2010 RPM Basic Ratemaking Workshop

Session 3: Introduction to Increased Limit Factors

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Agenda

- Increased vs. Basic Limits Ratemaking
- Loss Severity Distributions
- Effects of Trend
  - By Limit and Layer
- Components of ILF Calculation
- Mixed Exponential Methodology
- Deductible and Layer Pricing

CAS Exam 5 Reference:

Basic Ratemaking
Chapter 11: Special Classification *

Geoff Werner, FCAS, MAAA
Claudine Modlin, FCAS, MAAA
EMB America LLC

* Candidates studying for Exam 5 should refer to the CAS text, rather than this workshop presentation.

Prior CAS Exam 5 Paper:
Increased Limits Ratemaking For Liability Insurance *

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* This paper provides a description of the Mixed Exponential Methodology used by ISO, and is referenced in Chapter 11 of the Basic Ratemaking text mentioned in the previous slide.

Liability Lines of Business

- Premises/Operations and Products (GL)
- Medical Professional
- Commercial Auto
- Personal Auto
- Farm
- Personal (Individual or within Homeowner Policy)
- Management Protection (D&O)
- E-Commerce
- Lawyers Professional
- Business Owners
- Employment-Related Practices
- Other Professional

Basic Limits Ratemaking

- Use large volume of losses capped at basic limit for detailed, experience-based analysis.
- Able to produce relativities by
  - Class
  - Territory
  - Tiers

Increased Limits Ratemaking

- Need broader experience base
  - low claim volume at higher limits
- Group loss experience for credibility
  - Class Groups
  - State Groups
  - Countrywide

Increased Limit Factor Definition

$$\frac{\text{Expected Costs at the desired policy limit}}{\text{Expected Costs at the Basic Limit}}$$

KEY ASSUMPTION:

Claim Frequency is <u>independent</u> of Claim Severity

This allows for ILFs to be developed by an examination of the relative severities ONLY

$$ILF_k = \frac{\cancel{E(Frequency)} \times E(Severity_k)}{\cancel{E(Frequency)} \times E(Severity_B)}$$

$$= \frac{E(Severity_k)}{E(Severity_B)}$$

Limited Average Severity (LAS)

- Defined as the average size of loss, where all losses are limited to a particular value.
- Thus, the ILF can be defined as the ratio of two limited average severities.
- ILF (k) = LAS (k) ÷ LAS (B)

Example

Losses	@100,000 Limit	@1 Mill Limit
50,000		
75,000		
150,000		
250,000		
1,250,000		

Example (cont'd)

Losses	@100,000 Limit	@1 Mill Limit
50,000	50,000	
75,000	75,000	
150,000	100,000	
250,000	100,000	
1,250,000	100,000	
1,775,000	**425,000**	

Example (cont'd)

Losses	@100,000 Limit	@1 Mill Limit
50,000	50,000	50,000
75,000	75,000	75,000
150,000	100,000	150,000
250,000	100,000	250,000
1,250,000	100,000	1,000,000
1,775,000	**425,000**	**1,525,000**

Example – Calculation of ILF

Total Losses	$1,775,000
Limited to $100,000 (Basic Limit)	$425,000/5 = $85,000
Limited to $1,000,000	$1,525,000/5 = $305,000
Increased Limits Factor (ILF)	$305,000/85,000 = 3.588

Empirical Data - ILFs

Lower	Upper	Losses	Occs.	Average
1	100,000	25,000,000	1,000	25,000
100,001	250,000	75,000,000	500	150,000
250,001	500,000	60,000,000	200	300,000
500,001	1 Million	30,000,000	50	600,000
1 Million	-	15,000,000	10	1,500,000

Empirical Data - ILFs

LAS @ 100,000

$$(25{,}000{,}000 + 760 \times 100{,}000) \div 1760$$
$$= 57{,}386$$

LAS @ 1,000,000

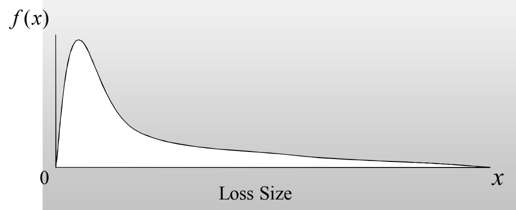
$$(\ 190{,}000{,}000 + 10 \times 1{,}000{,}000\ ) \div 1760$$
$$= 113{,}636$$

Empirical ILF = 1.98

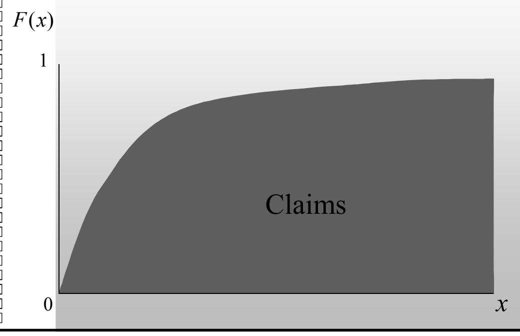
Insurance Loss Distributions

- Loss Severity Distributions are Skewed
- Many Small Losses/Fewer Larger Losses
- Yet Larger Losses, though fewer in number, are a significant amount of total dollars of loss.

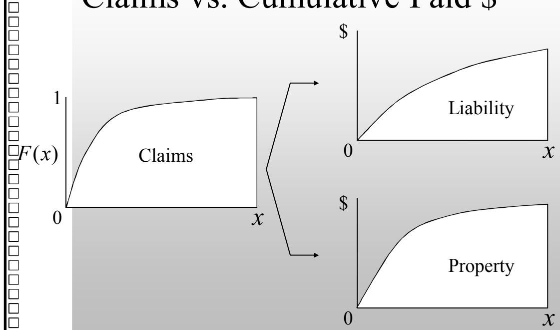
Loss Distribution - PDF



Claim Distribution - CDF



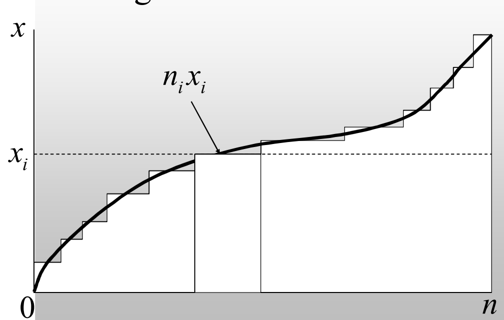
Claims vs. Cumulative Paid $



A Graphical Approach

A novel approach to understanding Increased Limits Factors was presented by Yoong S. Lee in the CAS Exam 9 paper – "The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach"

Lee Figure



Limited Average Severity

$$\int_0^k x dF(x) + k[1 - F(k)]$$

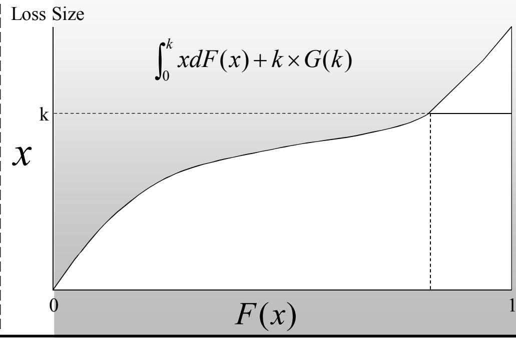
Size method; 'vertical'

$$\int_0^k [1 - F(x)] dx$$

Layer method; 'horizontal'

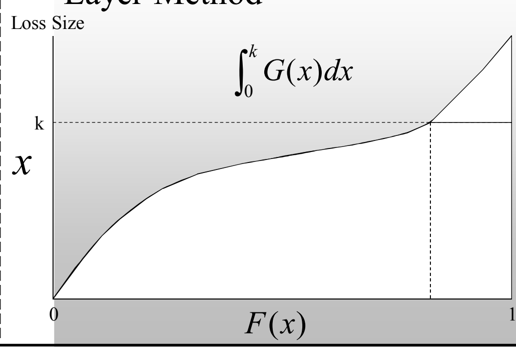
$* G(x) = 1 - F(x)$

Size Method



$* G(x) = 1 - F(x)$

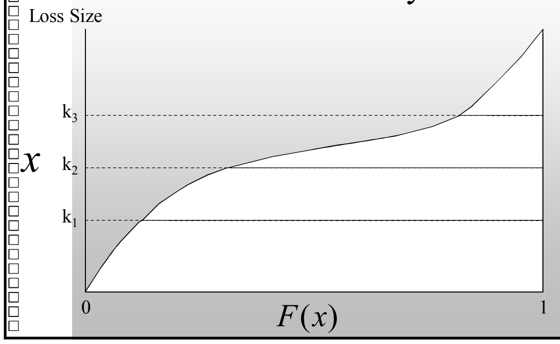
Layer Method



"Consistency" of ILFs

- As Policy Limit Increases
  - ILFs should increase
  - But at a decreasing rate
- Expected Costs per unit of coverage should not increase in successively higher layers.

Illustration: Consistency



"Consistency" of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40			
500,000	1.80			
1 Million	2.75			
2 Million	4.30			
5 Million	5.50			

"Consistency" of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	
500,000	1.80	250	0.40	
1 Million	2.75	500	0.95	
2 Million	4.30	1,000	1.55	
5 Million	5.50	3,000	1.20	

“Consistency” of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	.0027
500,000	1.80	250	0.40	.0016
1 Million	2.75	500	0.95	.0019
2 Million	4.30	1,000	1.55	.00155
5 Million	5.50	3,000	1.20	.0004

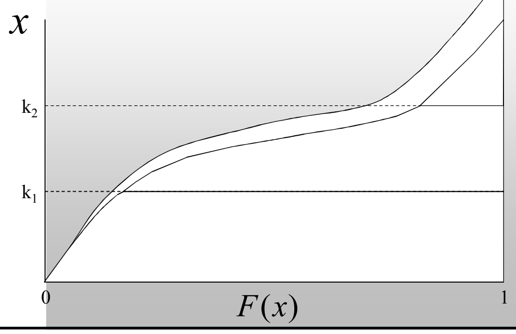
“Consistency” of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	.0027
500,000	1.80	250	0.40	.0016
1 Million	2.75	500	0.95	.0019*
2 Million	4.30	1,000	1.55	.00155
5 Million	5.50	3,000	1.20	.0004

Inflation – Leveraged Effect

- Generally, trends for higher limits will be higher than basic limit trends.
- Also, Excess Layer trends will generally exceed total limits trends.
- Requires steadily increasing trend.

Effect of Inflation



Example: Effect of +10% Trend @ $100,000 Limit

Loss Amount ($)	@ $100,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	100,000	100,000
490,000	100,000	100,000
750,000	100,000	100,000
925,000	100,000	100,000
1,825,000	100,000	100,000
Total	550,000	555,000
Realized Trend	+0.9%	

Example: Effect of +10% Trend @ $250,000 Limit

Loss Amount ($)	@ $250,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	250,000	250,000
490,000	250,000	250,000
750,000	250,000	250,000
925,000	250,000	250,000
1,825,000	250,000	250,000
Total	1,300,000	1,305,000
Realized Trend	+0.4%	

Example: Effect of +10% Trend @ $500,000 Limit

Loss Amount ($)	@ $500,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	250,000	275,000
490,000	490,000	500,000
750,000	500,000	500,000
925,000	500,000	500,000
1,825,000	500,000	500,000
Total	2,290,000	2,330,000
Realized Trend	+1.7%	

Example: Effect of +10% Trend @ $1,000,000 Limit

Loss Amount ($)	@ $1,000,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	250,000	275,000
490,000	490,000	539,000
750,000	750,000	825,000
925,000	925,000	1,000,000
1,825,000	1,000,000	1,000,000
Total	3,465,000	3,694,000
Realized Trend	+6.6%	

Example Summary
Trend Effect by Limit

- $100,000: + 0.9 %
- $250,000: + 0.4 %
- $500,000: + 1.7 %
- $1,000,000: + 6.6 %
- Overall: +10.0 %

Trends *generally* increase with the limit.

Example: Effect of +10% Trend $150,000 xs $100,000

Loss Amount ($)	$150,000 excess of $100,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	150,000	150,000
490,000	150,000	150,000
750,000	150,000	150,000
925,000	150,000	150,000
1,825,000	150,000	150,000
Total	750,000	750,000
Realized Trend	0.0%	

Example: Effect of +10% Trend $250,000 xs $250,000

Loss Amount ($)	$250,000 excess of $250,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	25,000
490,000	240,000	250,000
750,000	250,000	250,000
925,000	250,000	250,000
1,825,000	250,000	250,000
Total	990,000	1,025,000
Realized Trend	+3.5%	

Example: Effect of +10% Trend $500,000 xs $500,000

Loss Amount ($)	$500,000 excess of $500,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	-
490,000	-	39,000
750,000	250,000	325,000
925,000	425,000	500,000
1,825,000	500,000	500,000
Total	1,175,000	1,364,000
Realized Trend	+16.1%	

Example: Effect of +10% Trend
$1,000,000 xs $1,000,000

Loss Amount ($)	$1,000,000 excess of $1,000,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	-
490,000	-	-
750,000	-	-
925,000	-	17,500
1,825,000	825,000	1,000,000
Total	825,000	1,017,500
Realized Trend	+23.3%	

Example Summary
Trend Effect by Excess Layer

Layer	Net Trend
150 xs 100	+ 0.0%
250 xs 250	+ 3.5%
500 xs 500	+ 16.1%
1,000 xs 1,000	+ 23.3%
Overall	+ 10.0%

Commercial Automobile
ISO Aggregate Data - BI Trends

Calendar Year Data Through 3/31/2008
(Quarterly year-ending points)

Limit	12-point fit	24-point fit
$50,000	2.4%	3.0%
$100,000	3.1%	3.6%
$250,000	3.9%	4.5%
$500,000	4.5%	5.3%
$1,000,000	5.1%	5.9%
Total	4.8%	6.3%

Components of ILFs

- Expected Loss
- Allocated Loss Adjustment Expense (ALAE)
- Unallocated Loss Adjustment Expense (ULAE)
- Parameter Risk Load
- Process Risk Load

ALAE

- Claim Settlement Expense that can be assigned to a given claim --- primarily Defense Costs
- Loaded into Basic Limit
- Consistent with Duty to Defend Insured
- Consistent Provision in All Limits

ALAE Provision Determination

- Estimate ALAE/Total Limit Loss Ratio
- Find Average LAS (Limited Average Severity) Across Limits
- Multiply
  - 0.062 * 10,941 = 678
  - Use ALAE Provision at each limit

Unallocated LAE – (ULAE)

- Average Claims Processing Overhead Costs
  - e.g. Salaries of Claims Adjusters
- Percentage Loading into ILFs for All Limits
  - Average ULAE as a percentage of Losses plus ALAE
  - Loading Based on Financial Data
    - Ratio of ULAE to Incurred Loss + ALAE
    - 7.5% Loading in Upcoming Example

Process Risk Load

- Process Risk --- the inherent variability of the insurance process, reflected in the difference between actual losses and expected losses.
- Charge varies by limit

Parameter Risk Load

- Parameter Risk --- the inherent variability of the estimation process, reflected in the difference between theoretical (true but unknown) expected losses and the estimated expected losses.
- Charge varies by limit

Increased Limits Factors (ILFs)

ILF @ Policy Limit (k) is equal to:

$$\frac{LAS(k) + ALAE(k) + ULAE(k) + RL(k)}{LAS(B) + ALAE(B) + ULAE(B) + RL(B)}$$

Components of ILFs

Limit	LAS	ALAE	ULAE	PrRL	PaRL	ILF
100	7,494	678	613	76	79	1.00
250	8,956	678	723	193	94	1.19
500	10,265	678	821	419	108	1.37
1,000	11,392	678	905	803	123	1.55
2,000	12,308	678	974	1,432	135	1.74

Issues with Constructing ILF Tables

- Policy Limit Censorship
- Excess and Deductible Data
- Data is from several accident years
  - Trend
  - Loss Development
- Data is Sparse at Higher Limits

Use of Fitted Distributions

- May address these concerns
- Enables calculation of ILFs for all possible limits
- Smoothes the empirical data
- Examples:
  - Truncated Pareto
  - Mixed Exponential

Mixed Exponential Methodology

- Trend
- Construction of Empirical Survival Distributions
- Payment Lag Process
- Tail of the Distribution
- Fitting a Mixed Exponential Distribution
- Final Limited Average Severities

Trend

- Multiple Accident Years are Used
- Each Occurrence is trended from the average date of its accident year to one year beyond the assumed effective date.

Empirical Survival Distributions

- Paid Settled Occurrences are Organized by Accident Year and Payment Lag.
- After trending, a survival distribution is constructed for each payment lag, using discrete loss size layers.
- Conditional Survival Probabilities (CSPs) are calculated for each layer.
- Successive CSPs are multiplied to create ground-up survival distribution.

Conditional Survival Probabilities

- The probability that an occurrence exceeds the upper bound of a discrete layer, given that it exceeds the lower bound of the layer is a CSP.
- Attachment Point must be less than or equal to lower bound.
- Policy Limit + Attachment Point must be greater than or equal to upper bound.

Empirical Survival Distributions

- Successive CSPs are multiplied to create ground-up survival distribution.
- Done separately for each payment lag.
- Uses 52 (or more) discrete size layers.
- Allows for easy inclusion of excess and deductible loss occurrences.

Payment Lag Process

- Payment Lag =
  (Payment Year – Accident Year) + 1
- Loss Size tends to increase at higher lags
- Payment Lag Distribution is Constructed
- Used to Combine By-Lag Empirical Loss Distributions to generate an overall Distribution
- Implicitly Accounts for Loss Development

Payment Lag Process

- Payment Lag Distribution uses three parameters R1, R2, R3

$$R1 = \frac{\text{Expected \% of Occ. Paid in lag 2}}{\text{Expected \% of Occ. Paid in lag 1}}$$

$$R2 = \frac{\text{Expected \% of Occ. Paid in lag 3}}{\text{Expected \% of Occ. Paid in lag 2}}$$

$$R3 = \frac{\text{Expected \% of Occ. Paid in lag (n+1)}}{\text{Expected \% of Occ. Paid in lag (n)}}$$

(Note that lags 5 and higher are combined – C. Auto)

Payment Lag Process

Acc. Year	Lag 1 Occ	Lag 2 Occ	Ratio of Lag 2 / 1
2002		2,850	
2003	10,000	3,000	0.300
2004	11,000	3,100	0.282
2005	12,000	3,500	0.292
2006	13,000	3,750	0.288
2007	14,000		
Total 03-06	46,000	13,350	0.290

Lag Weights

- Lag 1 wt. = $1 \div k$
- Lag 2 wt. = $R1 \div k$
- Lag 3 wt. = $R1 \times R2 \div k$
- Lag 4 wt. = $R1 \times R2 \times R3 \div k$
- Lag 5 wt. = $R1 \times R2 \times [R3^2 \div (1 - R3)] \div k$
- Where $k = 1 + R1 + [\ R1 \times R2\ ] \div [\ 1 - R3\ ]$

Lag Weights

- Represent % of ground-up occurrences in each lag
- Umbrella/Excess policies not included
- R1, R2, R3 estimated via maximum likelihood.

Tail of the Distribution

- Data is sparse at high loss sizes
- An appropriate curve is selected to model the tail (e.g. a Truncated Pareto).
- Fit to model the behavior of the data in the highest credible intervals – then extrapolate.
- Smoothes the tail of the distribution.
- A Mixed Exponential is then fit to the resulting Survival Distribution Function

Simple Exponential

- Mean parameter: μ
- Policy Limit: PL

$$SDF(x) = e^{-x/\mu} = 1 - CDF(x)$$

$$LAS(PL) = \mu[1 - e^{-PL/\mu}]$$

Mixed Exponential

- Weighted Average of Exponentials
- Each Exponential has Two Parameters mean (μ_i) and weight (w_i)
- Weights sum to unity

$$SDF(x) = \sum_i [w_i e^{-x/\mu_i}]$$

*PL: Policy Limit

$$LAS(PL) = \sum_i w_i \mu_i [1 - e^{-PL/\mu_i}]$$

2008 Methodology Changes

- Expanded Number of Layers Evaluated
  - SDFs and CSPs for 68 – 75 layers,

    Varying by Line of Business (was 52)
  - Provides Enhanced Information and Flexibility for Smoothing the Tail of the Distribution
- Highest mean now limited to 100M
  - Allows smooth fits through the 100M limit
  - Previous maximum mean was 10M (most lines)

Mixed Exponential

2007 Commercial Auto I/L Review

- Number of individual exponentials vary by state group/table
- Range between four and seven exponentials
- Highest mean limited to 10,000,000

Mixed Exponential

2008 Commercial Auto I/L Review

- Number of individual exponentials vary by state group/table
- Range between nine and eleven exponentials
- Highest mean limited to 100,000,000
- Additional CSP layers evaluated (68 vs. 52)

Sample of Actual Fitted Distribution

Mean	Weight
2,763	0.824796
24,548	0.159065
275,654	0.014444
1,917,469	0.001624
10,000,000	0.000071

Calculation of "Raw" ILF

$$LAS(PL) = \sum_i w_i \mu_i [1 - e^{-PL/\mu_i}]$$

*PL: Policy Limit

$$LAS(100{,}000) = 7{,}494$$

$$LAS(1{,}000{,}000) = 11{,}392$$

$$ILF = \frac{LAS(1{,}000{,}000)}{LAS(100{,}000)} = \frac{11{,}392}{7{,}494} = 1.52$$

LAS Calculation Details

Mean	100K LAS	1M LAS	Weight
2,763	2,763	2,763	0.824796
24,548	24,130	24,548	0.159065
275,654	83,869	268,328	0.014444
1,917,469	97,437	779,227	0.001624
10,000,000	99,502	951,626	0.000071
Wtd. Average	7,494	11,392	1.000000

Deductibles

- Types of Deductibles
- Loss Elimination Ratio
- Expense Considerations

Types of Deductibles

- Reduction of Damages
  - Insurer is responsible for losses in excess of the deductible, up to the point where an insurer pays an amount equal to the policy limit
  - An insurer may pay for losses in layers above the policy limit (But, total amount paid will not exceed the limit)
- Impairment of Limits
  - The maximum amount paid is the policy limit minus the deductible

Impairment of Limits Example

Loss Size	# of Claims	Total Losses	Average Loss	Losses Net of Deductible		
				$100	$200	$500
0 to 100	500	30,000	60	0	0	0
101 to 200	350	54,250	155	19,250	0	0
201 to 500	550	182,625	332	127,625	72,625	0
501 +	335	375,125	1120	341,625	308,125	207,625
Total	1,735	642,000	370	488,500	380,750	207,625
Loss Eliminated				153,500	261,250	434,375
L.E.R.				0.239	0.407	.677

Deductibles (example 1)

Example 1:

Policy Limit: $100,000

Deductible: $25,000

Occurrence of Loss: $100,000

<u>Reduction of Damages</u>	<u>Impairment of Limits</u>
Loss - Deductible	**Loss does not exceed Pol. Limit, so:**
=100,000 - 25,000=75,000	**Loss - Deductible**
(Payment up to Policy Limit)	**=100,000 - 25,000=75,000**
Payment is $75,000	Payment is $75,000
Reduction due to Ded. is $25,000	Reduction due to Ded. is $25,000

Deductibles (example 2)

Example 2:

Policy Limit: $100,000

Deductible: $25,000

Occurrence of Loss: $300,000

Reduction of Damages	**Impairment of Limits**
Loss - Deductible	**Loss exceeds Policy Limit, so:**
= 300,000 - 25,000 = 275,000	**Policy Limit - Deductible**
(Payment up to Policy Limit)	**= 100,000 - 25,000 = 75,000**
Payment is $100,000	Payment is $75,000
Reduction due to Ded. is $0	Reduction due to Ded. is $25,000

Liability Deductibles

- Reduction of Damages Basis
- Apply to third party insurance
- Insurer handles all claims
  - Loss Savings
  - No Loss Adjustment Expense Savings
- Deductible Reimbursement
  - Risk of Non-Reimbursement
- Discount Factor

Deductible Discount Factor

- Two Components
  - Loss Elimination Ratio (LER)
  - Combined Effect of Variable & Fixed Expenses
    - This is referred to as the Fixed Expense Adjustment Factor (FEAF)

Loss Elimination Ratio

- Net Indemnity Costs Saved – divided by Total Basic Limit/Full Coverage Indemnity & LAE Costs
- Denominator is Expected Basic Limit Loss Costs

Loss Elimination Ratio (cont'd)

- Deductible (i)
- Policy Limit (j)
- Consider [LAS(i+j) - LAS(i)] ÷ LAS(j)
- This represents costs under deductible as a fraction of costs without a deductible.
- One minus this quantity is the (indemnity) LER
- Equal to

  [LAS(j) - LAS(i+j) + LAS(i)] ÷ LAS(j)

Pricing Liability Deductibles

- Can Use Fitted Indemnity Distributions
- Estimate Cost in Covered Layer
- Relate to Cost Without Deductible

$*G(x)=1-F(x)$

Limited Average Severity - Layer

$$\int_{k_1}^{k_2} x dF(x) + k_2 \times G(k_2) - k_1 \times G(k_1)$$

Size method; ‘vertical’

$$\int_{k_1}^{k_2} G(x) dx$$

Layer method; ‘horizontal’

$*G(x)=1-F(x)$

Size Method & LAS

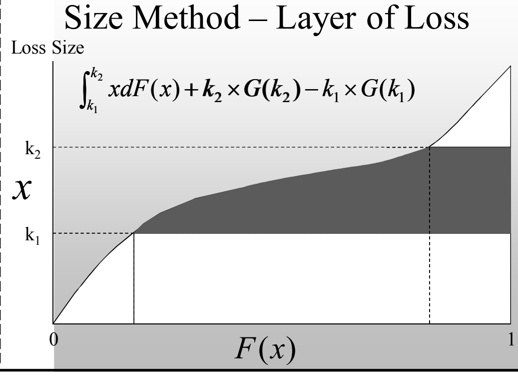
$$\int_{k_1}^{k_2} x dF(x) + k_2 \times G(k_2) - k_1 \times G(k_1)$$

is equal to

$$\left[\int_{0}^{k_2} x dF(x) + k_2 \times G(k_2)\right] - \left[\int_{0}^{k_1} x dF(x) + k_1 \times G(k_1)\right]$$

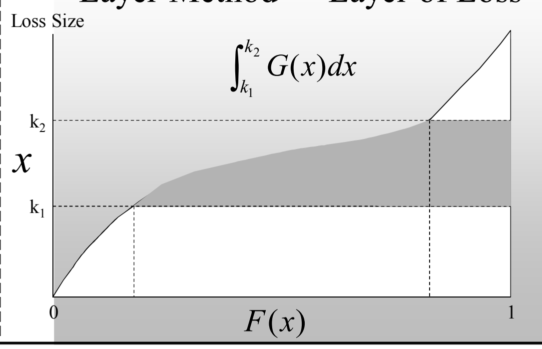
$*G(x)=1-F(x)$

Size Method – Layer of Loss



$$*G(x) = 1 - F(x)$$

"Layer Method" – Layer of Loss



Summary

- Increased vs. Basic Limits Ratemaking
- Loss Severity Distributions
- Effects of Trend
  - By Limit and Layer
- Components of ILF Calculation
- Mixed Exponential Methodology
- Deductible and Layer Pricing

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