Spatial Statistics

A Framework for Analyzing Geographically Referenced Data in Insurance Ratemaking

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Next 30 Minutes

An Introduction to Spatial Statistics For Territorial Ratemaking

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- ***Motivation***

  Spatial Statistics - An Improvement to the Territorial Ratemaking

  Location Matters - Foundation of Spatial Statistics

  Standard Regression vs. Spatial Regression

- ***Spatial Statistics Theory & Connection to Insurance Ratemaking***

  Stochastic Process, Random Fields and Different Types of Spatial Data

  Spatial Structure in GLM Residuals & Measures of Spatial Dependence

  Why Loss Ratio is So High in North Atlantis?

  Are Theft Claims Coming More From South Atlantis?

  Territorial Boundary Definition - What Territories to be used?

- ***A Case Study - A Spatial Econometric Model***

  Housing Price in California - Simultaneous Autoregressive (SAR) Error Model

  Diagnostics & Model Comparison with GLM & GAM

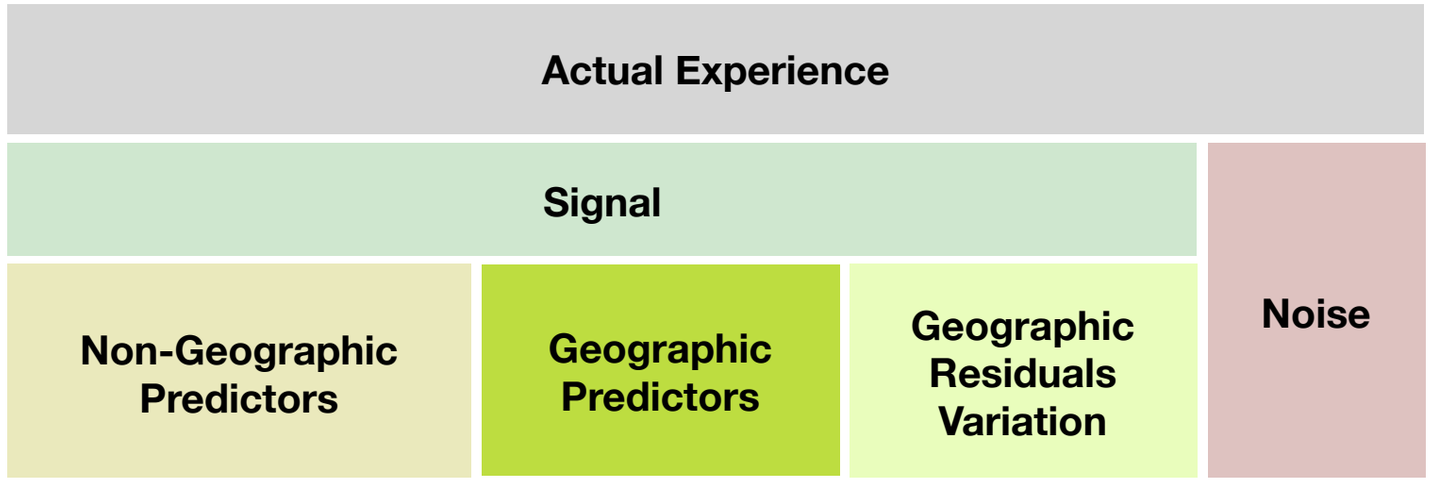
- ***An Evolution - Location in Insurance Ratemaking & Implementation***

  Three Different Assumptions, Three Different Framework and One Common Thread - Filtering

- ***Conclusion***

Territorial Ratemaking

Why We Want to Apply Spatial Statistics Methodologies?



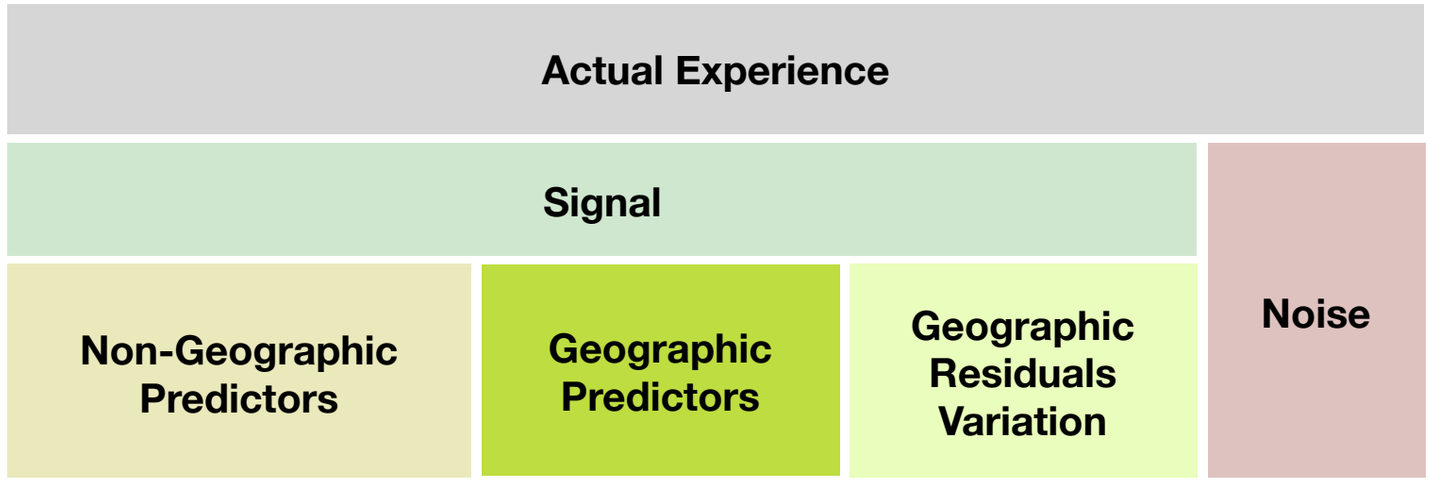
Elements of Territorial Ratemaking

1. ***Territorial Boundary Definition***
2. ***Setting up Territorial Relativities***

- ***Territorial Boundary Definition***
  - *Zip Code, Census Block, County*
  - *Territory acts as a proxy for many different variables that are hard to estimate*
  - ***Administrative Territories may not be optimal for insurance underwriting purpose***
  - *Same Territory may have inhomogeneous insured groups within; Different Territories may have homogeneous insured groups in between*
  - ***Spatial Models can "Filter-Out" this spatial overlap effect***

Territorial Ratemaking

Why We Want to Apply Spatial Statistics Methodologies?



- ***Setting Up Territorial Relativities***
  - ***Non-Geographic Predictors -*** *Age of Insured, Previous Loss History etc.*
  - ***Geographic Predictors -*** *Geo-demographic predictors (population density) as well as on Geo-physical predictors (average snow fall) etc.*
  - ***Geographic Residual Variation -*** *Accounts for possible left out Geographic Predictors*

Including Latitude-Longitude in the Model

- *Latitude-Longitude has a clear effect on Geographic Predictors. Generalized Additive Model (GAM) is the most intuitive way to include Latitude-Longitude in the Model that reduces Geographic Residual Variation.*

Including Spatial Correlation Structure in the Model

- *Practically, it is impossible to eliminate (Geographic) Residual Variation by including "all" possible predictors*
- *Spatial Statistics Methodologies have ability to include a Spatial Error Structure in the Model that accounts for the Geographic Residual Variation*

Motivation

Tobler's First Law of Geography, *Waldo R. Tobler, 1970*

- ***Idea*** *- "Everything is* ***related*** *to everything else, but* ***near*** *things are more related than* ***distant*** *things"*
    - **Location Matters -** Observed value at one location is influenced by the observed values at other locations in a geographic area
    - Influence declines with distance
    - Define "***near***" - Euclidean distance, Territory with common boundaries, Transit distance (Manhattan distance), Insured sharing the same fire station, Sphere of influence, other relationships e.g. Actuaries with a degree in Economics, Bostonians commuting in the green line T (subway)
- ***Theory and Computation***
    - Rapid theoretical development of Spatial Statistics in last few decades and widely available literature
    - Improved computation facility and advent of open source programming environment e.g. R, WinBugs
    - Application in the many fields - Epidemiology & Public Health, Political Science, Marketing, Real Estate, Economic Geography, Criminology
- ***Data*** *-* ***Cost effective and accurate geocoding process and easy availability of geocoded data***
    - Photos taken with most standard digital cameras, phones (e.g. iPhone) are geocoded
    - Different sources of Demographic and Geographic Data, Weather Data, Telematics data in coming days, Detailed and highly interactive GIS e.g. Google Earth

Mathematical Interpretation

Data Generating Process - Non-Spatial vs. Spatial

- ***Task - Regression in a Geographic Region -*** *Housing Prices in California, Area with high crime rate in Chicago (Crime Hotspot), Fire/ Water Insurance, Theft Insurance, Pollution Insurance, WC claims across a region*

- ***Non-Spatial Data Generating Process -*** *For location* ***i*** *and* ***k*** *in the region*

$$Y_i = X_i \beta + e_i$$

$$Y_k = X_k \beta + e_k$$

$$e_i \sim N(0, \sigma^2)$$

  - ***Conditional independence of the observed values*** *- observed value Y_i at location i is independent of observed value Y_k at location k (in a fully specified model)*

  - ***Independence of residuals -*** *e_i and e_k are independent*

- ***Spatial Data Generating Process -*** *For location* ***i*** *and* ***k*** *in the region*

$$Y_i = \alpha_k Y_k + X_i \beta + e_i$$

$$Y_k = \alpha_i Y_i + X_k \beta + e_k$$

$$e_i , e_k \sim N(0, \sigma^2)$$

  - ***Spatial dependence of the observed values*** *- observed value Y_i at location i is influenced by the observed value Y_k at location k*

  - ***Omitted Variable Bias (OVB) -*** *Observations are influenced by a "latent" or "unobservable" factor (e.g. goodness of a good society/ neighborhood can increase demand of houses in that area)*

  - ***Spatial Heterogeneity -*** *Relationship between X and Y changes over Geographic Region (not a constant β)*

Spatial Data & Analogy to Time Series

A Generic Stochastic Process and Three Types of Spatial Data

- ***Stochastic Process :*** ***{ Y(s) : s in D }*** *where* ***Y(s)*** *is* ***Random Observation, s*** *is an* ***Index*** *set from* ***D, a subset of R^r (r-dimensional Euclidean space)***
- ***Time Series -*** *Special case of stochastic process where index set s is 1-dimensional Euclidean space:* ***{ Y_t : t in {1,2,3,4,...}}***
- ***Random Field -*** *When the Domain D is from a multi-dimensional Euclidean space (* ***r > 1*** *)*
- ***In simple words:*** *Random Field is a list of correlated random observations that can be mapped onto a r-dimensional space*
- ***Spatial Data Generating Process - The Process generates spatial data for r = 2***

  { Y(s) : s in D } where D is a subset of R^2

  - ***Coordinate Reference System (CRS)*** *- Latitude, Longitude, Northing, Easting, Different Projections*
  - ***Induced Covariance Structure -*** *Observations are spatially correlated based on a covariance function*

Three Types of Spatial Data

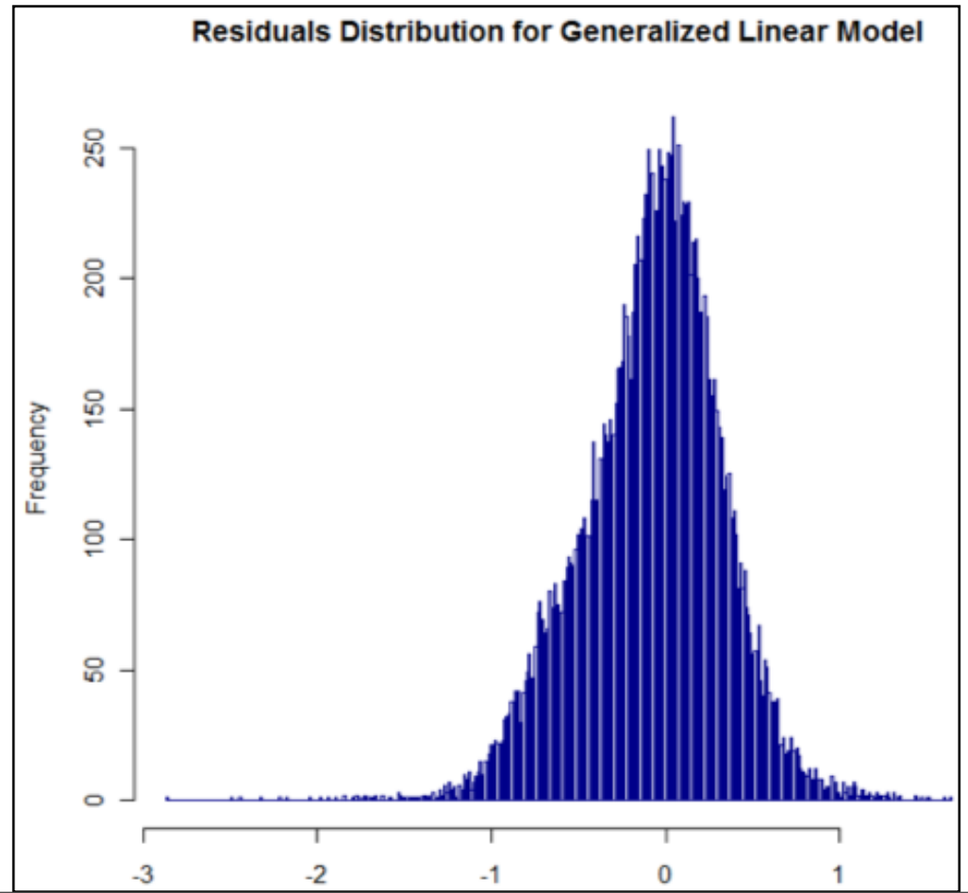
- *How s takes values in D (discrete/ continuous)?*
- *How D comes from R^2 (Fixed/ Random)?*
  - ***Point Referenced Data -*** *When s takes values in D continuously, D is a fixed subset of R^2*
  - *Temperature in Chicago (Possible to collect every point in Chicago)*
  - ***Lattice / Areal Data -*** *D is a fixed partitioned subset of R^2, D = {s_1, ..., s_n}, s assumes value from one of the partitions*
  - *Postal Zip Codes in Chicago - Non-overlapping Areal Unit*
  - ***Spatial Point Pattern Process -*** *The domain D itself is a random subset in R^2*
  - *Locations of Starbucks in Chicago - Are they more clustered in the Chicago Loop? Do their Cappuccinos taste better than the Starbucks at other places in the city?*

Why Loss Ratio Is So High In North Atlantis?

Point Referenced Data & Geostatistics

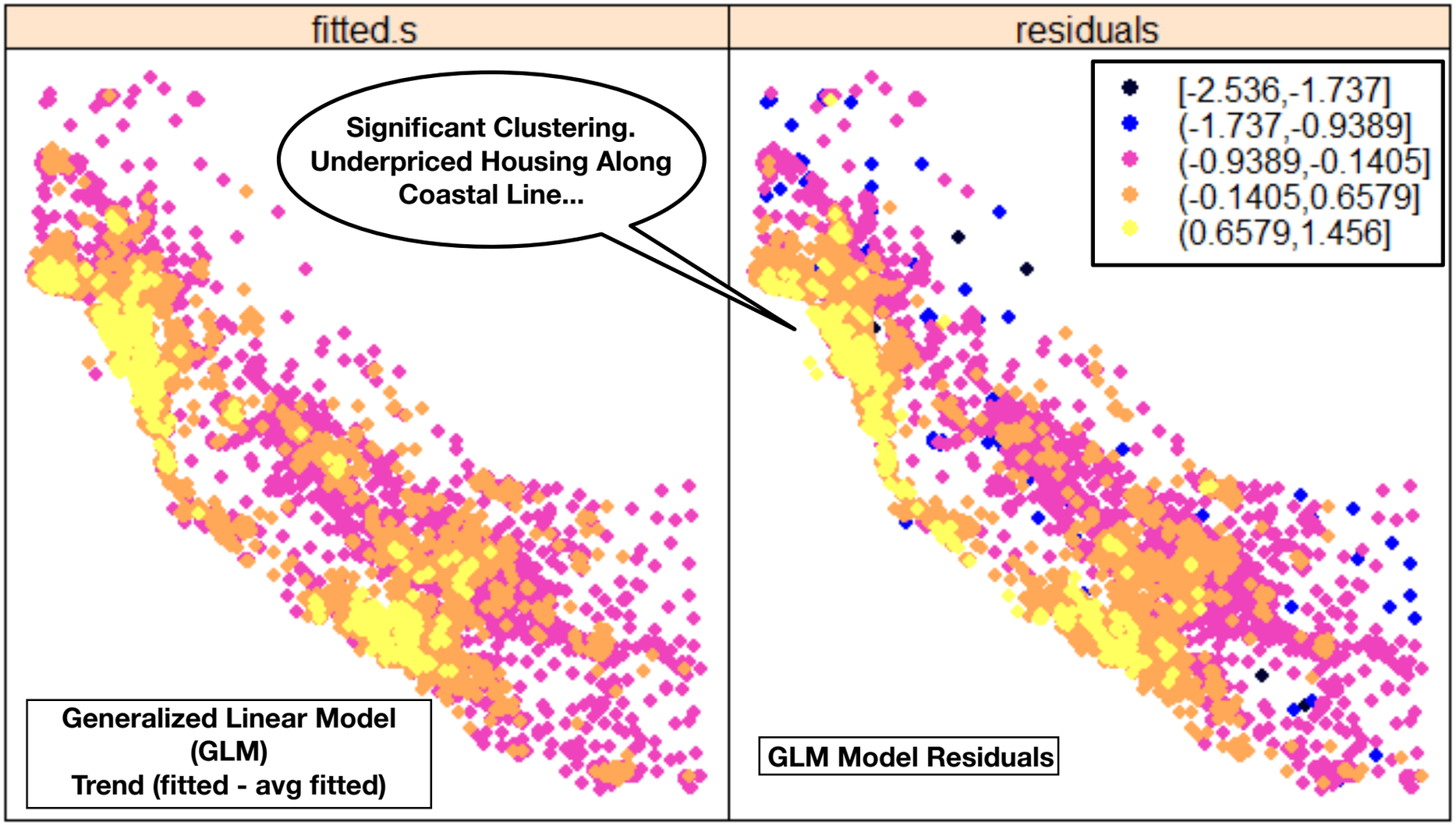
- ***Analysis and inference of Stochastic Process { Y(s) : s** runs **continuously** in **D } : D** is a **fixed subset** of **R^2***
- ***Common Practical Interest in Geostatistics***
  - *Given the observations in different location { $Y(s_1)$,,, $Y(s_n)$} : How to optimally predict Y(s) at a new location s*
  - *Estimation of spatial averages under spatially correlated data*
  - *Diagnostic of existing model: Spatial clustering of residuals in study region*
- ***A Simple Illustration -** California Housing Data (GAM example data) by Census Block*
  - *A typical example of Areal Data, but we will treat as Point Referenced Data*
  - *Assuming the data is a random selection of 20640 houses in California*
  - *Consider usual Generalized Linear Model (GLM) as in GAM Example*

```
glm(formula = value ~ income + I(income^2) + I(income^3) + log(age) +
     log(rooms) + log(bedrooms) + log(hh/pop) + log(hh), family = Gamma(link =
     log), data = ca.data)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.15154  -0.26238  -0.05152   0.15523   2.97976

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)   11.3680015  0.0556922 204.122  < 2e-16 ***
income         0.6587304  0.0244209  26.974  < 2e-16 ***
I(income^2)   -0.0488121  0.0048773 -10.008  < 2e-16 ***
I(income^3)    0.0015019  0.0002929   5.127 2.97e-07 ***
log(age)       0.1924867  0.0060165  31.993  < 2e-16 ***
log(rooms)    -0.8568208  0.0171994 -49.817  < 2e-16 ***
log(bedrooms)  1.0472060  0.0261859  39.991  < 2e-16 ***
log(hh/pop)    0.2696699  0.0218861  12.322  < 2e-16 ***
log(hh)        0.0244465  0.0038982   6.271 3.65e-10 ***

    Null deviance: 6394.0  on 20639  degrees of freedom
Residual deviance: 2627.7  on 20631  degrees of freedom
AIC: 515354
```



GLM & Geostatistics

Independence of Residuals - Spatial Perspective



- *Residuals from the simple model are not distributed randomly over CA*
- *Model underfits along coastline*
- *Model overfits in the locations away from coastline*
- *This example is an analogy to usual insurance adverse selection*
- *Can we show this Spatial Structure in a Quantitative Measure?*

Spatial Correlation

Measure of Spatial Correlation

- ***Variogram / Semi-Variogram***
  - *Quantitative measure of Spatial Correlation between two near-by values (observations / errors)*
  - *Mathematical Formulation:*

    $$\textbf{Variogram} = 2\gamma(h) = Var[\,Y(s+h) - Y(s)\,] = 2\,[\,Cov(0) + Cov(h)\,]$$

    Assumes: $E[\,Y(s+h) - Y(s)\,] = 0$; $E[\,Y(s+h) - Y(s)\,]^2$ *depends only on the separation vector h*
  - *Statistical packages provide with the graph between different distances and corresponding* $\gamma(h)$
  - *Empirical graph of* $\gamma(h)$ *or sample variogram is then compared with different theoretical covariance function*
  - $\gamma(h)$ *plays an important role in the geostatistical prediction as the key to spatial correlation*

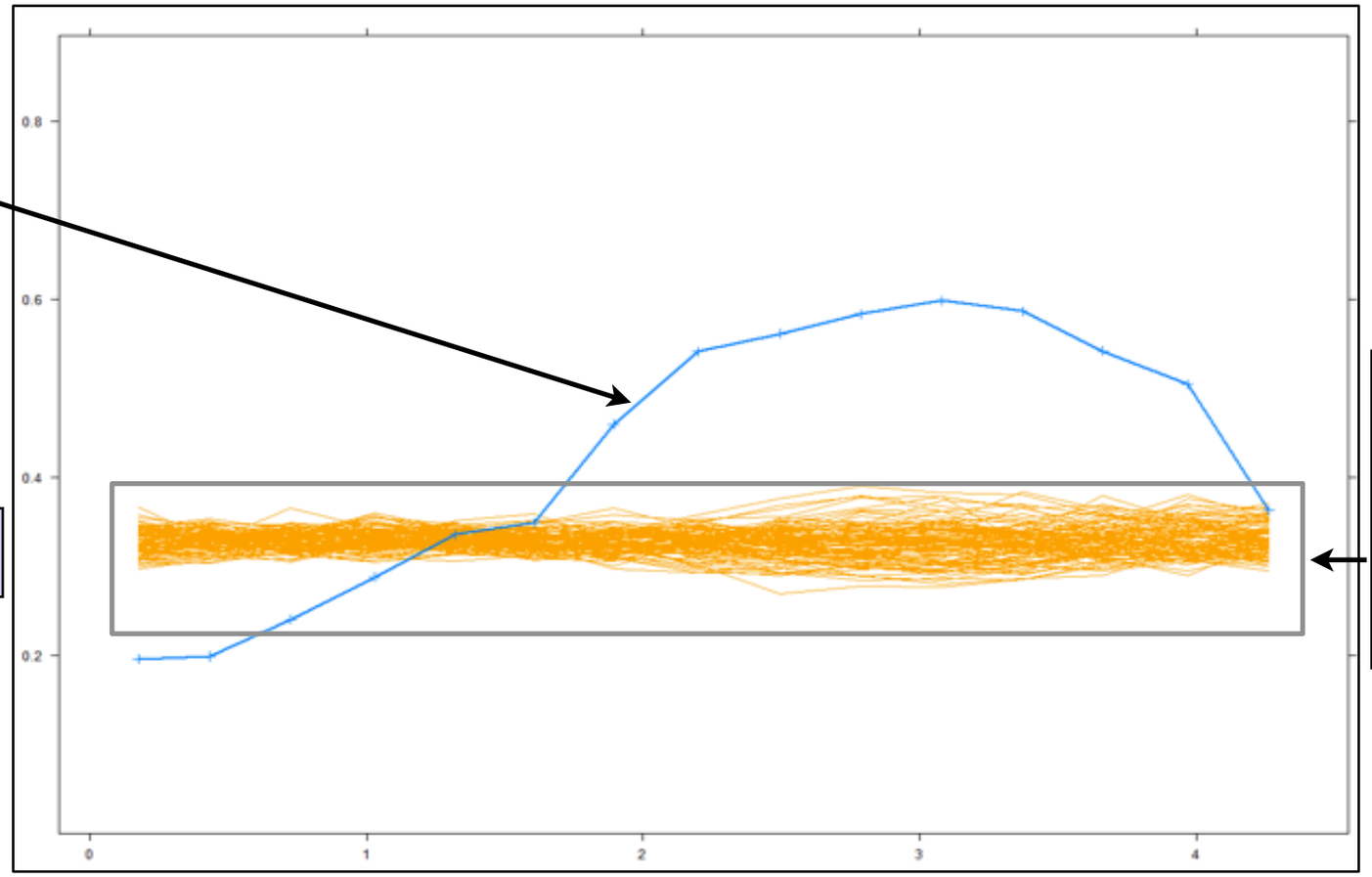
- ***Statistical Testing for Spatial Correlation***
  - ***Spatially Lagged Scatterplot:*** *A simple way to accept or reject spatial correlation is to check the scatter plots of pairs Y(s) and Y(s +h) for all possible separation vector h and grouped by the distances corresponding to h*
  - *In presence of spatial correlation Y(s) and Y(s+h) should show high correlation for lower degree of separation h and low correlation for higher degree of separation*
  - *In Simple words: In the scatter plot, observations in close proximity will show high pattern and observations at distant locations will show randomness.*

Spatial Homogeneity

Sample Variogram and Existence of Spatial Correlation

Recall Variogram: $\gamma(h) = \frac{1}{2} * Var[Y(s + h) - Y(s)]$ ⟶ ***Higher the Semivariance Lower the Homogeneity Among Observations***

Variogram of the True Data (CA Housing Price) shows Spatial Homogeneity for Close-by Data Points and Heterogeneity for Far-Apart Data Points



Sample Variograms from Location Shuffled Data Sets Are Showing No Changes in Homogeneity with Increasing Distance

Semivariance - a Measure of Heterogeneity

Distance

Close-by Observations

Far-apart Observations

- ***Sample Variogram & Estimation of Spatial Correlation***
  - *Calculate Variogram after re-assigning the observations (insured) randomly to different locations (street address) in the data (book of business) several times and obtain a 95% confidence interval*
  - *Spatial Patterns become evident if the sample Variogram from true data falls outside the confidence interval*
  - *Statistical packages can fit a parametric variogram to the sample variogram*
  - *Some important parametric variogram: Linear, Exponential, Spherical, Gaussian, Matern*

Are Theft Claims Coming More From South Atlantis?

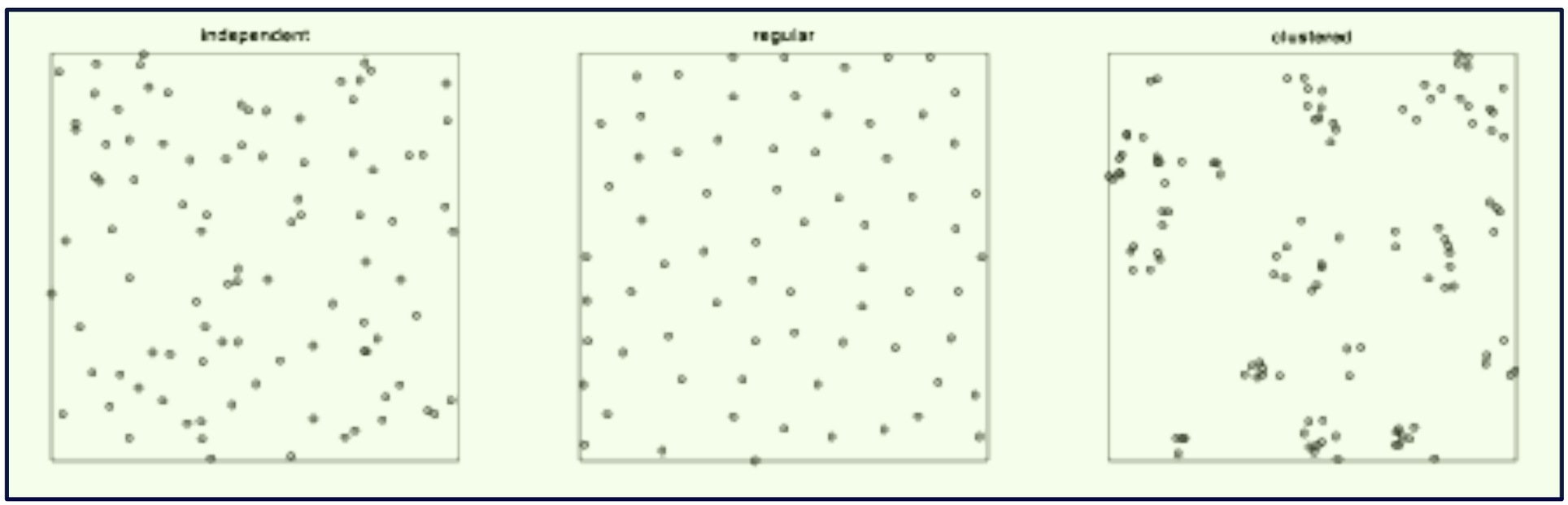
Spatial Point Pattern Process - Spatial Poisson Process

- ***Analysis and inference of Stochastic Process { Y(s) : s** in **D } : D** is a **random subset** of **R^2***

- ***Elements of Spatial Point Process:***
  - I. ***First Order Properties - Distribution:*** *Spatial Distribution of Events - Intensity of Event Occurrence, Spatial Density*
  - II. ***Second Order Properties - Interaction:*** *Clustering of Events, Independence*

- ***Complete Spatial Randomness (CSR) - Events occur independently and distributed uniformly over a geographic region***
  - I. ***Clustering of Events*** *- Attraction between points over the region*
  - II. ***Regularity of Events*** *- Presence of Inhibitation - Competition between points over the region*

- ***Spatial Poisson Process - Events occur independently and distributed according to a given intensity function $\lambda(.)$ over a geographic region***
  - I. ***Homogeneous Poisson Process (HPP)*** *- Intensity function is a constant :* **$\lambda(x) = \lambda$**
  - II. ***Inhomogeneous Poisson Process (IPP)*** *- Variable (often Parametric) Intensity function* **$\lambda(x)$**

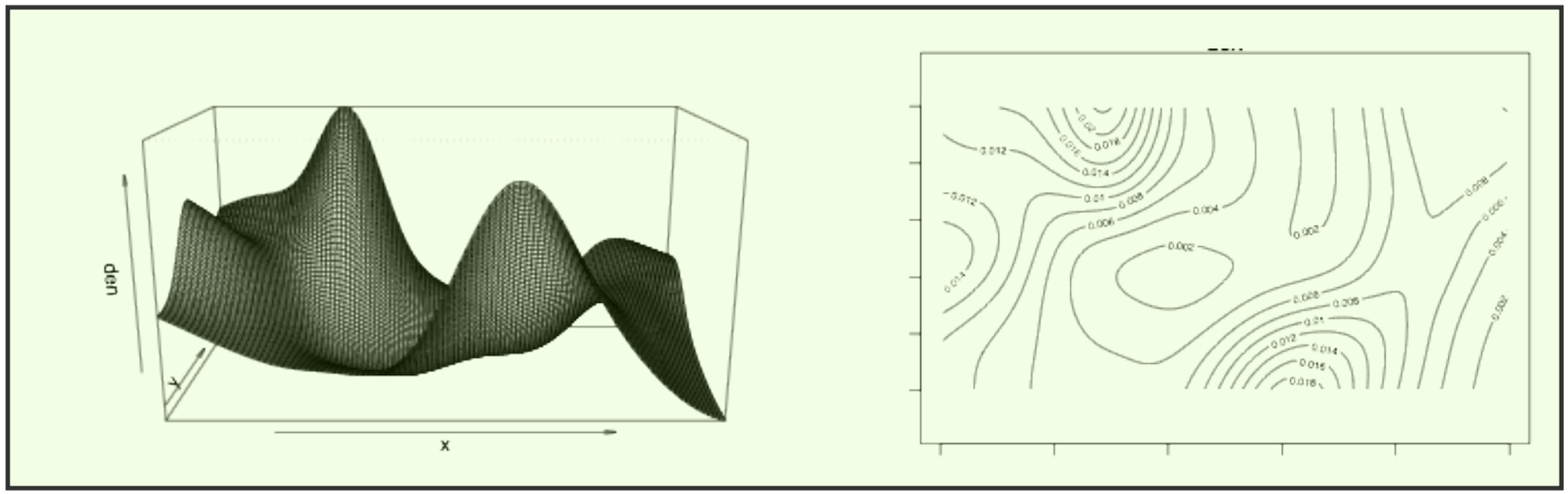


Spatial Point Pattern Process

Distribution of Events

- ***HPP - Homogeneous Poisson Process -*** *A formalization of Complete Spatial Randomness (CSR)*
  - ***The number of events in a region W with area A is Poisson distributed with mean λA, where λ is the constant intensity of the process***
  - ***Given there is n number of events observed in the region W, they are uniformly distributed***

- ***Inference on the Poisson Process and Estimation of λ(x)***
  - *In Homogeneous Poisson Process Estimated Intensity is:* **λ = (n / A)** *: n(x) = # points in region W with area A*
  - ***Statistical Test for CSR: Quadrant based Chi-Square Test and Spatial Kolmogorov-Smirnov Test***
  - *In Inhomogeneous Poisson Process usual Kernel estimation is used to estimate the intensity function* **λ(x)**
  - ***Perspective Plot*** *or* ***Contour Plot*** *are used as visual aid to understand intensity function*
  - *Maximum Likelihood Techniques are used to estimate a parametric intensity function in IPP*
  - *Estimated intensity function is used to fit Poisson Model and Residual Analysis takes place*



A Classic Illustration

1854 Broad Street Cholera - London

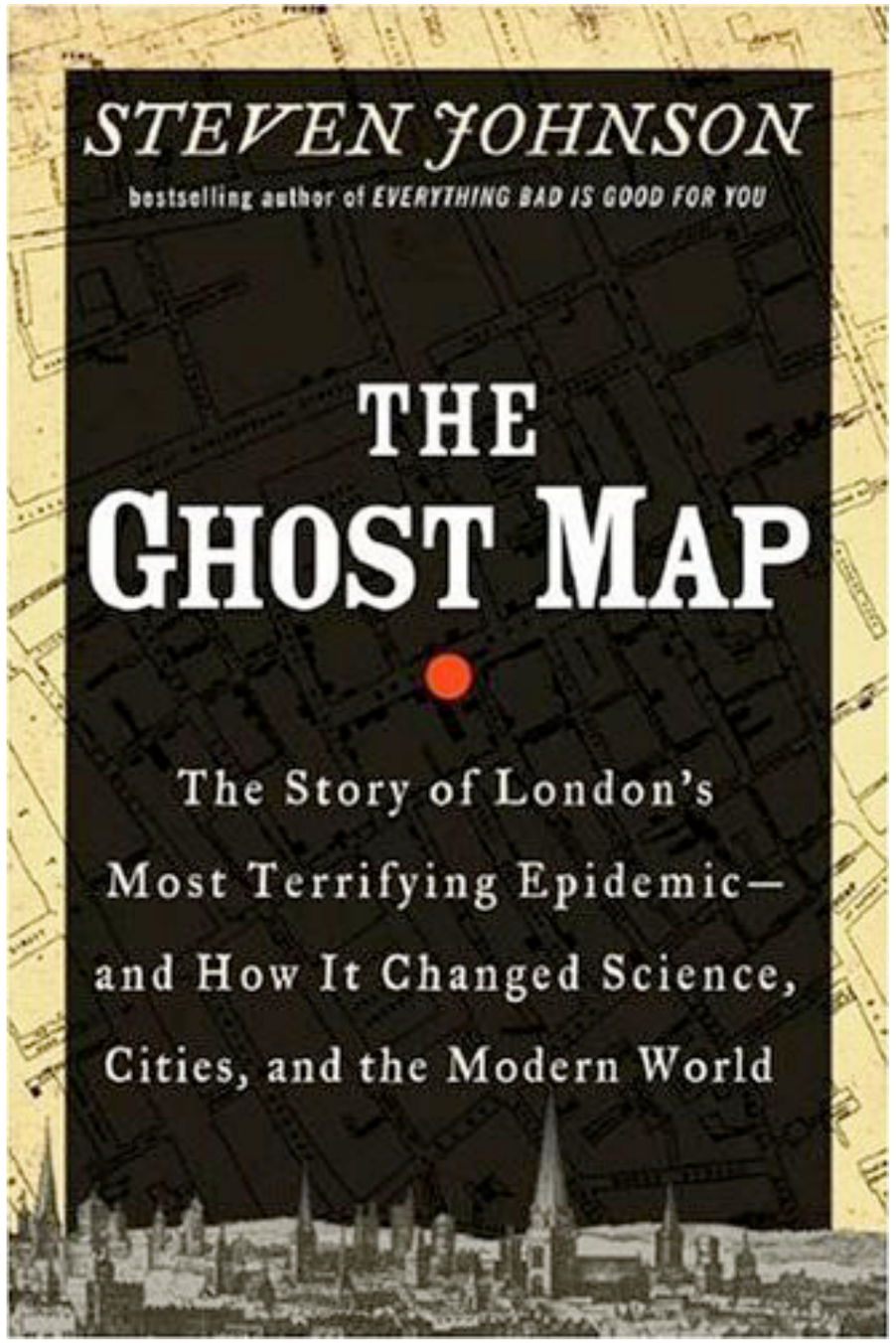
The Story - John Snow Example

- *Time: August, 1854*
- *Location: Soho District, London, UK*
- *Event: Cholera - Around 600 people died*

Dr. John Snow's Study & Spatial Interpretation

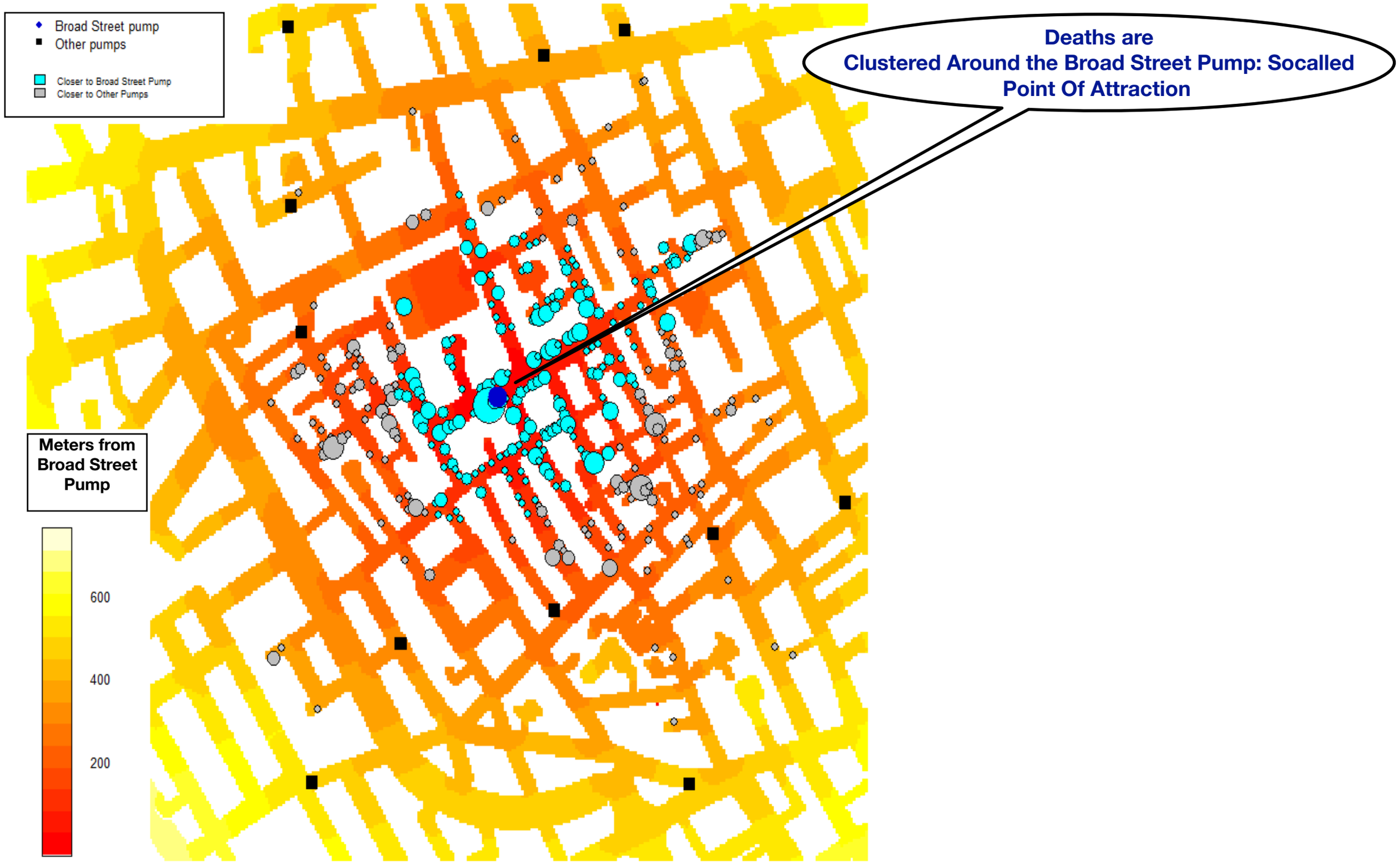
- *Miasma Theory - Disease such as Cholera/ Black Death were caused by noxious form of "bad air"*
- *Germ Theory - Disease is caused by Germs (micro-organisms)*
- *How Cholera deaths are distributed in Soho? Is there a Complete Spatial Randomness (CSR)?*
- *Snow draws a map to show that cholera deaths are clustered around Broad Street Pump and not Uniformly distributed*
- *Snow's visualization is considered to be the starting point of Modern Epidemiology and Disease Mapping*
- *Spatial Statistical Analysis can formally infer on the spatial distribution of cholera deaths*

- *For more Info:* http://en.wikipedia.org/wiki/1854_Broad_Street_cholera_outbreak



The Ghost Map

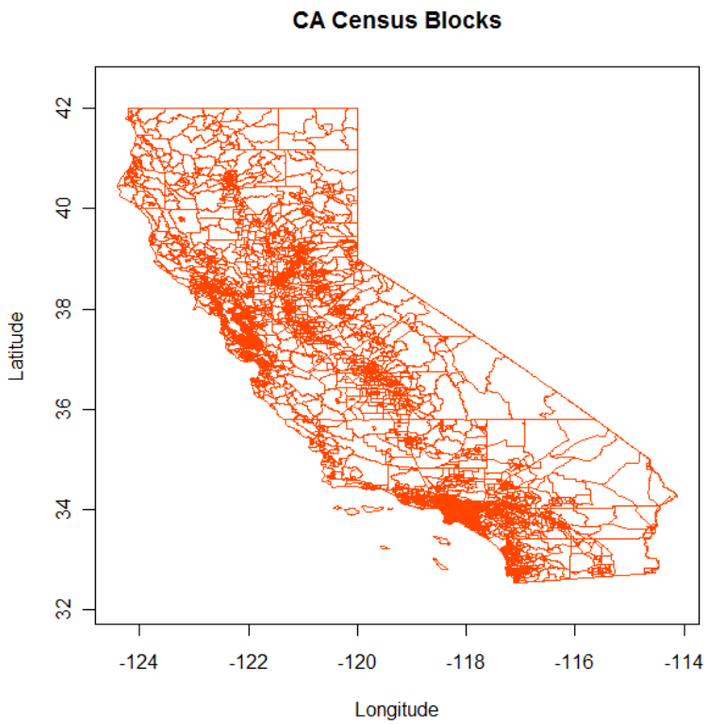
Spatial Concentration of Deaths Around Broad Street Pump



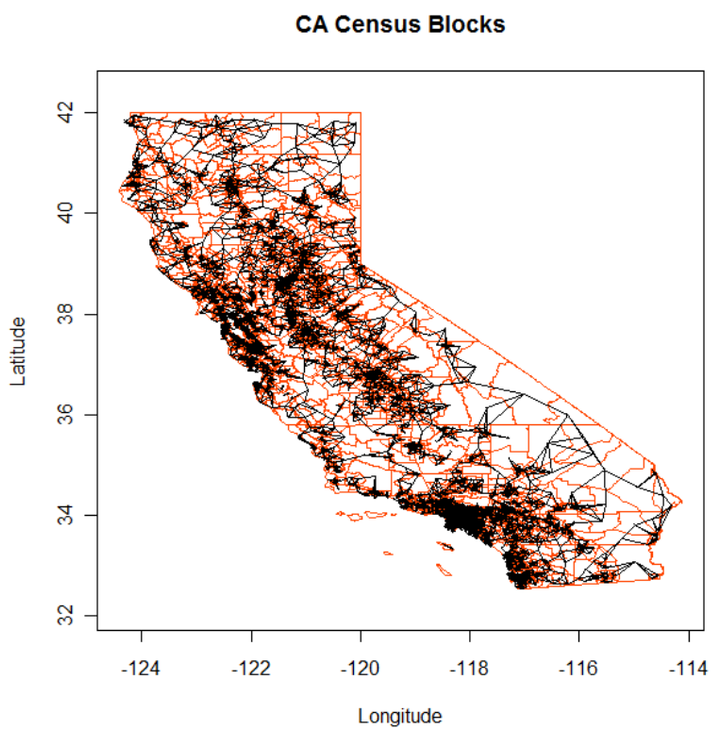
What Territories Should Be Used?

Lattice/ Areal Data

- ***Analysis and inference of Stochastic Process { Y(s) : s** in **D } : D = {s_1, ..., s_n}** is a **partitioned subset** of **R^2***
- ***Common Practical Interest:***
  - ***Spatial Correlation:** Spatial Correlations among territories/ areal units/ sub-regions and incorporating them into the model*
  - ***Model Based Smoothing:** Even out near-by Territories? How much smoothing should be done?*
  - ***Modifiable Areal Unit Problem (MAUP) -** How to re-allocate observations when there is a change in territorial definition (a new set of territory to be used)?*
- ***Correlation Quantification - Creation of Neighbors and Proximity Matrix W***
  - ***W - Proximity matrix - ((w_{ik})) - gets some value for each pair of locations (i,k)***
  - *Binary Proximity Matrix: **W = ((w_{ik})) = 1** if (i,k) has a common boundary; otherwise 0. Standardized for unit row sum.*
  - *Distance based neighbor criterion can be used (neighbors if within 50 miles of the Territory)*



California Proximity Matrix for 4 Nearest Census Tract Neighbors



A Spatial Econometric Model

Spatial Simultaneous Autoregressive Error Model

- ***Spatial Simultaneous Autoregressive (SAR) Error Model For Spatial Process - { Y(s) : s in D } : D = {s_1, ..., s_n}***

$$Y(s) = X(s)\,\beta + u(s) : \text{Regression Model}$$

$$u(s) = \lambda W\,u(s) + \varepsilon(s) : \text{Latent Spatial Lag Model}$$

X(s)β = Regression Covariate Structure (Mean)

u(s) = Spatial Error Structure

ε(s) = Pure Random Error

W = Proximity Matrix

λ = Spatial Lag Coefficient

λ = 0 leads to a purely non-spatial model

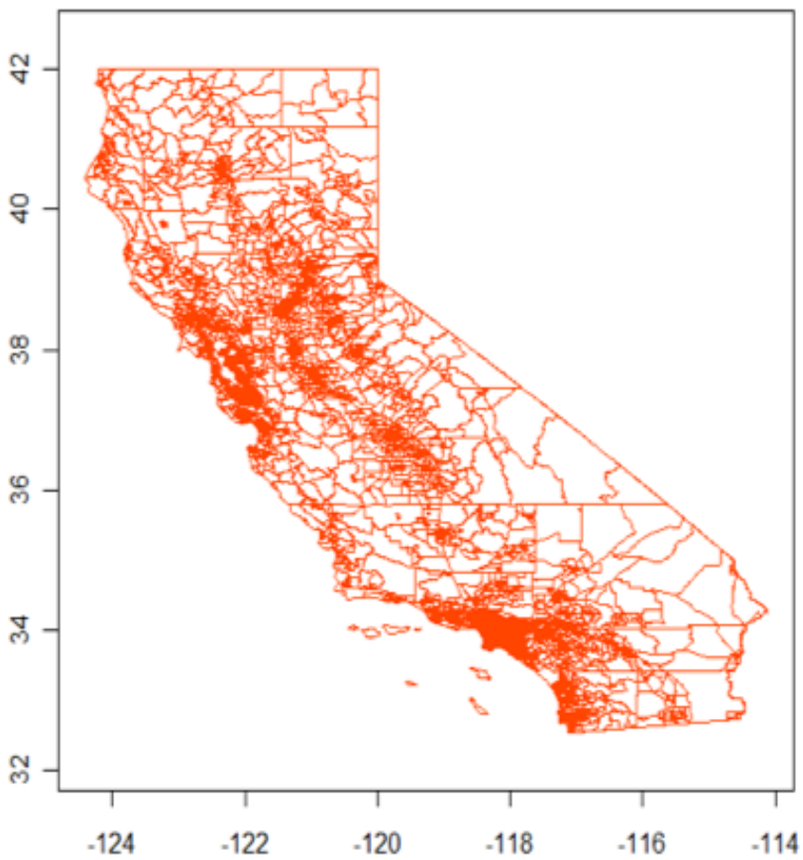
β = 0 leads to a purely spatial model

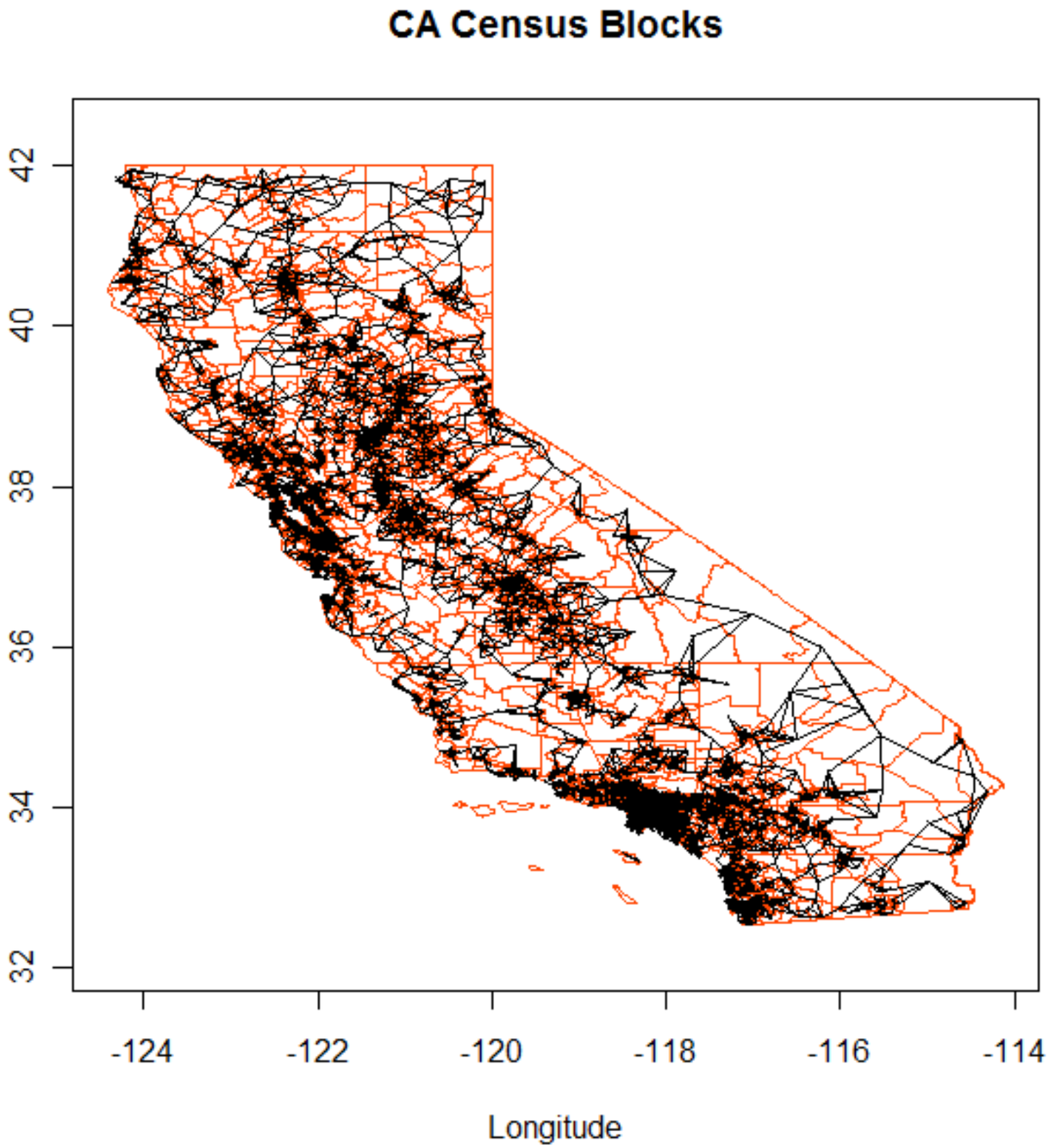
Actual Experience : *Y(s)*			
Signal : *X(s)β*			Noise *ε(s)*
Non-Geographic Predictors (Example: Age of Insured)	**Geographic Predictors** (Example: Avg. Snow Fall)	**Geographic Residuals Variation** ***u(s)***	

California Housing Price

Simultaneous Autoregressive Models - Neighborhood Creation

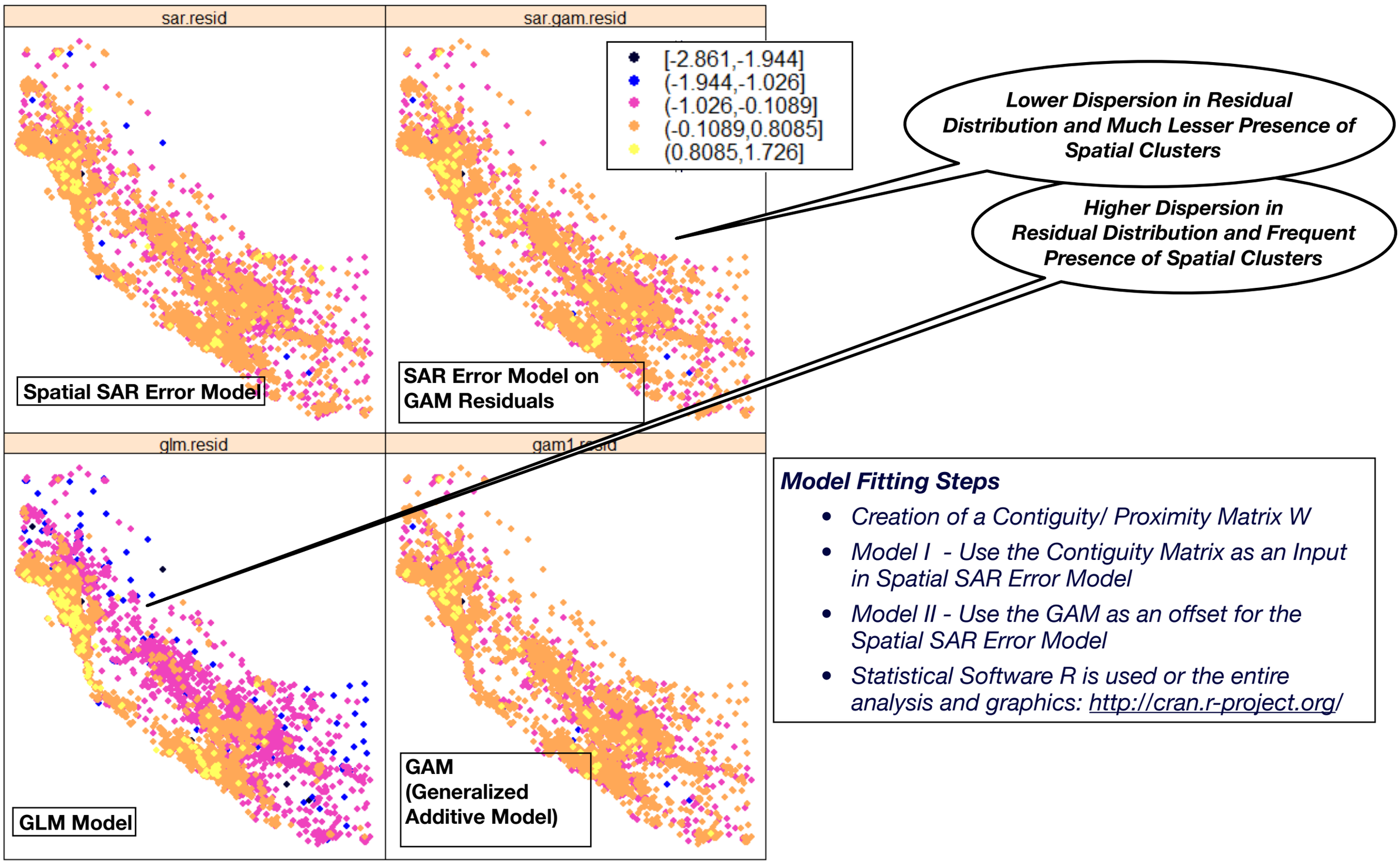
- **A 4-Closest Neighbors Contiguity Matrix has been created for California 1990 Census Blocks**
- **Map (Census Block) data source - US Census**
- **R program has been used to create this map**
- **A Common Border Contiguity Matrix May be Tried**





Diagnostics - Residual Mapping

Comparison of Different Models on California Housing Data

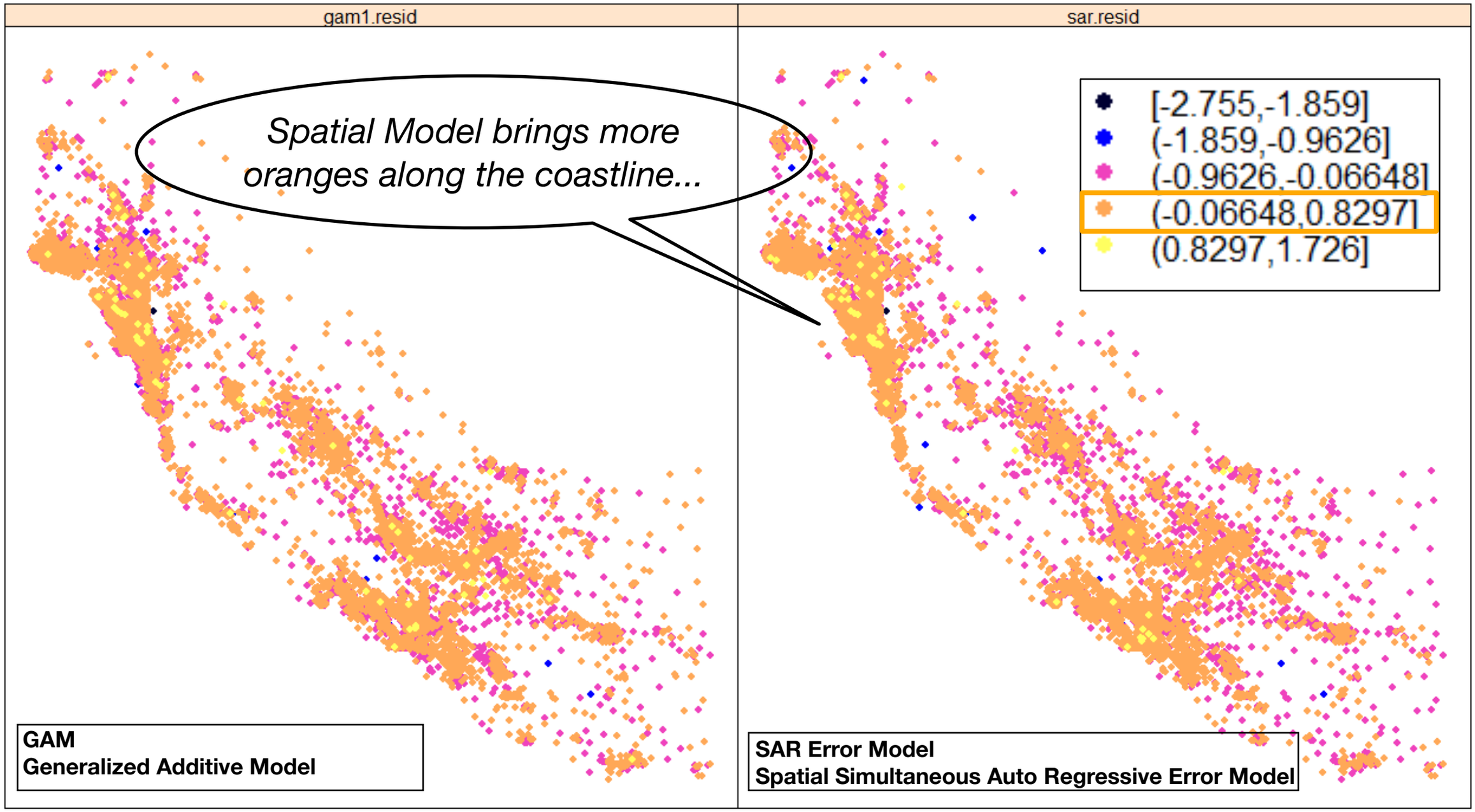


Model Fitting Steps

- *Creation of a Contiguity/ Proximity Matrix W*
- *Model I - Use the Contiguity Matrix as an Input in Spatial SAR Error Model*
- *Model II - Use the GAM as an offset for the Spatial SAR Error Model*
- *Statistical Software R is used or the entire analysis and graphics: http://cran.r-project.org/*

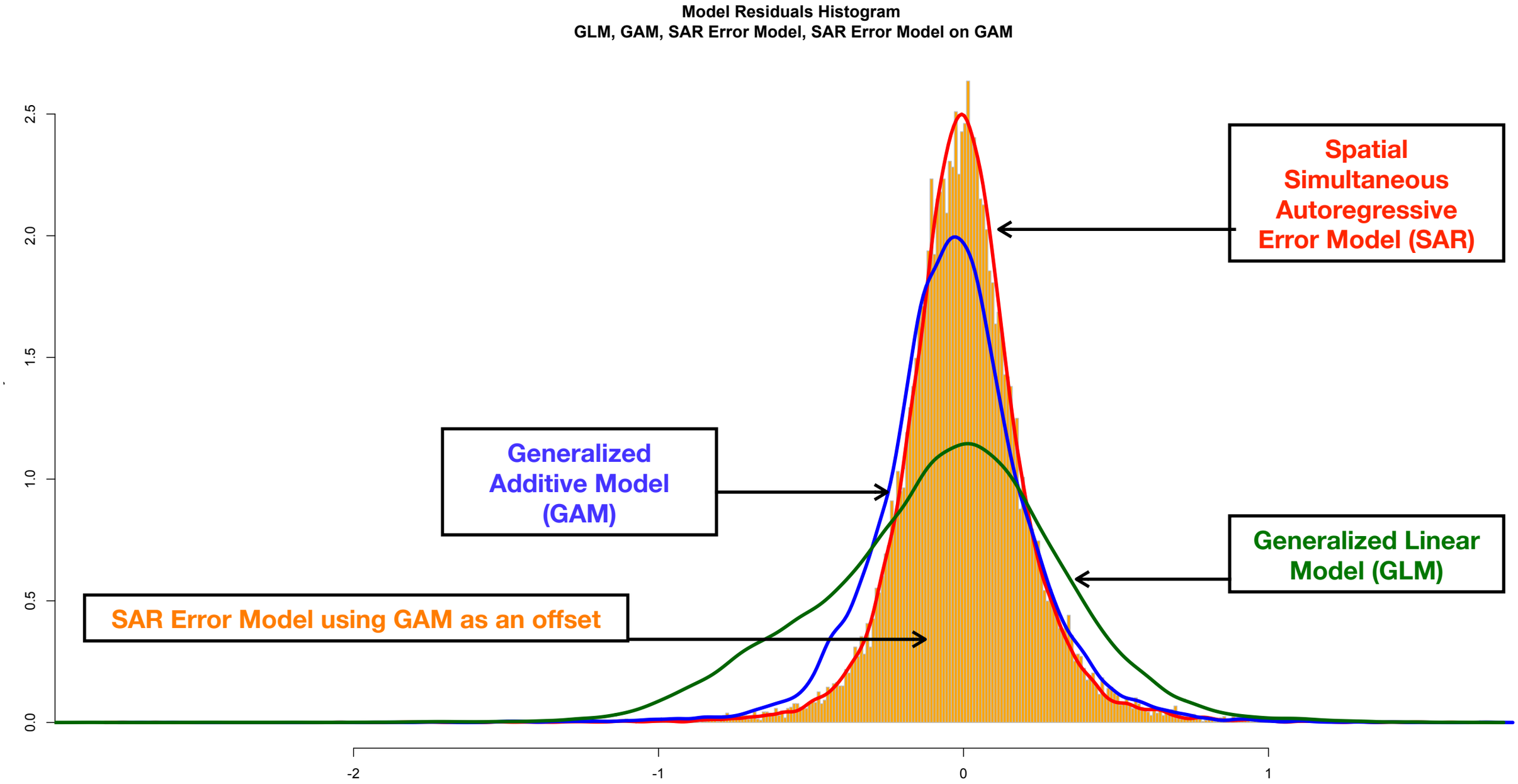
Diagnostics - Residual Mapping

Comparison GAM & SAR



Diagnostics - Residual Histograms

Comparison of GLM, GAM, Spatial SAR Error and GAM with SAR



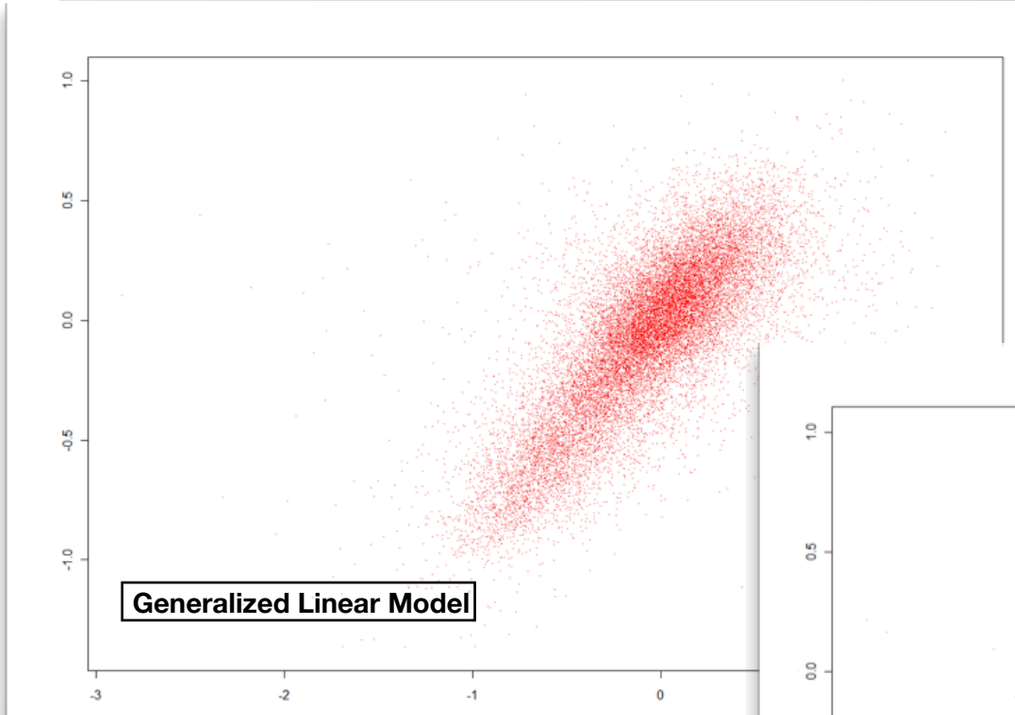
Spatial SAR Error Model shows lower dispersion and magnitude in model residuals distribution compare to GLM & GAM

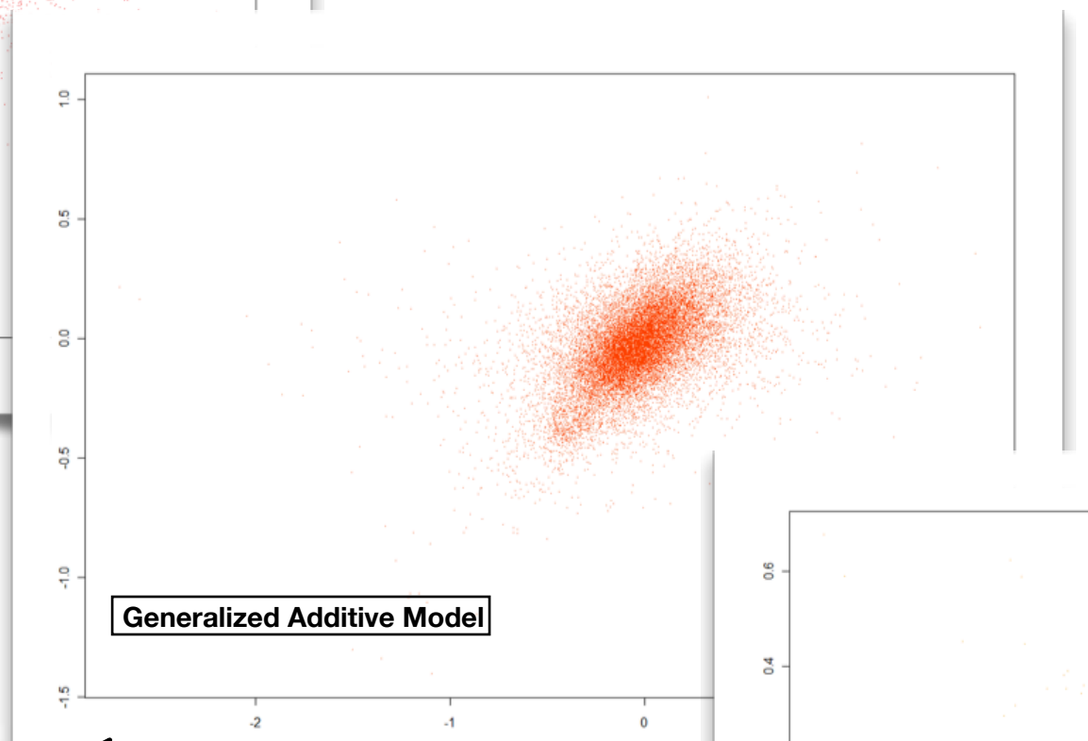
Further Diagnostics

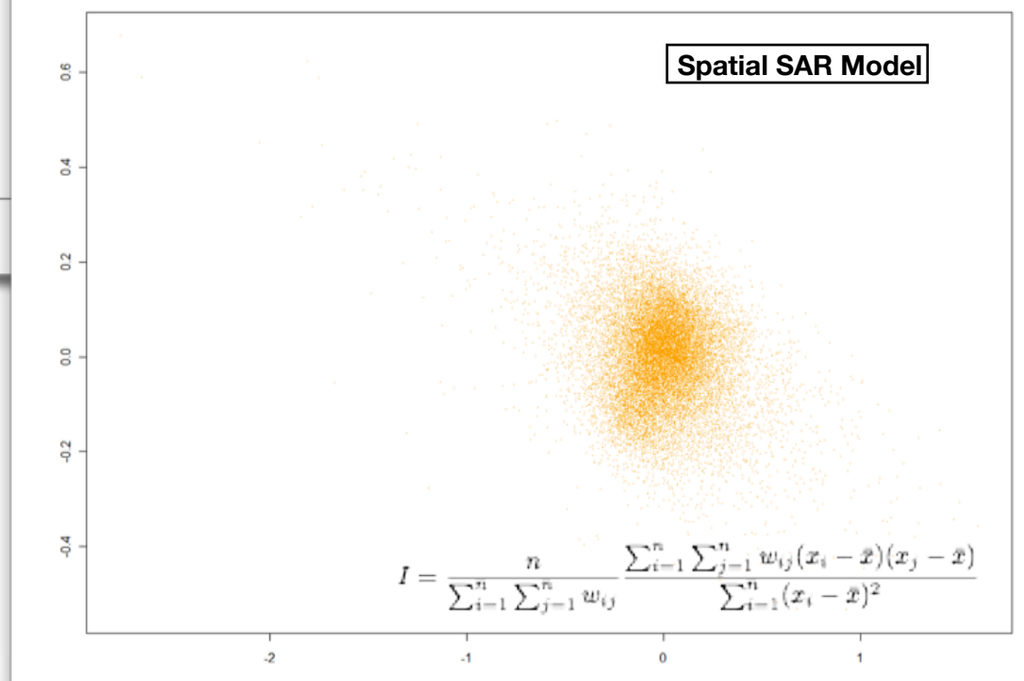
Correlations between Spatially Lagged Errors - Moran's I Statistics

I. Moran I, Measure of Strength Spatial Association among Areal Units

II. Time Series Analogous for Measuring Lagged Autocorrelation Coefficient







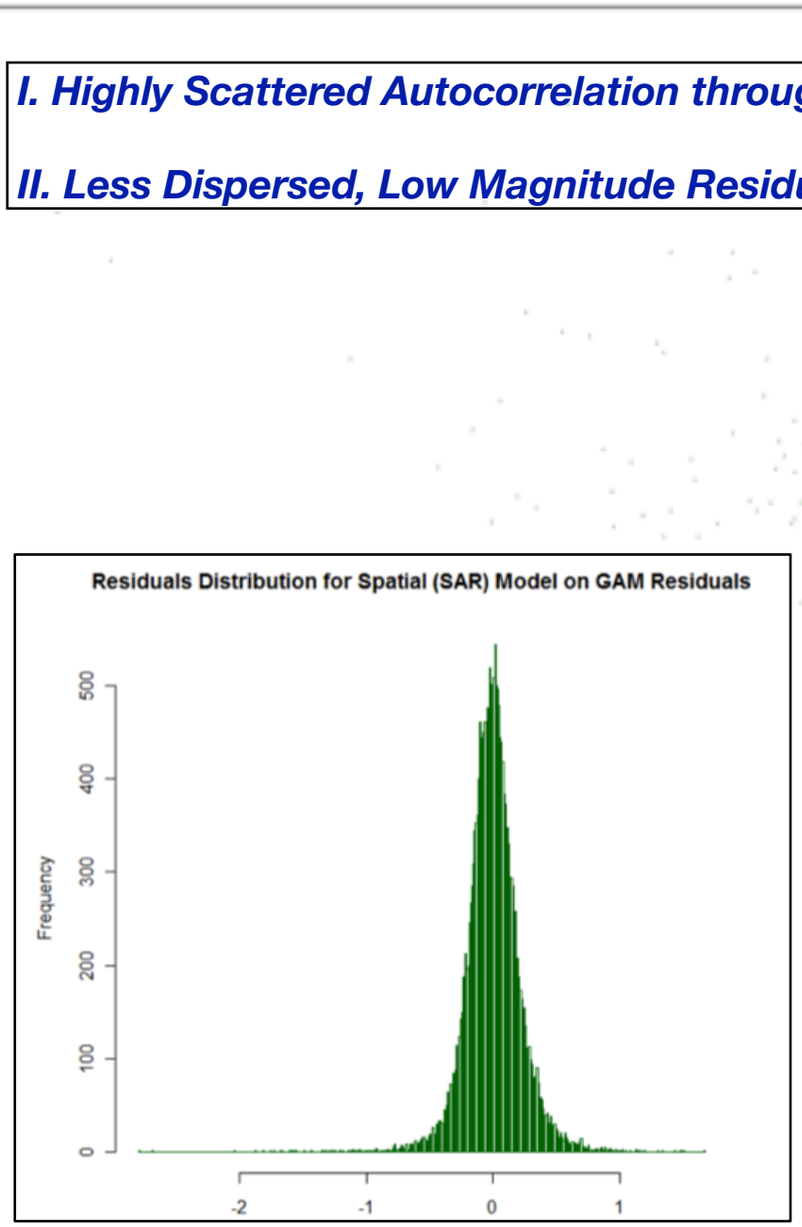
Filtering Spatial Dependence
GLM - Highly Patterned > GAM - Moderately Patterned > Spatial SAR Error Model - Least Patterned

Further Diagnostics

Correlations between Spatially Lagged Errors - Moran's I Statistics

I. Highly Scattered Autocorrelation through Moran-I Simulations

II. Less Dispersed, Low Magnitude Residuals



Spatial Simultaneous Autoregressive Error (SAR) Model Built on GAM

Residuals from SAR Error Model Built on GAM

Evolution

Location in Insurance Ratemaking & Implementation

- ***Generation I - Classical Territorial Ratemaking***
    - *Assumption: Complete Effect of location is captured in different location and demographic variables*
    - *Methods: Adding different proxy variables (Population Density, Other Geographic Variables, Different Location and Demographic Variables) in the GLM model, Credibility based approach (observed value, exposure, proximity), Kriging and Non-Geostatistical Smoothing (descriptive/ algorithmic opposed to model based)*
- ***Generation II - Latitude-Longitude in Predictive Models***
    - *Assumption: Latitude-Longitude holds significant predictive power*
    - *GLM – Use Latitude, Longitude as predictors (easting-northing effect – language, culture, food-habit). Not so promising in Insurance context.*
    - *GAM – Use a function of Latitude-Longitude as a predictor (location variables are function of latitude and longitude).*

Generation III - Spatially Correlated Observations (Insured) - Spatial Statistics Framework

- *Assumption: Unlike GLM or GAM set-up, underlying process has a spatial correlation structure that is only partially represented by GLM model*
- *Method: Filter the spatial effect to increase "correctness" in Model Estimation*
- *Consistent with GLM and GAM structure and can be built on existing GLM based Rating Tool*

Conclusion

"We're drowning in information and starving for knowledge" - Rutherford Rogers

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- ***Spatial Statistics - A Rigorous Statistical Framework For Analyzing Geographically Referenced Data***
  - *Complete Distributional Inference*
  - *Captures Predictive Variation*
- ***Computational Scope***
  - *Statistical software R (along with many well developed packages) offers extensive computational facilities and it has a high interaction capability with any standard GIS software*
  - *Entire analysis (including all graphics) in this presentation are done in R*
- ***Communicating Model Results***
  - *Extensive Visualization Techniques*
  - *Add-on to the GLM based Rating Tool*
  - *Model Results and Diagnostics are consistent with GLM*
- ***Text Book References:***
  - *"Applied Spatial Data Analysis with R" by Roger S. Bivand, Edzer J. Pebesma and V. Gómez-Rubio (UseR! Series, Springer 2008)*
  - *"Hierarchical Modeling and Analysis of Spatial Data", by Banerjee, S., Carlin, B.P. and Gelfand, A.E. (Chapman and Hall/CRC Press, 2004)*
  - *"Basic Ratemaking," Werner, G. and Modlin, C., Casualty Actuarial Society (January - 2010)*

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