GLM I
Introduction to Linear & Generalized Linear Models

Casualty Actuarial Society
Ratemaking and Product Management Seminar
March 21, 2011
New Orleans, LA

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Outline

I. Introduce our Data

II. Classical Linear Modeling

III. Generalized Linear Modeling

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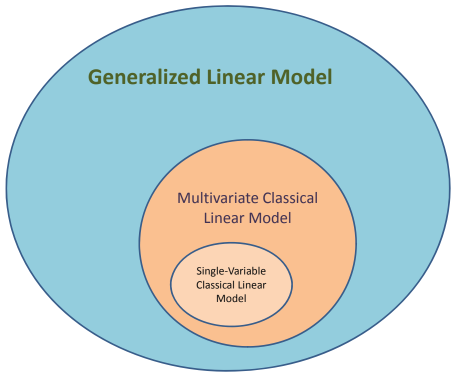

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Outline

1. **Introduce our Data**
2. Classical Linear Modeling
3. Generalized Linear Modeling

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Cracking open the data

veh_value	exposure	clm	numclaims	claimcst0	veh_body	veh_age	gender	area	agecat
1.06	0.303901437	0	0	0	HBACK	3	F	C	2
1.03	0.648870637	0	0	0	HBACK	2	F	A	4
3.26	0.569472964	0	0	0	UTE	2	F	E	2
4.14	0.317590691	0	0	0	STNWG	2	F	D	2
0.72	0.648870637	0	0	0	HBACK	4	F	C	2
2.01	0.854209446	0	0	0	HDTOP	3	M	C	4
1.6	0.854209446	0	0	0	PANVN	3	M	A	4
1.47	0.55578371	0	0	0	HBACK	2	M	B	6
0.52	0.361396304	0	0	0	HBACK	4	F	A	3
0.38	0.52019165	0	0	0	HBACK	4	F	B	4
1.38	0.854209446	0	0	0	HBACK	2	M	A	2
1.22	0.854209446	0	0	0	HBACK	3	M	C	4
1	0.492813142	0	0	0	HBACK	2	F	C	4
7.04	0.314852841	0	0	0	STNWG	1	M	A	5
1.66	0.484599589	1	1	669.5099993	SEDAN	3	M	B	6
2.35	0.391512663	0	0	0	SEDAN	2	M	C	4
1.51	0.993839836	1	1	806.6099987	SEDAN	3	F	F	4
0.76	0.539356605	1	1	401.8054514	HBACK	3	M	C	4
0.27	0.45174538	0	0	0	HBACK	4	F	D	2
0.89	0.594113621	0	0	0	HBACK	3	F	C	3
1.95	0.594113621	0	0	0	HBACK	1	M	A	1
0.39	0.536618754	0	0	0	SEDAN	4	M	C	5
3.86	0.594113621	0	0	0	STNWG	2	F	B	2
1.37	0.59137577	0	0	0	HBACK	1	F	B	1
1.3	0.999315537	0	0	0	HBACK	2	F	A	2
1.44	0.030116359	0	0	0	HBACK	2	F	C	1
1.349	0.462696783	0	0	0	HBACK	1	F	C	5
1.39	0.317590691	0	0	0	HBACK	3	F	C	2
1	0.287474333	0	0	0	STNWG	4	M	C	3
1.51	0.06844627	0	0	0	HBACK	2	F	C	2
4.45	0.594113621	0	0	0	STNWG	1	F	C	3
2.17	0.536618754	0	0	0	SEDAN	2	F	A	4
0.87	0.854209446	0	0	0	HBACK	3	M	D	3
4.09	0.848733744	0	0	0	UTE	1	M	A	2
1.31	0.405201917	0	0	0	HBACK	2	M	C	1

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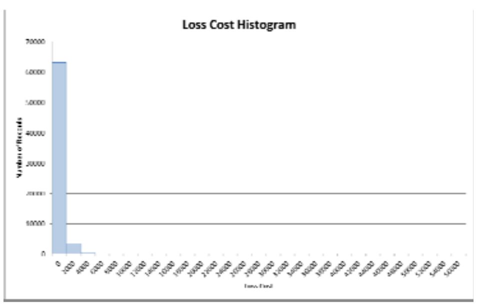
What is our target?

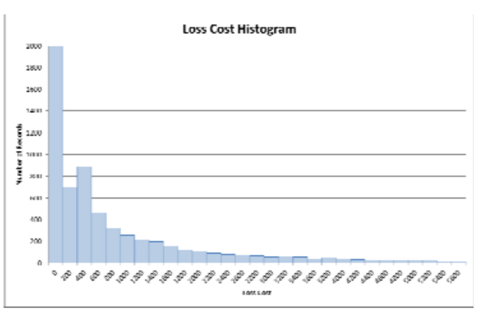
veh_value	exposure	clm	numclaims	claimcst0	veh_body	veh_age	gender	area	agecat
1.06	0.303901437	0	0	0	HBACK	3	F	C	2
1.03	0.648870637	0	0	0	HBACK	2	F	A	4
3.26	0.569472964	0	0	0	UTE	2	F	E	2
4.14	0.317590691	0	0	0	STNWG	2	F	D	2
0.72	0.648870637	0	0	0	HBACK	4	F	C	2
2.01	0.854209446	0	0	0	HDTOP	3	M	C	4
1.6	0.854209446	0	0	0	PANVN	3	M	A	4
1.47	0.55578371	0	0	0	HBACK	2	M	B	6
0.52	0.361396304	0	0	0	HBACK	4	F	A	3
0.38	0.52019165	0	0	0	HBACK	4	F	B	4
1.38	0.854209446	0	0	0	HBACK	2	M	A	2
1.22	0.854209446	0	0	0	HBACK	3	M	C	4
1	0.492813142	0	0	0	HBACK	2	F	C	4
7.04	0.314852841	0	0	0	STNWG	1	M	A	5
1.66	0.484599589	1	1	669.5099993	SEDAN	3	M	B	6
2.35	0.391512663	0	0	0	SEDAN	2	M	C	4
1.51	0.993839836	1	1	806.6099987	SEDAN	3	F	F	4
0.76	0.539356605	1	1	401.8054514	HBACK	3	M	C	4
0.27	0.45174538	0	0	0	HBACK	4	F	D	2
0.89	0.594113621	0	0	0	HBACK	3	F	C	3
1.95	0.594113621	0	0	0	HBACK	1	M	A	1
0.39	0.536618754	0	0	0	SEDAN	4	M	C	5
3.86	0.594113621	0	0	0	STNWG	2	F	B	2
1.37	0.59137577	0	0	0	HBACK	1	F	B	1
1.3	0.999315537	0	0	0	HBACK	2	F	A	2
1.44	0.030116359	0	0	0	HBACK	2	F	C	1
1.349	0.462696783	0	0	0	HBACK	1	F	C	5
1.39	0.317590691	0	0	0	HBACK	3	F	C	2
1	0.287474333	0	0	0	STNWG	4	M	C	3
1.51	0.06844627	0	0	0	HBACK	2	F	C	2
4.45	0.594113621	0	0	0	STNWG	1	F	C	3
2.17	0.536618754	0	0	0	SEDAN	2	F	A	4
0.87	0.854209446	0	0	0	HBACK	3	M	D	3
4.09	0.848733744	0	0	0	UTE	1	M	A	2

Number of Records =	67,856
Number of Claims =	4,937
Average Frequency =	0.0728
Average Claim Size =	$ 1,886.69
Max Claim Size =	$ 55,922.13
Min Claim Size =	$ 200.00

Exploring the Data



Exploring the Data



Frequency & Severity Component Modeling

veh_value	exposure	clm	numclaims	claimcst0	veh_body	veh_age	gender	area	agecat
1.06	0.303901437	0	0	0	HBACK	3	F	C	2
1.03	0.648870637	0	0	0	HBACK	2	F	A	4
3.26	0.569472964	0	0	0	UTE	2	F	E	2
4.14	0.317590691	0	0	0	STNWG	2	F	D	2
0.72	0.648870637	0	0	0	HBACK	4	F	C	2
2.01	0.854209446	0	0	0	HDTOP	3	M	C	4
1.6	0.854209446	0	0	0	PANVN	3	M	A	4
1.47	0.55578371	0	0	0	HBACK	2	M	B	6
0.52	0.361396304	0	0	0	HBACK	4	F	A	3
0.38	0.52019165	0	0	0	HBACK	4	F	B	4
1.38	0.854209446	0	0	0	HBACK	2	M	A	2
1.22	0.854209446	0	0	0	HBACK	3	M	C	4
1	0.492813142	0	0	0	HBACK	2	F	C	4
7.04	0.314852841	0	0	0	STNWG	1	M	A	5
1.66	0.484599589	1	1	669.5099993	SEDAN	3	M	B	6
2.35	0.391512663	0	0	0	SEDAN	2	M	C	4
1.51	0.993839836	1	1	806.6099987	SEDAN	3	F	F	4
0.76	0.539356605	1	1	401.8054514	HBACK	3	M	C	4
0.27	0.45174538	0	0	0	HBACK	4	F	D	2
0.89	0.594113621	0	0	0	HBACK	3	F	C	3
1.95	0.594113621	0	0	0	HBACK	1	M	A	1
0.39	0.536618754	0	0	0	SEDAN	4	M	C	5
3.86	0.594113621	0	0	0	STNWG	2	F	B	2
1.37	0.59137577	0	0	0	HBACK	1	F	B	1
1.3	0.999315537	0	0	0	HBACK	2	F	A	2
1.44	0.030116359	0	0	0	HBACK	2	F	C	1
1.349	0.462696783	0	0	0	HBACK	1	F	C	5
1.39	0.317590691	0	0	0	HBACK	3	F	C	2
1	0.287474333	0	0	0	STNWG	4	M	C	3
1.51	0.06844627	0	0	0	HBACK	2	F	C	2
4.45	0.594113621	0	0	0	STNWG	1	F	C	3
2.17	0.536618754	0	0	0	SEDAN	2	F	A	4
0.87	0.854209446	0	0	0	HBACK	3	M	D	3
4.09	0.848733744	0	0	0	UTE	1	M	A	2
1.31	0.405201917	0	0	0	HBACK	2	M	C	1

<u>Frequency Model</u>: Predicts Likelihood to file a claim.

<u>Severity Model</u>: Predicts size of claim given a claim is filed

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Our New Severity Dataset:

veh_value	exposure	clm	numclaims	claimcst0	veh_body	veh_age	gender	area	agecat
1.66	0.484599589	1	1	669.5099993	SEDAN	3	M	B	6
1.51	0.993839836	1	1	806.6099987	SEDAN	3	F	F	4
0.76	0.539356605	1	1	401.8054514	HBACK	3	M	C	4
1.89	0.654346338	1	2	1811.709997	STNWG	3	M	F	2
4.06	0.851471595	1	1	5434.439987	STNWG	2	M	F	3
1.39	0.317590691	1	1	865.789999	HBACK	3	F	A	4
2.66	0.317590691	1	1	1105.769999	STNWG	1	F	F	5
0.5	0.071184121	1	1	200	HBACK	4	F	A	5
1.16	0.596851472	1	2	739.2299995	STNWG	4	F	B	2
3.56	0.906228611	1	1	3230.599999	MCARA	3	M	F	4
2.15	0.654346338	1	1	200	SEDAN	3	F	A	5
3.03	0.90349076	1	1	200	TRUCK	1	M	C	1
2.41	0.339493498	1	1	407.8399997	STNWG	3	M	E	1
1.72	0.616016427	1	1	1619.829998	STNWG	2	F	A	6
1.92	0.616016427	1	1	1010.919998	STNWG	4	M	A	5
2.649	0.257357974	1	1	9424.349976	TRUCK	1	M	A	1
1.86	0.616016427	1	1	238.6400003	SEDAN	2	F	C	3
1.54	0.616016427	1	1	200	STNWG	2	F	B	3
1.88	0.750171116	1	1	369.1799998	PANVN	3	M	D	3
0.89	0.684462697	1	1	353.77	SEDAN	3	F	D	3
4.46	0.191649555	1	1	989.9199982	COUPE	2	M	B	2
4.32	0.646132786	1	1	736.8499985	STNWG	2	M	A	5
5.59	0.851471595	1	1	369.1499996	STNWG	1	M	C	3
1.44	0.665297741	1	1	500	UTE	4	M	F	2
2.81	0.851471595	1	1	353.77	STNWG	1	M	C	3
3	0.613278576	1	1	6372.029999	STNWG	3	F	C	2
2.29	0.353182752	1	1	500	STNWG	4	F	F	4
0.39	0.925393566	1	1	1379.039997	SEDAN	4	F	C	6
1.29	0.925393566	1	1	200	SEDAN	3	M	D	3
1.13	0.377823409	1	1	937.5599957	SEDAN	4	F	B	1
0	0.999315537	1	2	2724.339996	STNWG	4	M	A	3
2.41	0.750171116	1	1	12879.65997	HBACK	1	F	B	3
3.47	0.246406571	1	1	937.2799988	SEDAN	1	M	B	3
3.14	0.851471595	1	2	19142.13268	TRUCK	3	M	C	1
1.98	0.911704312	1	1	2659.369995	STNWG	4	M	C	6

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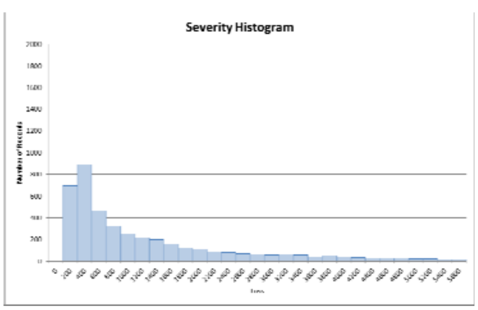
Exploring our new dataset



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Our Severity Dataset reorganized:

Predictor Variables						Target Variable
veh_value	veh_body	veh_age	gender	area	agecat	Severity
1.66	SEDAN	3	M	B	6	669.5099993
1.51	SEDAN	3	F	F	4	806.6099987
0.76	HBACK	3	M	C	4	401.8054514
1.89	STNWG	3	M	F	2	905.8549986
4.06	STNWG	2	M	F	3	5434.439987
1.39	HBACK	3	F	A	4	865.789999
2.66	STNWG	1	F	F	5	1105.769999
0.5	HBACK	4	F	A	5	200
1.16	STNWG	4	F	B	2	369.6149998
3.56	MCARA	3	M	F	4	3230.599999
2.15	SEDAN	3	F	A	5	200
3.03	TRUCK	1	M	C	1	200
2.41	STNWG	3	M	E	1	407.8399997
1.72	STNWG	2	F	A	6	1619.829998
1.92	STNWG	4	M	A	5	1010.919998
2.649	TRUCK	1	M	A	1	9424.349976
1.86	SEDAN	2	F	C	3	238.6400003
1.54	STNWG	2	F	B	3	200
1.88	PANVN	3	M	D	3	369.1799998
0.89	SEDAN	3	F	D	3	353.77
4.46	COUPE	2	M	B	2	989.9199982
4.32	STNWG	2	M	A	5	736.8499985
5.59	STNWG	1	M	C	3	369.1499996
1.44	UTE	4	M	F	2	500
2.81	STNWG	1	M	C	3	353.77
3	STNWG	3	F	C	2	6372.029999
2.29	STNWG	4	F	F	4	500
0.39	SEDAN	4	F	C	6	1379.039997
1.29	SEDAN	3	M	D	3	200
1.13	SEDAN	4	F	B	1	937.5599957
0	STNWG	4	M	A	3	1362.169998

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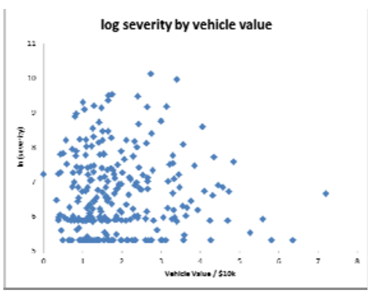
Variable Exploration (continuous)



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Variable Exploration (categorical)

Veh_Body	Claims	Av Severity
BUS	10	$ 1,336
CONVT	3	$ 2,296
COUPE	75	$ 2,503
HBACK	1330	$ 1,947
HDTOP	136	$ 2,168
MCARA	15	$ 712
MIBUS	45	$ 2,580
PANVN	68	$ 1,958
RDSTR	3	$ 456
SEDAN	**1598**	**$ 1,678**
STNWG	1248	$ 1,894
TRUCK	130	$ 2,458
UTE	276	$ 2,164

Veh_Age (1 is newest)	Claims	Av Severity
1	876	$ 1,775
2	1354	$ 1,836
3	**1446**	**$ 1,880**
4	1261	$ 2,026

Gender	Claims	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

Area	Claims	Av Severity
A	1181	$ 1,754
B	1021	$ 1,758
C	**1493**	**$ 1,919**
D	524	$ 1,739
E	413	$ 2,104
F	305	$ 2,629

Agecat (1 is youngest)	Claims	Av Severity
1	525	$ 2,490
2	1000	$ 1,985
3	**1189**	**$ 1,793**
4	1185	$ 1,810
5	648	$ 1,638
6	390	$ 1,753

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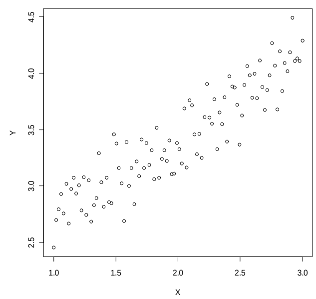
What is Linear Modeling?



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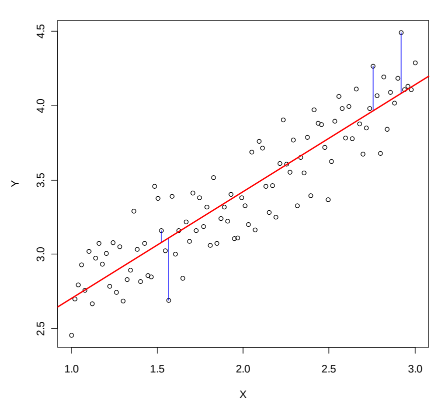

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Classical Linear Model; Moving Parts

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

1. Y_i is the (observed) value of response variable in the i^{th} trial
2. β_0 and β_1 are parameters
3. X_i is the (observed) value of the predictor variable in the i^{th} trial
4. ε_i is a random error term with mean 0 and variance σ^2
5. i= 1, 2, . . ., n

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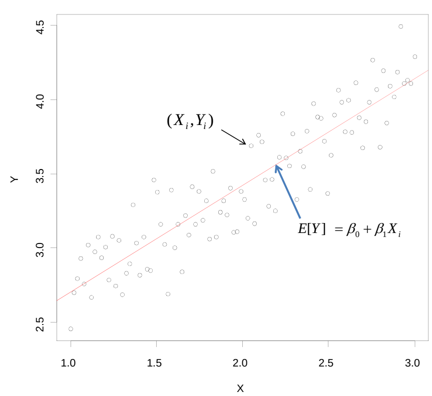

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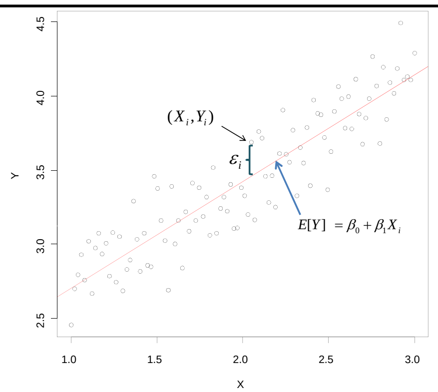

$$Y_i = E[Y_i] + \varepsilon_i$$

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Multivariate Classical Linear Model

$$E[Y_i] = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} \ldots$$

Or, in matrix notation:

$$E[Y] = \beta X$$

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How does this pertain to insurance modeling?

Gender	Weight	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

This categorical variable requires a two-parameter model.

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How does this pertain to insurance modeling?

Gender	Weight	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

This categorical variable requires a two-parameter model.

Predicted Severity

Binary Variable:
1 for male, 0 for female

$$E[Y] = \beta_0 + \beta_1 X_1$$

Base (female) Severity

Additional Severity for being male

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Gender		
	Weight	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

This categorical variable requires a two-parameter model.

$$E[Y] = \beta_0 + \beta_1 X_1$$

Parameter Estimates

β_0 = 1733
β_1 = 360

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Let's add another variable

Gender		
	Weight	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

Area		
	Weight	Av Severity
A	1181	$ 1,754
B	1021	$ 1,758
C	**1493**	**$ 1,919**
D	524	$ 1,739
E	413	$ 2,104
F	305	$ 2,629

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Let's add another variable

Gender		
	Weight	Av Severity
M	2105	$ 2,093
F	**2832**	**$ 1,733**

Area		
	Weight	Av Severity
A	1181	$ 1,754
B	1021	$ 1,758
C	**1493**	**$ 1,919**
D	524	$ 1,739
E	413	$ 2,104
F	305	$ 2,629

$$E[Y] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6$$

X1: Binary variable (Male = 1, Female = 0)
X2: Binary variable (Area A = 1, else = 0)
X3: Binary variable (Area B = 1, else = 0)
X4: Binary variable (Area D = 1, else = 0)
X5: Binary variable (Area E = 1, else = 0)
X6: Binary variable (Area F = 1, else = 0)

β0: Base severity (female from Area C)
β1: Add'l severity from being male
β2: Add'l severity from Area A
β3: Add'l severity from Area B
β4: Add'l severity from Area D
β5: Add'l severity from Area E
β6: Add'l severity from Area F

Example: Female from Area F $E[Y] = \beta_0 + \beta_6$

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Modeling Software Output

[GLM fit: Identity Link Function, Normal Error Structure]

Parameter Number	Name	Value	Standard Error	Standard Error (%)	Weight	Weight (%)
1	Mean	1,769.64	99.14477	5.6	4,937	100
-	gender (F)				2,832	57.4
2	gender (M)	361.864	100.18706	27.7	2,105	42.6
3	area (A)	-174.419	135.53147	77.7	1,181	23.9
4	area (B)	-170.408	141.33498	82.9	1,021	20.7
-	area (C)				1,493	30.2
5	area (D)	-174.622	176.69103	101.2	524	10.6
6	area (E)	173.704	193.48309	111.4	413	8.4
7	area (F)	707.8558	218.65201	30.9	305	6.2

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How good is our fit?

Claims

Area	Gender M	Gender F
A	519	662
B	449	572
C	618	875
D	208	316
E	183	230
F	128	177

Actual Severity

Area	Gender M	Gender F
A	$ 1,899	$ 1,641
B	$ 1,939	$ 1,616
C	$ 2,100	**$ 1,792**
D	$ 1,666	$ 1,787
E	$ 2,579	$ 1,726
F	$ 3,386	$ 2,082

Predicted Severity

Area	Gender M	Gender F
A	$ 1,957	$ 1,595
B	$ 1,961	$ 1,599
C	$ 2,132	**$ 1,770**
D	$ 1,957	$ 1,595
E	$ 2,305	$ 1,943
F	$ 2,839	$ 2,477

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Classical Linear Model; Assumptions

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

$$Y_i = \mu_{Y_i} + \varepsilon_i$$

1. Y_i is the sum of a constant term and a random term
2. $E\{Y_i\} = \beta_0 + \beta_1 X_i$ = "Linear Predictor"
   1. This implies a linear relationship between X_i and Y_i
3. The error terms ε_i are random variables which;
   1. Are independent
   2. Are normally distributed
   3. Have constant variance, σ^2.
4. Therefore, the responses, Y_i are also independent normally distributed random variables with constant variance, σ^2.

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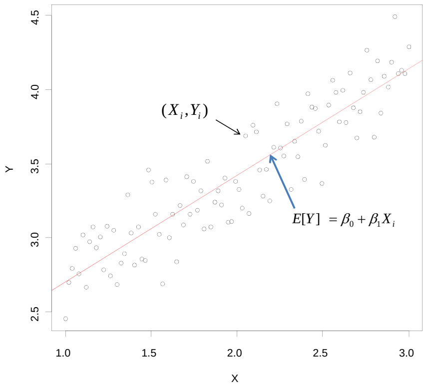

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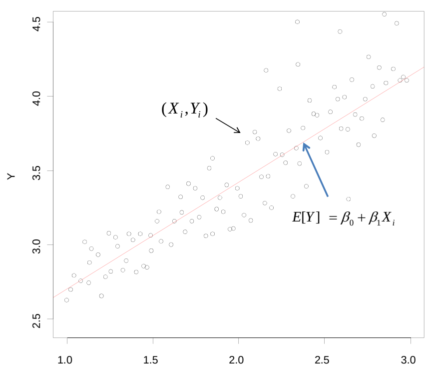
A slightly different scatterplot



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Outline

I. Introduce our Data

II. Classical Linear Modeling

III. Generalized Linear Modeling

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Linear vs. Generalized Linear Model

Assumption	Linear Regression Model	Generalized Linear Model
Relationship between X and Y	Y is a linear combination of X	Y is a function of a linear combination of X
Distribution of Y	Normal	Any distribution from the Exponential family
Variance of Y	Constant	Function of the mean

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Flexibility of Relationship between X & Y

- Recall that the Multiple Linear Regression Model can be written as:

$$E[\,Y_i\,] = X_i\beta$$

(Or… $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_n X_{in}$)

35

Flexibility of Relationship between X & Y

- Recall that the Multiple Linear Regression Model can be written as:

$$E[\,Y_i\,] = X_i\beta$$

(Or… $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_n X_{in}$)

- Generalized Linear Models assume a more general relationship between X and Y:

$$E[\,Y_i\,] = h(X_i\beta)$$

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Flexibility of Relationship between X & Y

- Generalized Linear Models assume a more general relationship between X and Y:

$$g(Y_i) = X_i\beta$$

Link Function

Examples:

- $Y_i = X_i\beta$ (Identity Link)
- $\ln(Y_i) = X_i\beta$ (Log Link)
- $\ln(Y_i/(1-Y_i)) = X_i\beta$ (Logit Link)
- $1/Y_i = X_i\beta$ (Reciprocal Link)

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Identity Link vs. Log Link

Identity Link:

– $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_n X_{in}$

Log Link:

– $\ln(Y_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_n X_{in}$

– $Y_i = \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_n X_{in})$

– $Y_i = \exp(\beta_0) \cdot \exp(\beta_1 X_{i1}) \cdot \exp(\beta_2 X_{i2}) \cdot \ldots \cdot \exp(\beta_n X_{in})$

Identity Link produces additive factors; Log Link produces multiplicative factors

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Why choose log link?

- Convenience: match structure of rating plan/scorecard
- Intuition: Do rating variables have additive or multiplicative effects on severity?
- Evaluation: We can test the appropriateness of the link function

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Model output: normal error / log link

[GLM fit: Log Link Function, Normal Error Structure]

Parameter Number	Name	Value	Standard Error	Standard Error (%)	Weight	Weight (%)	Exp(Value)
1	Mean	7.47	0.0539	0.7	4,937	100	1,749.90
-	gender (F)				2,832	57.4	
2	gender (M)	0.2051	0.05196	25.3	2,105	42.6	1.2277
3	area (A)	-0.0959	0.07417	77.3	1,181	23.9	0.9085
4	area (B)	-0.0918	0.0774	84.3	1,021	20.7	0.9123
-	area (C)				1,493	30.2	
5	area (D)	-0.1069	0.09972	93.3	524	10.6	0.8986
6	area (E)	0.098	0.09284	94.7	413	8.4	1.103
7	area (F)	0.3303	0.08776	26.6	305	6.2	1.3914

$E[Y] = \exp(\beta_1) \cdot \exp(\beta_2 X_2) \cdot \exp(\beta_3 X_3) \cdot \ldots \cdot \exp(\beta_7 X_7)$

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Model output: normal error / log link

[GLM fit: Log Link Function, Normal Error Structure]

Parameter Number	Name	Value	Standard Error	Standard Error (%)	Weight	Weight (%)	Exp(Value)
1	Mean	7.47	0.0539	0.7	4,937	100	1,749.90
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6	area (E)	0.098	0.09284	94.7	413	8.4	1.103
7	area (F)	0.3303	0.08776	26.6	305	6.2	1.3914

<u>For Gender=F, Area=C</u>

$E[Y] = \exp(\beta_1) \cdot \exp(\beta_2 \cdot 0) \cdot \exp(\beta_3 \cdot 0) \cdot \ldots \cdot \exp(\beta_7 \cdot 0)$

$= \exp(7.47) = 1{,}749.90$

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Model output: normal error / log link

[GLM fit: Log Link Function, Normal Error Structure]

Parameter Number	Name	Value	Standard Error	Standard Error (%)	Weight	Weight (%)	Exp(Value)
1	Mean	7.47	0.0539	0.7	4,937	100	1,749.90
-	~~gender (F)~~				2,832	57.4	
2	gender (M)	0.2051	0.05196	25.3	2,105	42.6	1.2277
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6	area (E)	0.098	0.09284	94.7	413	8.4	1.103
7	area (F)	0.3303	0.08776	26.6	305	6.2	1.3914

<u>For Gender=M, Area=A</u>

$E[Y] = \exp(\beta_1) \cdot \exp(\beta_2 \cdot 1) \cdot \exp(\beta_3 \cdot 1) \cdot \ldots \cdot \exp(\beta_7 \cdot 0)$

$= \exp(7.47) \cdot \exp(0.2051) \cdot \exp(-0.0959)$

$= 1{,}749.90 \cdot 1.2277 \cdot .9085 = 1{,}951.78$

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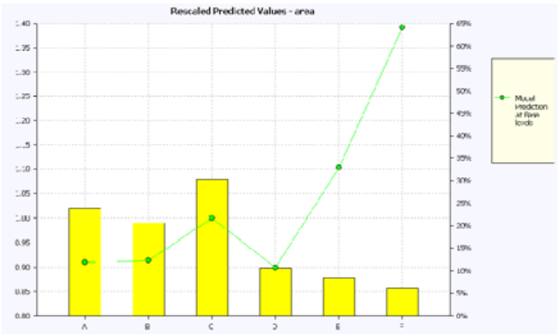

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Do estimates match the data?

Actual Severity

Area	Gender M	Gender F
A	$ 1,899	$ 1,641
B	$ 1,939	$ 1,616
C	$ 2,100	$ 1,792
D	$ 1,666	$ 1,787
E	$ 2,579	$ 1,726
F	$ 3,386	$ 2,082

Predicted Severity (additive)

Area	Gender M	Gender F
A	$ 1,957	$ 1,595
B	$ 1,961	$ 1,599
C	$ 2,132	$ 1,770
D	$ 1,957	$ 1,595
E	$ 2,305	$ 1,943
F	$ 2,839	$ 2,477

Predicted Severity (multiplicative)

Area	Gender M	Gender F
A	$ 1,952	$ 1,590
B	$ 1,960	$ 1,596
C	$ 2,148	$ 1,750
D	$ 1,931	$ 1,573
E	$ 2,370	$ 1,930
F	$ 2,989	$ 2,435

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Linear vs. Generalized Linear Model

Assumption	Linear Regression Model	Generalized Linear Model
Relationship between X and Y	Y is a linear combination of X	Y is a function of a linear combination of X
Distribution of Y	Normal	Any distribution from the Exponential family
Variance of Y	Constant	Function of the mean

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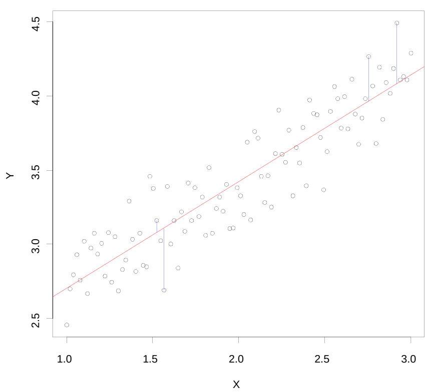
Flexibility of Distribution of Y

- Least-squares estimation implicitly assumes observations come from normal distribution

46



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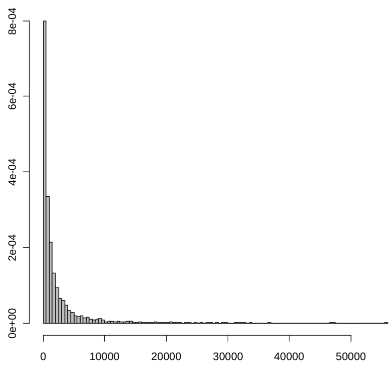
Histogram of loss



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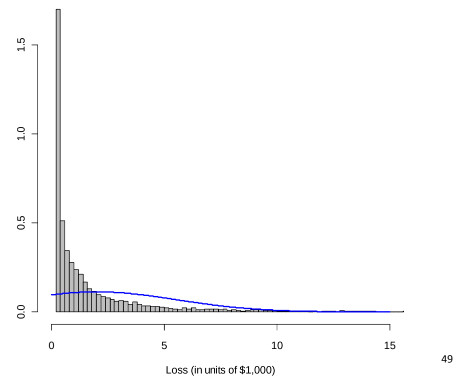
Histogram of Loss with normal distribution



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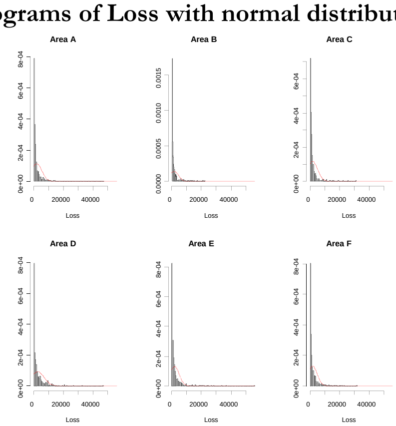
Histograms of Loss with normal distribution



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Flexibility of Distribution of Y

- Least-squares estimation implicitly assumes observations come from normal distribution
- Problems with normal distribution assumption
  - Severity distributions usually skewed to right
  - Higher mean of Y associated with higher variance
  - Values of response may be restricted to positive

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Exponential Family of Distributions

- In a GLM, Y_i may be distributed according to any member of the Exponential family of distributions
- Two Key Features of the Exponential Family:
  - The distribution is completely specified in terms of its mean and variance
  - The variance of Y_i is a function of the mean
- Familiar Examples: Normal, Poisson, Gamma, Inverse Gaussian

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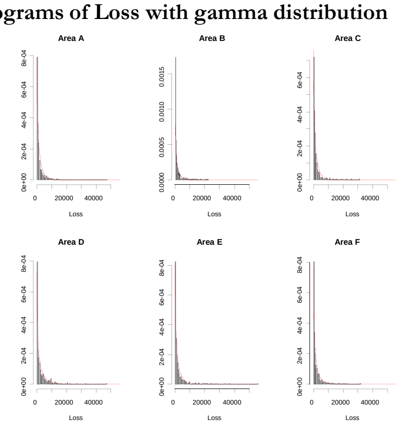
Histograms of Loss with gamma distribution



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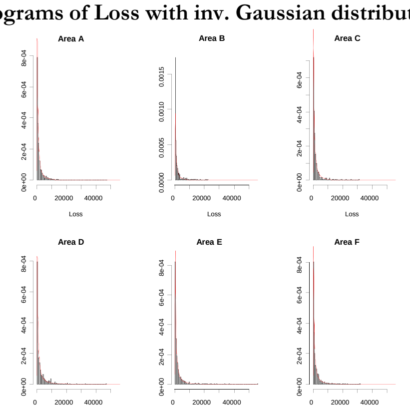
Histograms of Loss with inv. Gaussian distributions



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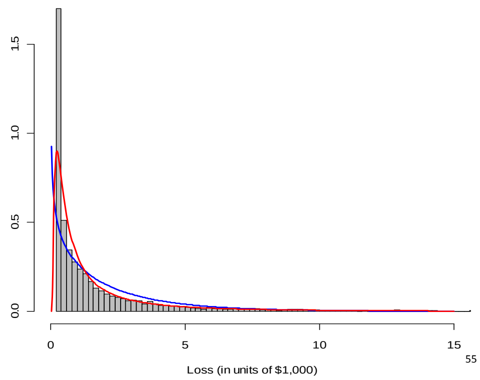
Histogram of Loss with gamma and inv Gaussian



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Least Squares vs. Maximum Likelihood

- For each observation (X_i, Y_i), consider the probability of Y_i based on assumed distribution.
- Further, consider the product of the n probabilities.
- The estimators (β) are those values that maximize the product of the n probabilities.
- (If a normal distribution is assumed, maximum likelihood is equivalent to minimizing sum of squared errors.)

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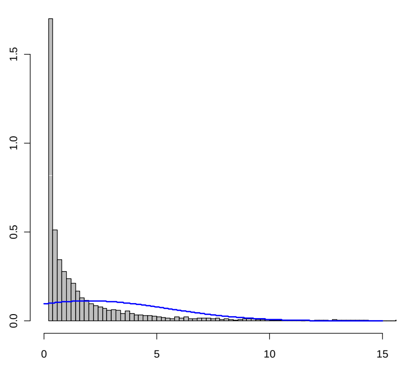
Histogram of Loss with normal distribution



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Linear vs. Generalized Linear Model

Assumption	Linear Regression Model	Generalized Linear Model
Relationship between X and Y	Y is a linear combination of X	Y is a function of a linear combination of X
Distribution of Y	Normal	Any distribution from the Exponential family
Variance of Y	Constant	Function of the mean

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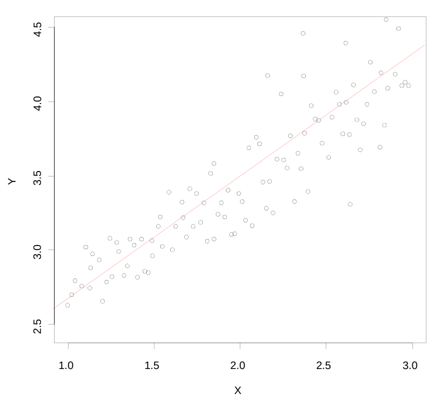

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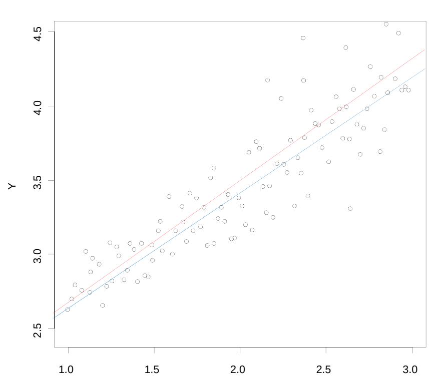

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Flexibility of Variance of Y

- The variance of Y_i is allowed to vary with the expected value of Y_i (μ)
- Variance functions link the variability of Y_i to the expected value of Y_i (μ)

Distribution of Y	Variance Function
Normal	1 (variance is constant across cells)
Poisson	μ (variance is proportional to mean)
Gamma	μ^2 (CV is constant across cells)
Inverse Gaussian (Normal)	μ^3
Binomial	$\mu(1-\mu)$
More General Case	μ^p (Tweedie if $p < 0$, $1 < p < 2$, $p > 2$)

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Error structure Diagnostics

- Deviance residuals against fitted value
  - *Deviance:* in a GLM, more weight given to differences in fitted vs. actual when variance function is small
  - *Deviance residual*: square root of an observation's contribution to total deviance
  - Plotting *deviance residual* against fitted value can highlight problems with error structure assumption
- Histogram of deviance residuals

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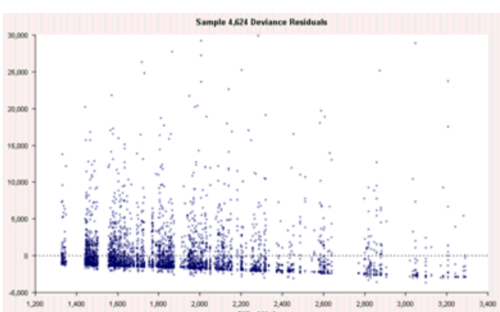
Error structure diagnostics: Normal



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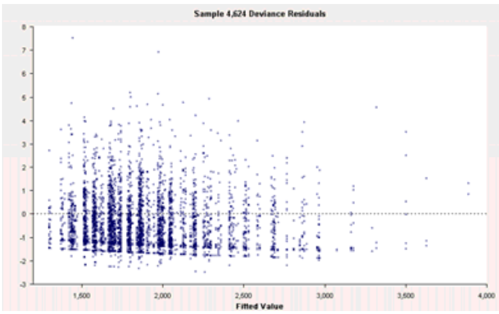
Error structure diagnostics: Gamma



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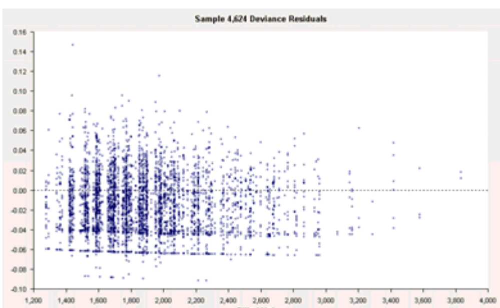
Error structure diagnostics: Inv. Gaussian



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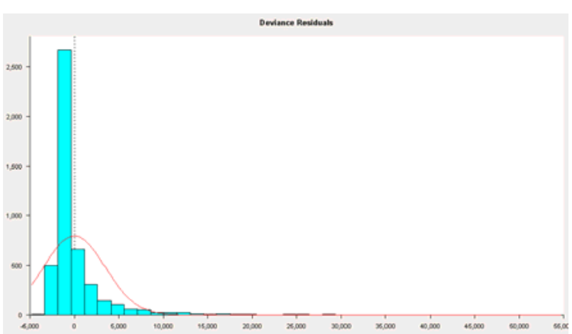
Error structure diagnostics: Normal



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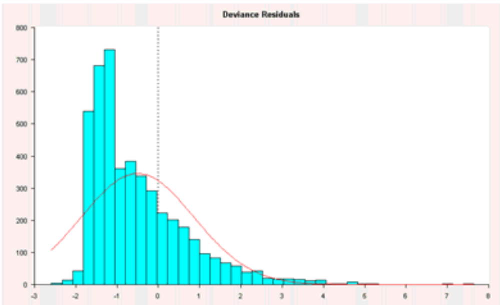
Error structure diagnostics: Gamma



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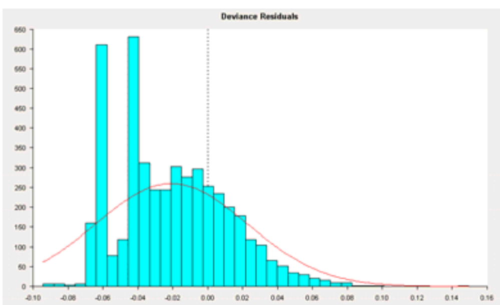
Error structure diagnostics: Inv. Gaussian



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Model comparison: normal vs. inverse Gaussian

		[Log Link, Normal Error Structure]			[Log Link, Inv Gaussian Error Structure]		
Parameter	Name	Value	Standard Error	Exp(Value)	Value	Standard Error	Exp(Value)
1	Mean	7.47	0.0539	1,749.90	7.49	0.0507	1,783.61
-	gender (F)						
2	gender (M)	0.2051	0.05196	1.2277	0.1711	0.05212	1.1867
3	area (A)	-0.0959	0.07417	0.9085	-0.0934	0.06874	0.9108
4	area (B)	-0.0918	0.0774	0.9123	-0.0941	0.07155	0.9102
-	area (C)						
5	area (D)	-0.1069	0.09972	0.8986	-0.0783	0.08922	0.9247
6	area (E)	0.098	0.09284	1.103	0.0664	0.10333	1.0687
7	area (F)	0.3303	0.08776	1.3914	0.2866	0.12842	1.3319

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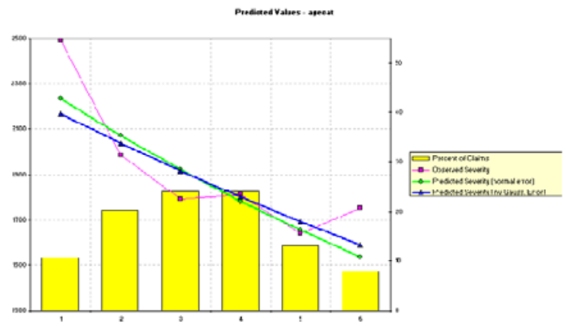
Parameters: Normal and Inv. Gaussian Error



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Common choices for some model types

Target	Link Function	Error
Claim Frequency	log	Poisson
Claim Severity	log	gamma
Loss Costs	log	Tweedie
Probability of Renewal	logit	binomial

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Further modeling

- Explore significance of other variables
- Group levels on our chosen variables
- Add interactions
- (see GLM II)

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References/Resources

De Jong, P., and Heller, G.Z. 2008.*Generalized Linear Models for Insurance Data.* Cambridge University Press

Anderson, D., Feldblum, S., Modlin, C., Schirmacher, D., Schirmacher, E., Thandi, N. 2007. *A Practitioner's Guide to Generalized Linear Models.* CAS Discussion Paper Program

Hardin, J. and Hilbe, J. 2001. *Generalized Linear Models and Extensions.* College Station, Texas: Stata Press

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