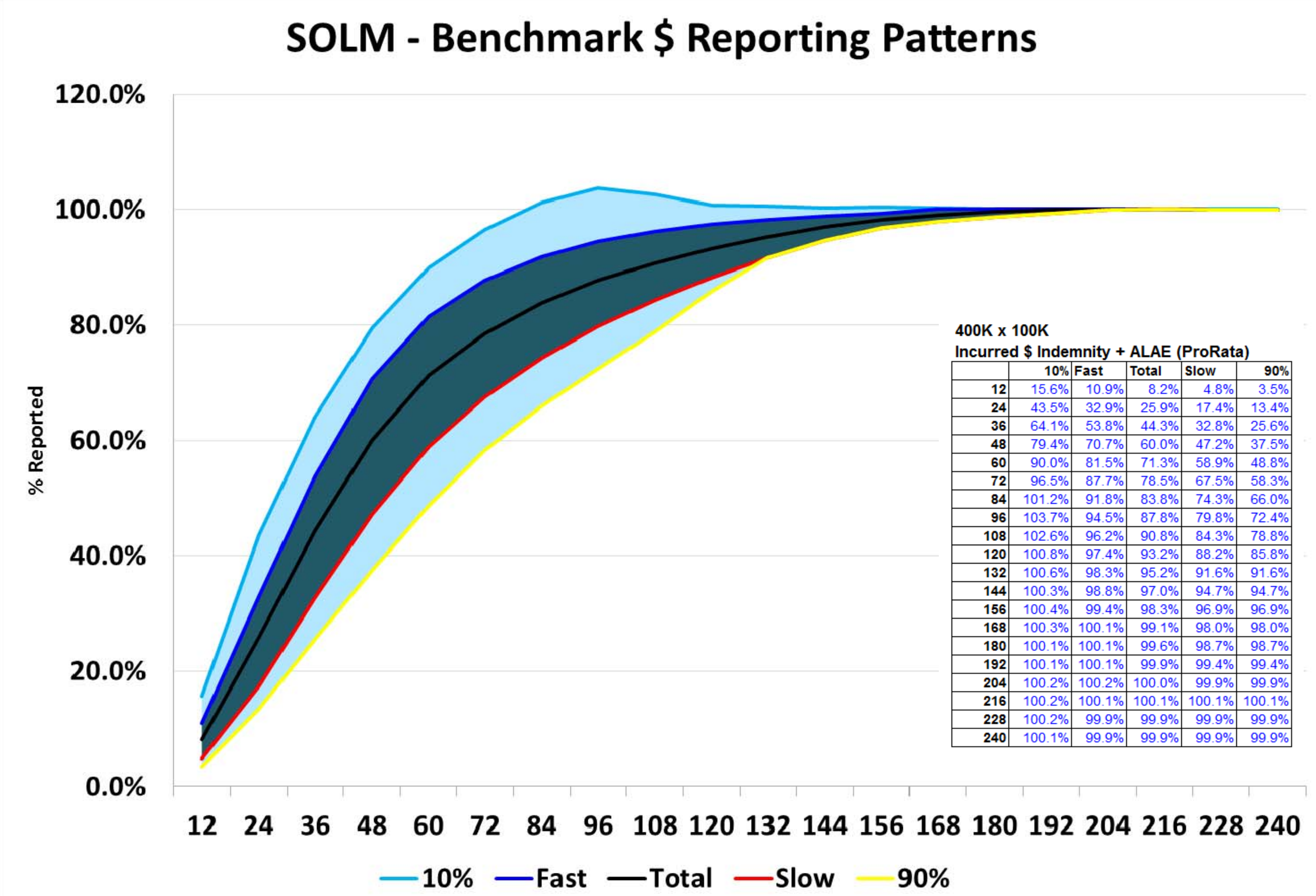

400K x 100K
Incurred $ Indemnity + ALAE (ProRata)

	10%	Fast	Total	Slow	90%
12	15.6%	10.9%	8.2%	4.8%	3.5%
24	43.5%	32.9%	25.9%	17.4%	13.4%
36	64.1%	53.8%	44.3%	32.8%	25.6%
48	79.4%	70.7%	60.0%	47.2%	37.5%
60	90.0%	81.5%	71.3%	58.9%	48.8%
72	96.5%	87.7%	78.5%	67.5%	58.3%
84	101.2%	91.8%	83.8%	74.3%	66.0%
96	103.7%	94.5%	87.8%	79.8%	72.4%
108	102.6%	96.2%	90.8%	84.3%	78.8%
120	100.8%	97.4%	93.2%	88.2%	85.8%
132	100.6%	98.3%	95.2%	91.6%	91.6%
144	100.3%	98.8%	97.0%	94.7%	94.7%
156	100.4%	99.4%	98.3%	96.9%	96.9%
168	100.3%	100.1%	99.1%	98.0%	98.0%
180	100.1%	100.1%	99.6%	98.7%	98.7%
192	100.1%	100.1%	99.9%	99.4%	99.4%
204	100.2%	100.2%	100.0%	99.9%	99.9%
216	100.2%	100.1%	100.1%	100.1%	100.1%
228	100.2%	99.9%	99.9%	99.9%	99.9%
240	100.1%	99.9%	99.9%	99.9%	99.9%

In addition to the “client” data for Skipper Insurance Company, we have “industry” data showing the range of patterns collected by ISO.

For example:
10% = the average of the quickest 10% of companies in the SOLM database.

The “variance of hypothetical means” would be narrower than this range if we could control for the variance from individual companies.

All numbers for illustration only

Credibility Theory: Creating a Prior Distribution

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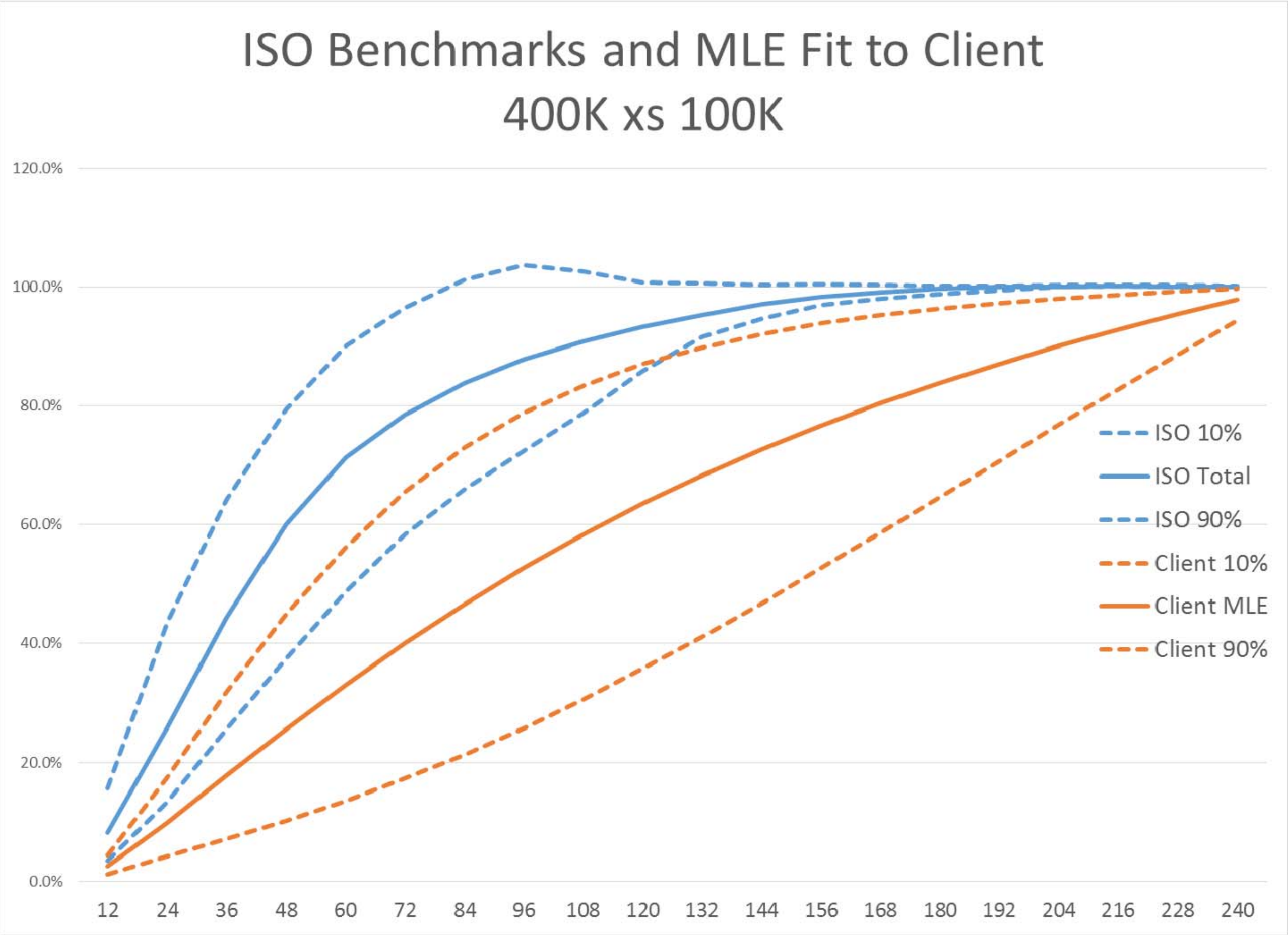

The “penguin” from industry data (shown in **blue**) represents approximately a 90% confidence interval.

From a curve-fit to the client triangle, we can calculate comparable numbers (shown in **orange**).

For the 400K xs 100K layer, the client is much slower than the range of industry patterns.

Credibility Theory: Creating a Prior Distribution

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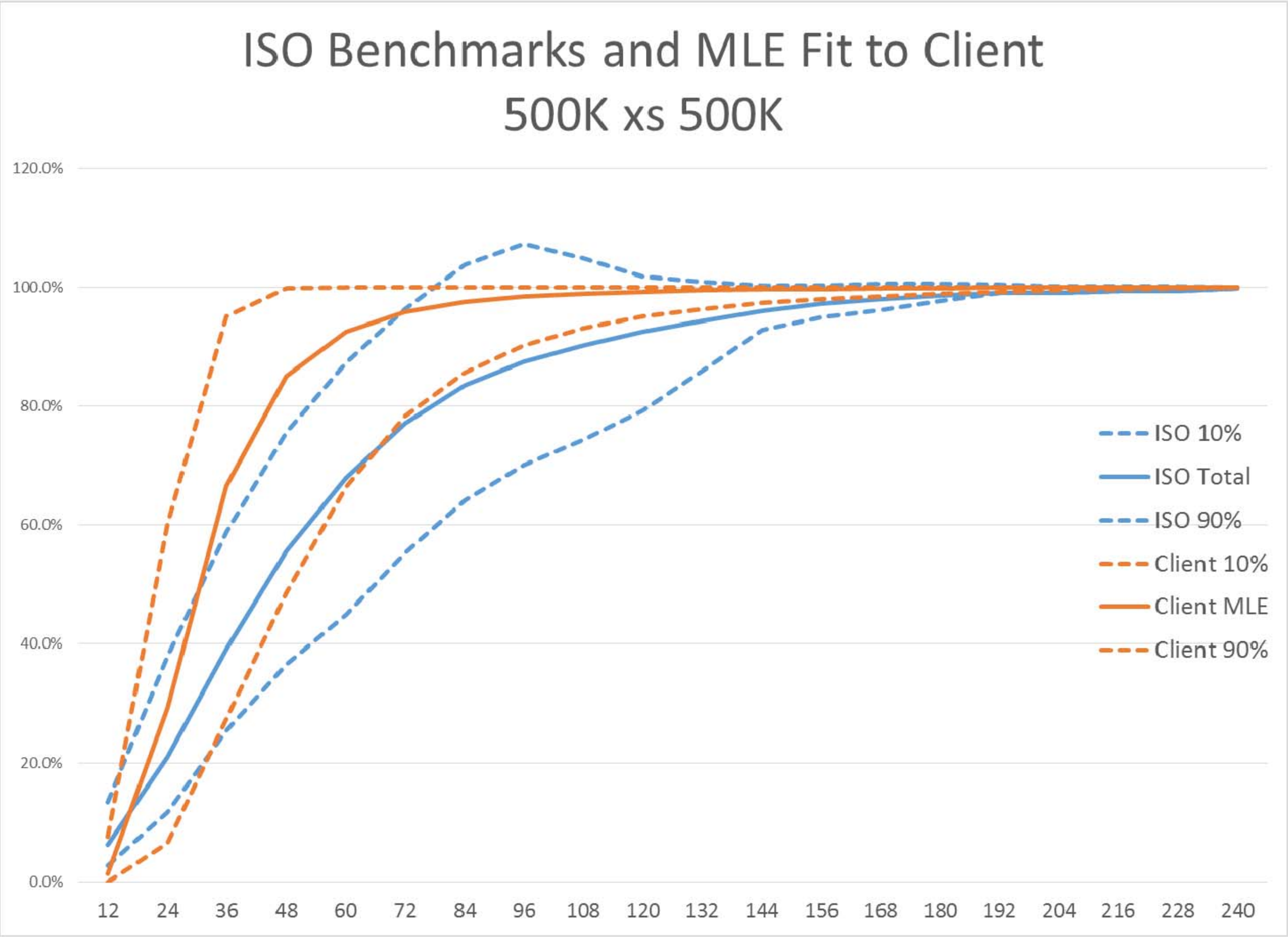

The "penguin" from industry data (shown in **blue**) represents approximately a 90% confidence interval.

From a curve-fit to the client triangle, we can calculate comparable numbers (shown in **orange**).

For the 500K xs 500K layer, the client is much faster than the range of industry patterns.

Credibility Theory: Bayesian Philosophy

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Bayes' Theorem:

$$\pi(\theta|X) = \frac{f(X|\theta) \cdot \pi(\theta)}{\int f(X|\theta) \cdot \pi(\theta)\, d\theta}$$

This formula has three components:

$\pi(\theta)$ A distribution representing "prior" knowledge of the parameters θ

$f(X|\theta)$ A likelihood function representing the probability of observing the actual data X given a certain parameter set.

$\pi(\theta|X)$ The "posterior" probability of the parameters, revised based on the data

Credibility Theory: Conjugate Priors

When the prior distribution $\pi(\theta)$ and likelihood $f(X|\theta)$ are chosen such that the posterior distribution $\pi(\theta|X)$ has the same distribution form as the prior, then we have a *conjugate* relationship.

Common examples from the Exponential Family are:

$\pi(\theta)$ => $f(X|\theta)$

Gamma => Poisson

Beta => Binomial

For the loss development pattern problem, we need a multivariate conjugate relationship.

Dirichlet => Multinomial

Shi/Hartman "Credibility in Loss Reserving" (2014) https://www.casact.org/pubs/forum/14sumforumv2/Shi_Hartman.pdf

Clark "Introduction to Bayesian Loss Development" (2016) http://www.casact.org/pubs/forum/16sforum/Clark.pdf

Credibility Theory: Conjugate Priors

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"Conjugate priors… have the desirable feature that prior information can be viewed as 'fictitious sample information' in that it is combined with the sample in exactly the same way that additional sample information would be combined.

"The only difference is that the prior information is 'observed' in the mind of the researcher, not in the real world."

- Bayesian Econometric Methods; Koop, Poirier & Tobias

For actuaries, our "prior" knowledge comes from:

- Understanding the loss-generating process
- Having reviewed many, many triangles in the past (this should take the form of "as if" observed data)

Credibility Theory: Application

For our example, the creation of the credibility model follows these steps:

- Select Expected Fast/Medium/Slow patterns
- Create distribution around these patterns to mimic the range of the "penguins"
- Estimate the "process" variance/mean ratio for the excess layers
    - Dispersion parameter in ODP model
    - Variance/Mean from exposure rating $\approx E[X^2] / E[X]$

Credibility Theory: Application

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The benchmark factors are selected for three representative levels.

Because we know nothing about the risk characteristics for this client, our a priori weights for each benchmark curve are equal at 33.33%.

Loss Development Factors (LDF to Ultimate)

	12	24	36	48	60	72	84	96	108	120
Fast	7.547	2.618	1.696	1.332	1.166	1.086	1.043	1.028	1.019	1.013
Medium	12.195	3.861	2.257	1.667	1.403	1.274	1.193	1.139	1.101	1.073
Slow	24.096	6.494	3.425	2.361	1.857	1.590	1.426	1.314	1.226	1.149
Average	11.719	3.774	2.265	1.691	1.422	1.285	1.201	1.149	1.109	1.076

	A Priori Weights
Fast	33.33%
Medium	33.33%
Slow	33.33%

All numbers for illustration only

Credibility Theory: Application

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400 xs 100 Reported Loss

	12	24	36	48	60	72	84	96
2009	14,700	462,500	1,082,700	1,675,200	2,156,100	2,458,500	3,347,000	4,296,200
2010	196,900	1,033,300	1,758,900	2,517,000	3,455,800	3,891,300	4,423,300	
2011	275,800	946,400	1,738,400	1,956,200	2,077,100	2,383,000		
2012	215,700	527,800	1,192,300	2,126,000	2,009,200			
2013	332,100	1,447,500	2,562,800	3,170,400				
2014	284,800	1,141,400	1,758,600					
2015	132,800	262,100						
2016	20,100							

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Client								
Column 1	1,452,800	5,558,900	8,335,100	8,274,400	7,689,000	6,349,800	3,347,000	
Column 2	5,821,000	10,093,700	11,444,800	9,698,200	8,732,800	7,770,300	4,296,200	
ATA	4.007	1.816	1.373	1.172	1.136	1.224	1.284	
Benchmark (Medium)								
Column 1	3,166,023	5,846,501	7,383,333	8,415,147	9,082,803	9,367,542	9,544,419	8,780,000
Column 2	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000
ATA	3.159	1.710	1.354	1.188	1.101	1.068	1.048	1.139
Credibility-Weighted								
Column 1	4,618,823	11,405,401	15,718,433	16,689,547	16,771,803	15,717,342	12,891,419	8,780,000
Column 2	15,821,000	20,093,700	21,444,800	19,698,200	18,732,800	17,770,300	14,296,200	10,000,000
ATA	3.425	1.762	1.364	1.180	1.117	1.131	1.109	1.139
LDF	15.499	4.525	2.568	1.883	1.595	1.428	1.263	1.139

The credibility blending becomes a simple dollar-weighted average.

The “Column 1” and “Column 2” amounts shown are the basis for the all-year weighted average factors.

The Benchmark pattern is averaged with the triangle as though it also had dollar weights.

Credibility Theory: Application

400 xs 100 Reported Loss

	12	24	36	48	60	72	84	96
2009	14,700	462,500	1,082,700	1,675,200	2,156,100	2,458,500	3,347,000	4,296,200
2010	196,900	1,033,300	1,758,900	2,517,000	3,455,800	3,891,300	4,423,300	
2011	275,800	946,400	1,738,400	1,956,200	2,077,100	2,383,000		
2012	215,700	527,800	1,192,300	2,126,000	2,009,200			
2013	332,100	1,447,500	2,562,800	3,170,400				
2014	284,800	1,141,400	1,758,600					
2015	132,800	262,100						
2016	20,100							

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Client								
Column 1	1,452,800	5,558,900	8,335,100	8,274,400	7,689,000	6,349,800	3,347,000	
Column 2	5,821,000	10,093,700	11,444,800	9,698,200	8,732,800	7,770,300	4,296,200	
ATA	4.007	1.816	1.373	1.172	1.136	1.224	1.284	
Benchmark (Slow)								
Column 1	2,694,805	5,273,973	6,894,923	7,864,438	8,561,208	8,966,500	9,218,134	7,610,000
Column 2	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000
ATA	3.711	1.896	1.450	1.272	1.168	1.115	1.085	1.314
Credibility-Weighted								
Column 1	4,147,605	10,832,873	15,230,023	16,138,838	16,250,208	15,316,300	12,565,134	7,610,000
Column 2	15,821,000	20,093,700	21,444,800	19,698,200	18,732,800	17,770,300	14,296,200	10,000,000
ATA	3.814	1.855	1.408	1.221	1.153	1.160	1.138	1.314
LDF	24.315	6.374	3.437	2.441	2.000	1.735	1.495	1.314

This procedure is repeated for each of the benchmark patterns.

The “slow” benchmark is closest to the empirical data so we will increase the probability of the client being in the “slow” development category.

Credibility Theory: Application

Our a priori weights for the three benchmark curves were equal at 33.33%.

These weights are revised to reflect the fact that the client data most likely came from the "slow" curve.
This revision of weights is a direct application of Bayes' Theorem.
The revised weights can be the "prior" when we move to the 500x500 layer.

Bayesian Updating of Probabilities

	LogLikelihood	Difference in LL	Relative Likelihood	Original Weights	Revised Weights
	A	B=Max(A)-A	C=exp(B)	D	E=C*D/Avg(C)
Fast	-22.7256	-6.20	0.002	33.33%	0.18%
Baseline	-18.5356	-2.01	0.134	33.33%	11.82%
Slow	-16.5285	0.00	1.000	33.33%	88.00%
			0.379	100.00%	100.00%

All numbers for illustration only

Credibility Theory: Application

The final credibility-weighted pattern for the 400x100 layer is an average of the individual benchmark patterns weighted with the client data.

Loss Development Factors (LDF to Ultimate)

	12	24	36	48	60	72	84	96	108	120
Fast	11.274	3.507	2.101	1.591	1.366	1.240	1.113	1.028	1.019	1.013
Medium	15.499	4.525	2.568	1.883	1.595	1.428	1.263	1.139	1.101	1.073
Slow	24.315	6.374	3.437	2.441	2.000	1.735	1.495	1.314	1.226	1.149
Average	22.739	6.072	3.301	2.356	1.940	1.691	1.462	1.290	1.210	1.140

	A Posteriori Weights
Fast	0.18%
Medium	11.82%
Slow	88.00%

Credibility Theory: Application

The same procedure is followed for the 500x500 layer.

Instead of the initial 33.33% weights for each benchmark, however, we can start with the result from the 400x100 layer. Because of the low credibility for the 500x500 layer, the final pattern is close to the “slow” benchmark.

Loss Development Factors (LDF to Ultimate)

	12	24	36	48	60	72	84	96	108	120
Fast	9.909	3.242	1.866	1.399	1.203	1.084	1.038	1.025	1.020	1.015
Medium	16.705	4.811	2.474	1.760	1.462	1.286	1.195	1.143	1.109	1.081
Slow	33.051	7.635	3.480	2.416	1.965	1.638	1.454	1.343	1.267	1.201
Average	29.273	7.087	3.303	2.303	1.880	1.582	1.414	1.313	1.244	1.184

	A Posteriori Weights
Fast	0.16%
Medium	12.81%
Slow	87.03%

Final Pricing: Experience Rating 400x100 Layer

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The experience rating for the 400x100 layer makes use of the credibility-weighted LDFs.

Experience Rating 400K xs 100K

Accident Year	Onlevel Premium	Exposure Trend	Trended Premium	LDF	Premium / LDF	400x100 Reported	Severity Trend	Frequency Trend	Policy Limit Drift	400x100 Trended	Rate
2009	18,432,700	1.083	19,959,973	1.290	15,473,231	4,296,200	1.267	1.000	0.995	5,413,578	34.99%
2010	17,258,900	1.072	18,503,877	1.462	12,652,832	4,423,300	1.230	1.000	0.995	5,414,567	42.79%
2011	17,916,600	1.062	19,018,832	1.691	11,250,307	2,383,000	1.194	1.000	0.996	2,835,398	25.20%
2012	18,544,100	1.051	19,490,035	1.940	10,047,197	2,009,200	1.159	1.000	0.997	2,322,369	23.11%
2013	18,470,700	1.041	19,220,684	2.356	8,158,792	3,170,400	1.126	1.000	0.998	3,559,919	43.63%
2014	19,199,500	1.030	19,781,264	3.301	5,992,680	1,758,600	1.093	1.000	0.998	1,918,277	32.01%
2015	19,157,800	1.020	19,542,872	6.072	3,218,461	262,100	1.061	1.000	0.999	277,898	8.63%
2016	19,374,100	1.010	19,567,841	22.739	860,542	20,100	1.030	1.000	1.000	20,703	2.41%
	148,354,400		155,085,378		67,654,043	18,322,900				21,762,710	32.17%
Prospective	20,000,000									6,433,528	32.17%

All numbers for illustration only

Final Pricing: Experience Rating 500x500 Layer

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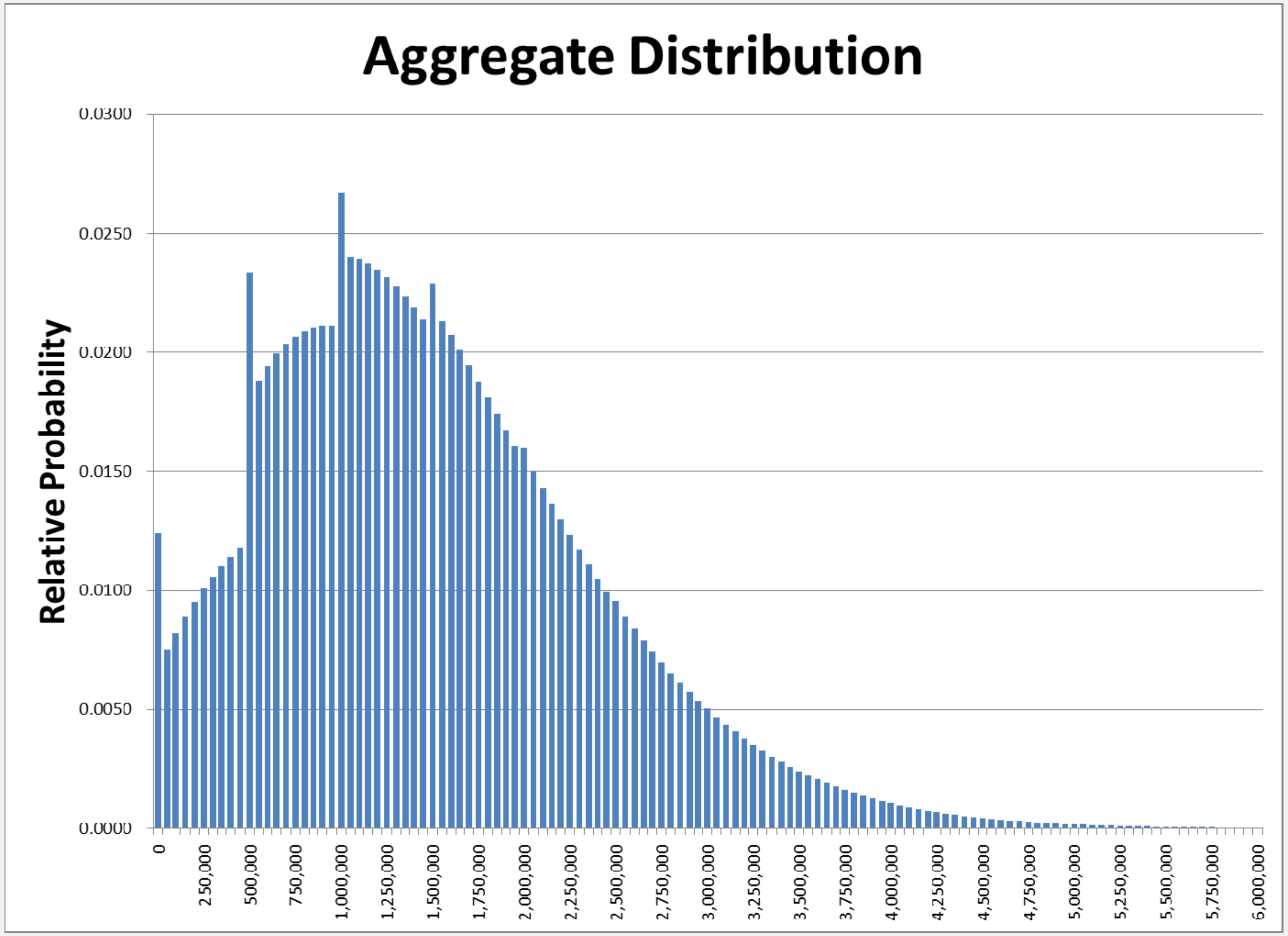
Experience Rating 500K xs 500K

Accident Year	Onlevel Premium	Exposure Trend	Trended Premium	LDF	Premium / LDF	500x500 Reported	Severity Trend	Frequency Trend	Policy Limit Drift	500x500 Trended	Rate
2009	18,432,700	1.083	19,959,973	1.313	15,201,243	468,000	1.267	1.000	1.037	615,038	4.05%
2010	17,258,900	1.072	18,503,877	1.414	13,086,268	371,400	1.230	1.000	1.033	471,909	3.61%
2011	17,916,600	1.062	19,018,832	1.582	12,025,363	361,700	1.194	1.000	1.025	442,533	3.68%
2012	18,544,100	1.051	19,490,035	1.880	10,365,628	531,900	1.159	1.000	1.020	629,230	6.07%
2013	18,470,700	1.041	19,220,684	2.303	8,345,310	678,500	1.126	1.000	1.016	776,103	9.30%
2014	19,199,500	1.030	19,781,264	3.303	5,988,474	482,900	1.093	1.000	1.012	534,101	8.92%
2015	19,157,800	1.020	19,542,872	7.087	2,757,550	0	1.061	1.000	1.004	0	0.00%
2016	19,374,100	1.010	19,567,841	29.273	668,468	0	1.030	1.000	1.000	0	0.00%
	148,354,400		155,085,378		68,438,304	2,894,400				3,468,914	5.07%
Prospective	20,000,000									1,013,735	5.07%

400xs100 Rate:	32.17%
Exposure-Rating Relativity:	0.461
Expected 500xs500 Rate:	14.83%
Credibility:	75%
Selected 500xs500 Rate:	7.51%
Selected 500xs500 Expected Loss:	**1,501,765**

All numbers for illustration only

Final Pricing: Aggregate Distribution



Aggregate distribution model can be created using any one of our available tools (simulation, FFT, Panjer, etc.).

This distribution allows for the evaluation of loss sensitive treaty features:

- Annual Aggregate Deductible
- No Claim Bonus

Final Pricing

The selection of expected losses to the 500K xs 500K layer is the starting point for our pricing.

- Additional Costs for ECO/XPL?
- Aggregate Distribution for loss-sensitive features
- Investment Income and NPV calculation
- Profit and Expense loads

Thank You

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