



INTERMEDIATE PRICING
Loss-Sensitive Features in Reinsurance Contracts
Seminar on Reinsurance

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Loss-Sensitive Features

Agenda

- Basic Definition of Loss-Sensitive Features
- References
- Reasons to use Loss-Sensitive Features
- Types of Loss-Sensitive Features
- What is needed (data, models)
- Examples

Loss-Sensitive Features Definitions

- Working definition of loss-sensitive feature

“Any feature of a reinsurance contract which causes the ultimate (net) ceded premium to depend upon the losses ceded to the contract”

- “Net Premium” to mean net of ceding or other commissions
- “Adjustable Premium” could simply mean that ceded premium adjusts based on a rate to SPI, as SPI develops from estimated to ultimate

Loss-Sensitive Features Definitions

- We can broaden this a little to loss-*sharing* feature

“Any feature of a reinsurance contract which causes some of the losses that would be ceded to the reinsurer to instead be retained by the cedant”

- Conveniently we can now say LSF to mean either or both!

Loss-Sensitive Features

References

- *Basics of Reinsurance Pricing*, by David Clark (still on the exam syllabus!)
- A few others of note, notably (1)

[1] Bear, Robert A. and Kenneth J. Nemlick Pricing the Impact of Adjustable Features and Loss Sharing Provisions of Reinsurance Treaties 1990 PCAS Vol. LXXVII

[2] Feldblum, Sholom Risk Loads For Insurers 1990 PCAS Vol. LXXVII

[3] Heckman, Philip E. and Glenn G. Meyers The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions 1983 PCAS Vol. LXX

[4] Ludwig, Stephen J. An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance 1991 PCAS Vol. LXXVIII

[5] Mashitz, Isaac and Gary Patrik Credibility for Treaty Reinsurance Excess Pricing 1990 CAS Discussion Paper Program

[6] Panjer, Harry H. and Gordon E. Wilmot Insurance Risk Models Published by the Society of Actuaries 1992

[7] Robertson, John P. The Computation of Aggregate Loss Distributions 1992 PCAS Vol. LXXIX

[8] Venter, Gary Transformed Beta and Gamma Distributions and Aggregate Losses 1983 PCAS Vol. LXX

Loss-Sensitive/Sharing Features Examples

- Sliding Scale Ceding Commissions
- Profit Commissions
- Paid Reinstatements
- Other types of Additional Premiums
- Swing Rates
- Annual Aggregate Deductibles (AADs)
- Corridors, caps. etc.
- Structured Risk (out of scope for today)

Loss-Sensitive Features

Why are these used in reinsurance?

- Reduce up-front premium
- Avoid “dollar-trading”
- Better sharing of loss experience between parties
- Volatility/uncertainty makes flat-pricing more difficult
- New company/new program with little or no experience to use in pricing

Loss-Sensitive Features Data and Modeling Needs

- Generally speaking, pricing for these features requires more than point estimates (of layer loss, or an expected loss ratio)
- Usually a collective risk model or aggregate model is used that provides the per-claim or aggregate distribution needed
 - Poisson/Pareto (old favorite)
 - Lognormal (quick and easy for aggregate)
- Various methods/models available
- Assume for all our examples we have an easy to use simulation model that can handle the reinsurance terms (there are several vendor models in the market or you can use @Risk or VBA in Excel)

Loss-Sensitive Features

Example 1

- Profit Commission on a Quota Share
 - 60% mean loss ratio with 25% CV (lognormal)
 - 30% underlying expenses
 - Flat 25% ceding commission
 - Profit Commission: 50% of profit after 10% expense allowance (margin)
 - The PC goes back to the cedant, like an additional ceding commission

$$\text{Profit} = \text{Premium} - \text{Loss} - \text{Ceding Commission} - .10 * \text{Premium}$$

$$\text{PC} = \max(0, .5 * \text{Profit})$$

Loss-Sensitive Features

Example 1

- Similar to sliding-scale ceding commission
- More explicit profit-share
- Allows additional commission to recover original expense or obtain override in favorable years

								Quota Share = 50%	
Premium	Expense	Loss	CC	Margin	Profit	PC	PC+CC	Reins Margin*	Cedant Margin
100	30	50	25	10	15	7.5	32.5	8.75	11.25
100	30	55	25	10	10	5.0	30.0	7.50	7.50
100	30	60	25	10	5	2.5	27.5	6.25	3.75
100	30	65	25	10	0	0.0	25.0	5.00	0.00
100	30	70	25	10	-5	0.0	25.0	2.50	-2.50
100	30	75	25	10	-10	0.0	25.0	0.00	-5.00
100	30	80	25	10	-15	0.0	25.0	-2.50	-7.50

**Excluding brokerage/other expense*

Loss-Sensitive Features

Example 1

Mean Statistics

Subject Premium	1,000,000
Ceded Premium	500,000
Ceded Loss	300,000
Loss Ratio	60.0%
Ceding Commission	125,000
Mean Profit Commission	22,374
Total Commission	29.5%
PC at Mean Loss Ratio	12,500

Net Profit Distribution

	Cedant	Reinsurer
Mean	-47,294	-52,547
10.0%	-169,552	-106,517
25.0%	-118,007	-89,336
50.0%	-50,608	-66,869
75.0%	19,145	-30,855
80.0%	33,271	-16,729
90.0%	73,955	23,955
95.0%	111,833	61,833
98.0%	157,666	107,666
99.0%	190,388	140,388
99.5%	224,091	174,091
99.6%	236,607	186,607
99.8%	270,317	220,317

Loss-Sensitive Features

Example 1

Try new terms: 60% PC after 7.5% margin

Mean Statistics		Net Profit Distribution		
			Cedant	Reinsurer
Subject Premium	1,000,000			
Ceded Premium	500,000	Mean	-57,006	-42,835
Ceded Loss	300,080	10.0%	-188,355	-87,714
Loss Ratio	60.0%	25.0%	-133,374	-73,969
		50.0%	-61,482	-55,995
Ceding Commission	125,000	75.0%	19,145	-30,855
Mean Profit Commission	32,085	80.0%	33,271	-16,729
Total Commission	31.4%	90.0%	73,955	23,955
		95.0%	111,833	61,833
PC at Mean Loss Ratio	22,452	98.0%	157,666	107,666
		99.0%	190,388	140,388
		99.5%	224,091	174,091
		99.6%	236,607	186,607
		99.8%	270,317	220,317

Loss-Sensitive Features

Example 2

- Low attaching XOL
- For example, Layer 1 of a Per Risk tower
- 10M xs 10M with 80M aggregate limit (7 Free)
- Flat price is 40M (agreed upon as a base)
- Consider two alternatives
 - Use a 10M annual aggregate deductible (AAD)
 - Switch to paid reinstatements (some or all)

Loss-Sensitive Features

Example 2

- Quick Review
- Paid Reinstatements
 - Pays additional premium when there is a loss according to

$$\text{Reinstatement Premium} = \text{Loss/Limit} * \text{Base Premium} * \text{Reinst. \%}$$

- For example “1 @ 100” pays for up to one more limit as a portion of full premium
 - Reinstatements imply finite # of limits available
- Annual Aggregate Deductible (AAD)
 - Sometimes known as “otherwise recoverable”
 - Typically on a per-occurrence excess of loss
 - The cedant retains the first \$X of layer loss on an aggregate basis
 - Reduces ceded loss and increases volatility
 - There may be an aggregate limit as well

Loss-Sensitive Features

Example 2

10M XS 10M

Mean	30,204
Stdev	17,716
CV	58.7%

5.00%	4,574
10.00%	9,971
20.00%	14,339
25.00%	16,995
40.00%	23,475
50.00%	28,115
75.00%	41,107
80.00%	44,636
90.00%	54,636
95.00%	63,694
98.00%	74,370
99.00%	80,000
99.50%	80,000
99.60%	80,000

- Loss distribution with aggregate limit leads to selected premium of 40M
- 80M aggregate limit “kicks in” at 99% level (1 in 100 year event) so there is plenty of aggregate limit
- Distribution suggests an AAD of 10m
- Usually like to set AAD around 10%ile level to try to get 1-1 premium credit
- Typically will reduce aggregate limit when adding AAD

Loss-Sensitive Features

Example 2

	10M XS 10M	10M XS 10M XS 10M
Mean	30,204	20,756
Stdev	17,716	16,945
CV	58.7%	81.6%
5.00%	4,574	0
10.00%	9,971	0
20.00%	14,339	4,337
25.00%	16,995	6,995
40.00%	23,475	13,474
50.00%	28,115	18,114
75.00%	41,107	31,107
80.00%	44,636	34,637
90.00%	54,636	44,638
95.00%	63,694	53,689
98.00%	74,370	64,370
99.00%	80,000	70,000
99.50%	80,000	70,000
99.60%	80,000	70,000

- Expected loss decreases by almost 10M
- Probability of attaching decreases
- New aggregate limit of 70M hits at same point
- Volatility *increases*
- *What is the new “correct” price?*

Loss-Sensitive Features

Example 2

- We would like the new price to be 30M (1-1 credit) or better
- Does this make sense technically?

	10M XS 10M	10M XS 10M XS 10M
Expected Loss	30,204	20,756
CV	58.7%	81.6%
Premium	40,000	30,000
Ceded ELR	75.5%	69.2%

- Since the volatility increased, it makes sense for the ELR to decrease
- The two prices are technically consistent (within reason)
- From the cedant point of view, the layer with AAD with 1-1 credit is favorable in every scenario (take the deal!)

Loss-Sensitive Features

Example 2

10M XS 10M

Mean	30,204
Stddev	17,716
CV	58.7%
5.00%	4,574
10.00%	9,971
20.00%	14,339
25.00%	16,995
40.00%	23,475
50.00%	28,115
75.00%	41,107
80.00%	44,636
90.00%	54,636
95.00%	63,694
98.00%	74,370
99.00%	80,000
99.50%	80,000
99.60%	80,000

- Now consider paid reinstatements
- The loss distribution is identical
- The 80M aggregate limit = 7 Free
- Now consider 4 free and 3@100
- How can we adjust the 40M premium for this feature?

Reinstatement	Percentage
1	Free
2	Free
3	Free
4	Free
5	100.00 %
6	100.00 %
7	100.00 %

Loss-Sensitive Features

Example 2

- How can we adjust the 40m premium for this feature?
- Approach 1: try to get a similar expected ceded loss ratio, allowing for total premium collected

	10M XS 10M	10M XS 10M Paid Reinst		10M XS 10M	10M XS 10M Paid Reinst
Expected Loss	30,204	30,204	Expected Loss	30,204	30,204
CV	58.7%	58.7%	CV	58.7%	58.7%
Deposit Premium	40,000	28,000	Deposit Premium	40,000	30,000
Expected Premium	40,000	37,628	Expected Premium	40,000	40,315
Loss Ratio on Deposit	75.5%	107.9%	Loss Ratio on Deposit	75.5%	100.7%
Loss Ratio on Total	75.5%	80.3%	Loss Ratio on Total	75.5%	74.9%
Expected Loss Ratio	75.5%	81.4%	Expected Loss Ratio	75.5%	75.9%
Expected Profit	4,996	3,390	Expected Profit	4,996	5,790

- By these metrics, it looks “cheap” (or the 28m should be a bit more to be technically equivalent to the 40m, so try 30m e.g.)
- But is this really true?

Loss-Sensitive Features

Example 2

- Consider a return on allocated capital point of view
- Based on IERP by Rodney Kreps (simplified)

	10M XS 10M	10M XS 10M Paid Reinst
Expected Loss	30,204	30,204
Expected Premium	40,000	40,315
Expected Profit	4,996	5,790
Allocated Capital	44,800	13,301
Return on Capital	11.2%	43.5%

- Allocated capital based on a 99%ile contract loss but is reduced by premium received
- With paid reinstatements, this is a far lower capital need and results in a very high return on capital
- You could try pushing the premium down, even past 28m, say 26m

Loss-Sensitive Features

Example 2

- Conclusions
- Paid Reinstatements “don’t really work” with low attaching XOL combined with high rates on line
 - Too much premium paid too quickly (lack of benefit in mult. losses)
 - Try adding paid reinst. after a few free but this can fail too
 - Try using factors less than 100%
 - You can always find a price that works in some way but in this case it may approach a structured deal (margin + additional premiums) which is not the intent
- In this example (or similar ones) the AAD is likely a better choice (to reduce ceded premium and dollar trading)

Loss-Sensitive Features

Example 3

- Paid Reinstatements tend to work with
 - Higher attaching layers with limit losses further in the tail
 - Standard cat contracts are prototypical examples
 - Cat often attaches at 1 in 10 or greater
- For this example, try one “in-between”
- 25M xs 25M layer with 50M aggregate limit (one reinstatement, free then convert to paid)
- How to adjust the price for the reinstatement?

Loss-Sensitive Features

Example 3

25M XS 25M

Mean	7,474
Stdev	10,617
CV	0
5.00%	0
10.00%	0
20.00%	0
25.00%	0
40.00%	0
50.00%	1,106
75.00%	12,890
80.00%	15,911
90.00%	22,775
95.00%	29,300
98.00%	38,673
99.00%	44,933
99.50%	50,000
99.60%	50,000

- Attaches around 1 in 2 years
- First limit exhaustion ~ 1 in 14 years
- Suppose 10m is a “fair price” with one free reinstatement
- What is an equivalent price for 1 @ 100?
- What does equivalent even mean?

Loss-Sensitive Features

Example 3

- Try an expected premium approach
- Set new premium so that $E(\text{premium}) = 10\text{M}$

	25M XS 25M Free	25M XS 25M Paid
Mean Loss	7,474	7,474
Loss Standard Deviation	10,617	10,617
Loss Volatility	142.0%	142.0%
Loss On Line	29.9%	29.9%
Deposit Premium	10,000	7,900
Rate On Line	40.0%	31.6%
Loss Ratio on Deposit Premium	74.7%	94.6%
Expected Reinstatement Premium	0	2,140
Expected Total Premium	10,000	10,040
Loss Ratio on Total Premium	74.7%	74.4%

Loss-Sensitive Features

Example 3

- But, this will typically overstate the premium need based on a return on allocated capital approach

	25M XS 25M Free	25M XS 25M Paid
Expected Loss	7,474	7,474
Deposit Premium	10,000	7,900
Expected Premium	10,000	10,040
Expected Profit*	1,526	1,562
Allocated Capital**	15,000	9,200
Return on Capital	10.2%	17.0%

	25M XS 25M Free	25M XS 25M Paid
Expected Loss	7,474	7,474
Deposit Premium	10,000	7,450
Expected Premium	10,000	9,468
Expected Profit*	1,526	1,047
Allocated Capital**	15,000	10,100
Return on Capital	10.2%	10.4%

* Using 10% brokerage

** Limit loss less premium at a limit loss

Loss-Sensitive Features

Example 3

- The full profit/loss distribution will not be the same: typically a bit “tighter” for the layer with paid reinstatements and less deposit premium

Distribution of Profit/Loss*		
	25M XS 25M Free	25M XS 25M Paid
10.0%	-10,000	-7,450
25.0%	-10,000	-7,450
50.0%	-8,894	-6,674
75.0%	2,890	1,599
80.0%	5,911	3,719
90.0%	12,775	8,538
95.0%	19,300	14,400
98.0%	28,673	23,773
99.0%	34,933	30,033
99.5%	40,000	35,100
99.6%	40,000	35,100
99.8%	40,000	35,100
99.9%	40,000	35,100

* Loss - Deposit Premium - Reinstatement Premium



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