# Estimate Attrition Using Survival Analysis

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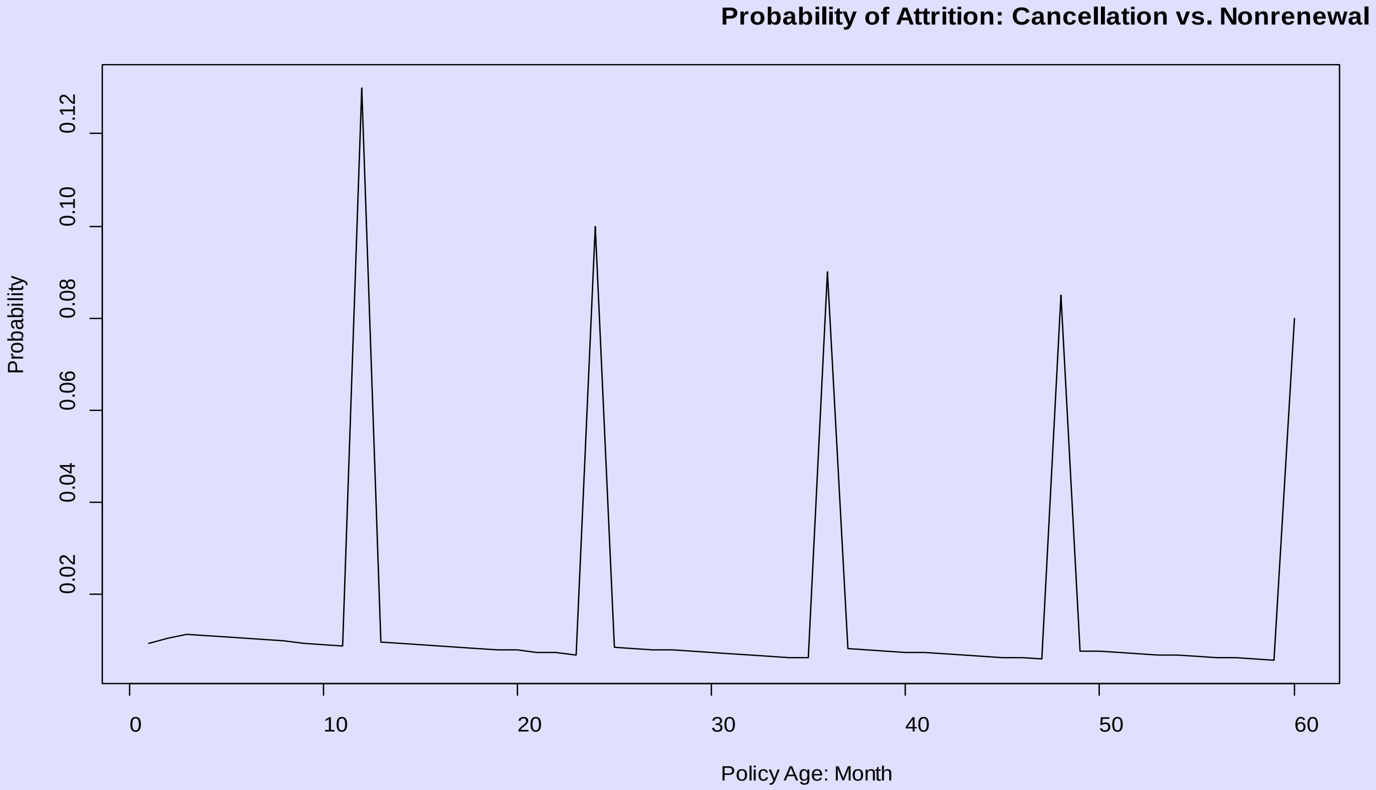
# Agenda

- Introduction
- Survival Analysis
- Cox Proportional Hazard Model
- A case study
- Q&A

# Introduction

# Two Ways of Attrition

- Mid-term cancellation
- End-of-term nonrenewal



# Snapshot View of Retention/Attrition

- If there were 10,000 inforced policies at 12/31/2009, how many of them were still with the company at 12/31/2010?
- Variable of interest: yes or no
- Do not separate cancellation and nonrenewal.
- Static view

# Dynamic View of Retention/Attrition

- If there were 10,000 inforced policies at 12/31/2009, how many of them left by cancelation and non-renewal, and when they left?
- Variable of interest: t (time of attrition)
- Cancellation and non-renewal occurs sequentially and dynamically.
- Time-varying variables (Unemployment, GDP change, Premium Change …) impact retention.

# Why Survival Analysis?

- Better estimation of life time value: not just whether a policy will leave, but when it will leave.
- Estimate cancellation and non-renewal sequentially and simultaneously.
- Measure the impacts of time-variant macroeconomic variables on attrition by incorporating monthly macroeconomic data in the regression.

# Survival Analysis

# What is Survival Analysis?

- Another name for *time to event* analysis
- Statistical methods for analyzing survival data.
- Primarily developed in the medical and biological sciences (death or failure time analysis)
- Widely used in the social and economic sciences, as well as in Insurance (longevity, time to claim analysis).

# What is Survival Time?

- Refers to a variable *t* which measures the time from a particular starting time (e.g., time initiated the treatment) to a particular endpoint of interest (e.g., attaining certain functional abilities).
- Examples:

  *Insurance Policy*: Started at Jan2005, terminated at Aug2008.

  *Products*: Bought at Dec2006, failed at Feb2007.

# Censoring

- Occurs when the value of a measurement or observation is only partially known.
- Left Censoring:

  Example: Subject's lifetime is known to be less than a certain duration.
- Right Censoring:

  Example: Subjects still active when they are lost to follow-up or when the study ends.

# Survival Analysis Functions

- Survival Function *S(t)* :

  $S(t)$ = Prob{T ≥ t}, here t ≥ 0 ;

- Lifetime Distribution Function *F(t)* :

  $F(t) = 1 - S(t)$ ;

- Event Density Function *f(t)* :

  Prob{t≤ T ≤ t+δt} = f(t)δt, $\frac{dF(t)}{dt} = f(t)$

- Hazard Function *h(t)* :

  $h(t) = f(t)/S(t)$

  or h(t)δt = Prob{t ≤ T≤ t+δt |T≥t};

# Survival Analysis Functions

All those functions are connected.

- Density function is the negative of the derivative of the survival function;
- Hazard function is the negative of the derivative of the log of the survival function.

$$f(t) = F'(t) = -S'(t)$$

$$h(t) = -\,{}^{d(\ln S(t))}\!/_{dt}$$

$$S(t) = \exp\left\{-\int_0^t h(s)ds\right\}$$

$$f(t) = h(t)\exp\left\{-\int_0^t h(s)ds\right\}$$

# Survival Analysis Functions

- The most popular distributions are exponential, Weibull, etc.
- Exponential: $S(t) = exp(-\lambda t) \quad \lambda > 0$ ;

  $f(t)= \lambda exp(-\lambda t);$

  $h(t) =\lambda$ ; ( so no ageing)
- Weibull; $S(t) =exp\ (-\beta t^{\alpha}) \qquad \alpha\ ,\ \beta > 0$ ;

  $f(t) = \alpha\beta t^{\alpha-1} (exp(-\beta t^{\alpha} ));$

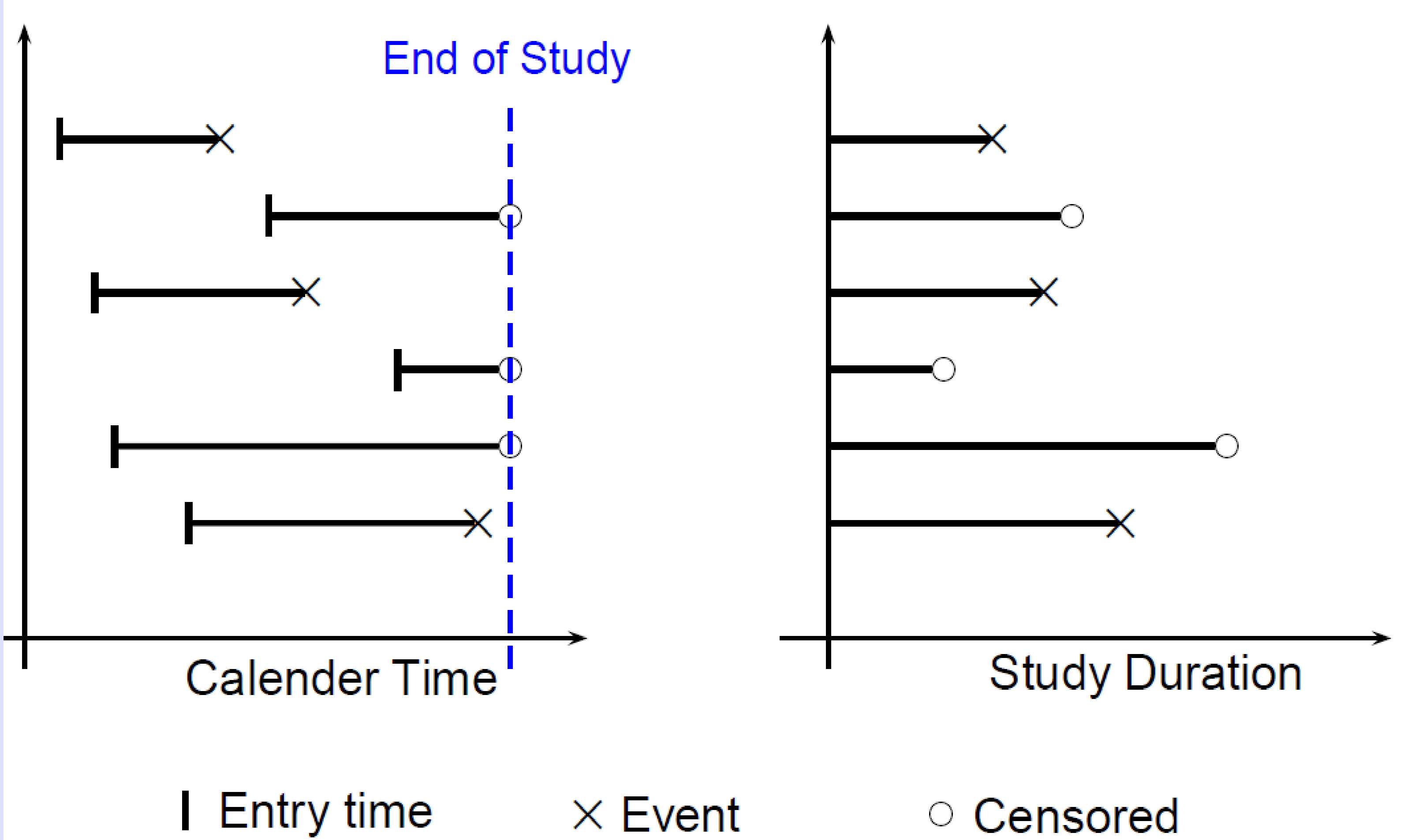
  $h(t) = \alpha\beta t^{\alpha-1}$ ;

  $\alpha > 1$ (increasing hazard) , $\alpha < 1$ (decreasing hazard)

# Survival Analysis Data

- Calendar time of whole study (Starting day, Ending day of the whole study period)
- Study Duration of each individual.
- Define the *censored observations.*
- Time measure units (Month, Year … )
- Define the dependent variable and independent.

# Survival Analysis Data



# Examples

| Duration Times of Interest in Marketing | | |
|---|---|---|
| **Subdiscipline** | **Decision/Forecasting** | **Duration Time** |
| **Pricing/Promotion** | Timing of price chinages or promotions;<br>Measuring effect of promotion | Interpurchase duration; Timing of coupon redemption |
| **Salesforce Management** | Forecasting and managing salesforce turnover | Salesperson job duration |
| **New Product Development** | Forecasting trial, adoption, depth of repeat purchase | Duration time from new product introduction until initial trial;<br>Interpurchase times |
| **Marketing Research** | Forecasting response rates;<br>Forecasting size and composition of firm's customer base; | Time until survey response;<br>Time until customer becomes inactive or disaffected;<br>Time until cancellation of service contract; |

Sources: Kristiaan H. and D. C. Schmittlein, 1993, Analyzing Duration Times in Marketing: Evidence for the Effectiveness of Hazard Rate Models; *Marketing Science*, Vol. 12, No. 4, page 396 .

# Cox Proportional Hazard Model

# Advantages

- The dependent variable of interest (survival/failure time) is most likely not normally distributed.
- Censoring(especially right censoring) of the Data.
- Baseline hazard function is unknown.
- Whether and when the customer will leave.
- Dynamics covariates and duration

# Cox Proportional Hazard Model Equation

Let $h(t \mid x_t)$ denote the resultant hazard rate at time $t$ for an individual have covariate value $x_t$,

$$h(t \mid x_t) = h_0(t) e^{\beta' x_t}$$

Here $x_t = (x_{1t}, x_{2t}, \ldots\ldots, x_{kt})$ $\beta = (\beta_1, \beta_2, \ldots\ldots, \beta_k)$

$k$ is the total number of the covariates,

$\beta_j$ *is the constant Proportional effect of* $x_j$

The term *$h_0(t)$* is called the *baseline hazard*; it is the hazard for the respective individual when all independent variable values are equal to zero.

# Cox Proportional Hazard Model Equation

We can linearize this model by dividing both sides of the equation by $h_0(t)$ and then taking the natural logarithm of both sides:

$$\ln\{h(t \mid x_t)/h_0(t)\} = \beta' x_t$$

Taking partial derivative we have

$$\partial \ln h(t \mid x_t, \beta)/\partial x_{jt} = \beta_j$$

# Partial Likelihood Estimation of *β*

$$L(i \mid t, j_1, j_2, \ldots\ldots, j_{n(t)}) = \frac{h_i(t)}{\sum_{k=1}^{n(t)} h_{j_k}(t)} \qquad (1)$$

$$L(i \mid t, j_1, j_2, \ldots\ldots, j_{n(t)}) = \frac{h_0(t) e^{\beta' x_{it}}}{\sum_{k=1}^{n(t)} h_0(t) e^{\beta' x_{j_k t}}} \qquad (2)$$

$$L(i \mid t, j_1, j_2, \ldots\ldots, j_{n(t)}) = \frac{e^{\beta' x_{it}}}{\sum_{k=1}^{n(t)} e^{\beta' x_{j_k t}}} \qquad (3)$$

Estimation of ***β*** is obtained by Maximizing the Product of Expression (3) over all observed duration times.

# Literatures

- Kristiaan H. and D. C. Schmittlein, 1993, Analyzing Duration Times in Marketing: Evidence for the Effectiveness of Hazard Rate Models; *Marketing Science*, Vol. 12, No. 4, pp. 395-414 .
- Graves S, D. Kletter, W. B. Hetzel, R. N. Bolton, 1998, A Dynamic Model of the Duration of the Customer’s Relationship with a Continuous Service Provider: The Role of Satisfaction, *Marketing Science*, Vol. 17, No. 1, pp. 45-65.
- Andreeva G., 2006, European Generic Scoring Models Using Survival Analysis, *Journal of the Operational Research Society*, Vol. 57, No. 10, pp. 1180-1187.
- Bellotti T. and J. Crook, 2009, Credit Scoring With Macroeconomic Variables Using Survival Analysis; *Journal of the Operational Research Society*, Vol. 60, pp. 1699–1707.

# A Case Study

# Case Study Data

- 6.5 years Commercial Line Policies.
- The Dependent Variable:

  Duration = The time until the policy cancellation
- If a policy is still alive at the end of study, it is right censored ( i.e. Censor = 1)
- Monthly policy data and economic data are stacked together to get the final model data.

# Annual Attrition Summary

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 24,570 | 156,478 | 16,907 | 197,955 | 12.41% | 79.05% | 8.54% |
| 200601 | 25,101 | 158,794 | 17,529 | 201,424 | 12.46% | 78.84% | 8.70% |
| 200701 | 24,756 | 159,079 | 18,057 | 201,892 | 12.26% | 78.79% | 8.94% |
| 200801 | 24,951 | 160,688 | 19,697 | 205,336 | 12.15% | 78.26% | 9.59% |
| 200901 | 27,398 | 162,875 | 20,787 | 211,061 | 12.98% | 77.17% | 9.85% |

The data is for illustration purpose.

# Annual Attritions by Policy Type

**Line1**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 10,708 | 63,270 | 7,283 | 81,262 | 13.18% | 77.86% | 8.96% |
| 200601 | 11,292 | 65,190 | 7,924 | 84,407 | 13.38% | 77.23% | 9.39% |
| 200701 | 11,657 | 64,801 | 8,336 | 84,793 | 13.75% | 76.42% | 9.83% |
| 200801 | 11,525 | 64,178 | 9,539 | 85,242 | 13.52% | 75.29% | 11.19% |
| 200901 | 12,860 | 63,911 | 10,469 | 87,241 | 14.74% | 73.26% | 12.00% |

**Line2**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 9,630 | 67,757 | 7,101 | 84,488 | 11.40% | 80.20% | 8.40% |
| 200601 | 9,514 | 66,928 | 7,076 | 83,518 | 11.39% | 80.14% | 8.47% |
| 200701 | 8,666 | 66,705 | 6,799 | 82,170 | 10.55% | 81.18% | 8.27% |
| 200801 | 8,615 | 68,238 | 7,280 | 84,133 | 10.24% | 81.11% | 8.65% |
| 200901 | 9,611 | 70,428 | 7,516 | 87,555 | 10.98% | 80.44% | 8.58% |

**Line3**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 4,232 | 25,450 | 2,523 | 32,206 | 13.14% | 79.02% | 7.83% |
| 200601 | 4,295 | 26,676 | 2,529 | 33,500 | 12.82% | 79.63% | 7.55% |
| 200701 | 4,433 | 27,574 | 2,922 | 34,930 | 12.69% | 78.94% | 8.37% |
| 200801 | 4,810 | 28,272 | 2,878 | 35,960 | 13.38% | 78.62% | 8.00% |
| 200901 | 4,927 | 28,536 | 2,803 | 36,265 | 13.59% | 78.69% | 7.73% |

# Annual Attritions by Premium Change

**Annual Premium change < -x%**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 330 | 2,247 | 113 | 2,690 | 12.27% | 83.53% | 4.21% |
| 200601 | 3,657 | 22,867 | 880 | 27,405 | 13.35% | 83.44% | 3.21% |
| 200701 | 4,317 | 31,587 | 1,417 | 37,321 | 11.57% | 84.64% | 3.80% |
| 200801 | 5,103 | 37,126 | 1,602 | 43,831 | 11.64% | 84.70% | 3.66% |
| 200901 | 4,041 | 24,618 | 892 | 29,551 | 13.67% | 83.31% | 3.02% |

**Annual Premium change (-x%, 0%)**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 110 | 1,194 | 72 | 1,376 | 7.99% | 86.76% | 5.25% |
| 200601 | 1,514 | 12,866 | 676 | 15,056 | 10.06% | 85.45% | 4.49% |
| 200701 | 2,181 | 18,409 | 958 | 21,548 | 10.12% | 85.43% | 4.45% |
| 200801 | 2,306 | 18,315 | 829 | 21,450 | 10.75% | 85.38% | 3.87% |
| 200901 | 1,348 | 10,783 | 493 | 12,625 | 10.68% | 85.42% | 3.91% |

**Annual Premium change (0%, x%)**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 23,656 | 153,279 | 11,864 | 188,800 | 12.53% | 81.19% | 6.28% |
| 200601 | 13,448 | 85,676 | 7,538 | 106,661 | 12.61% | 80.33% | 7.07% |
| 200701 | 12,725 | 77,042 | 7,157 | 96,924 | 13.13% | 79.49% | 7.38% |
| 200801 | 13,844 | 84,623 | 9,115 | 107,582 | 12.87% | 78.66% | 8.47% |
| 200901 | 18,302 | 109,942 | 11,085 | 139,329 | 13.14% | 78.91% | 7.96% |

**Annual Premium change > x%**

| BaseMonth | nonRenewed | Renewed | Midterm_canceled | Total | nonRenewedPer | RenewedPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200501 | 474 | 4,232 | 478 | 5,184 | 9.15% | 81.64% | 9.21% |
| 200601 | 6,482 | 41,619 | 4,355 | 52,456 | 12.36% | 79.34% | 8.30% |
| 200701 | 5,533 | 36,853 | 3,928 | 46,313 | 11.95% | 79.57% | 8.48% |
| 200801 | 3,698 | 25,252 | 3,723 | 32,674 | 11.32% | 77.29% | 11.40% |
| 200901 | 3,708 | 21,809 | 4,235 | 29,752 | 12.46% | 73.30% | 14.24% |

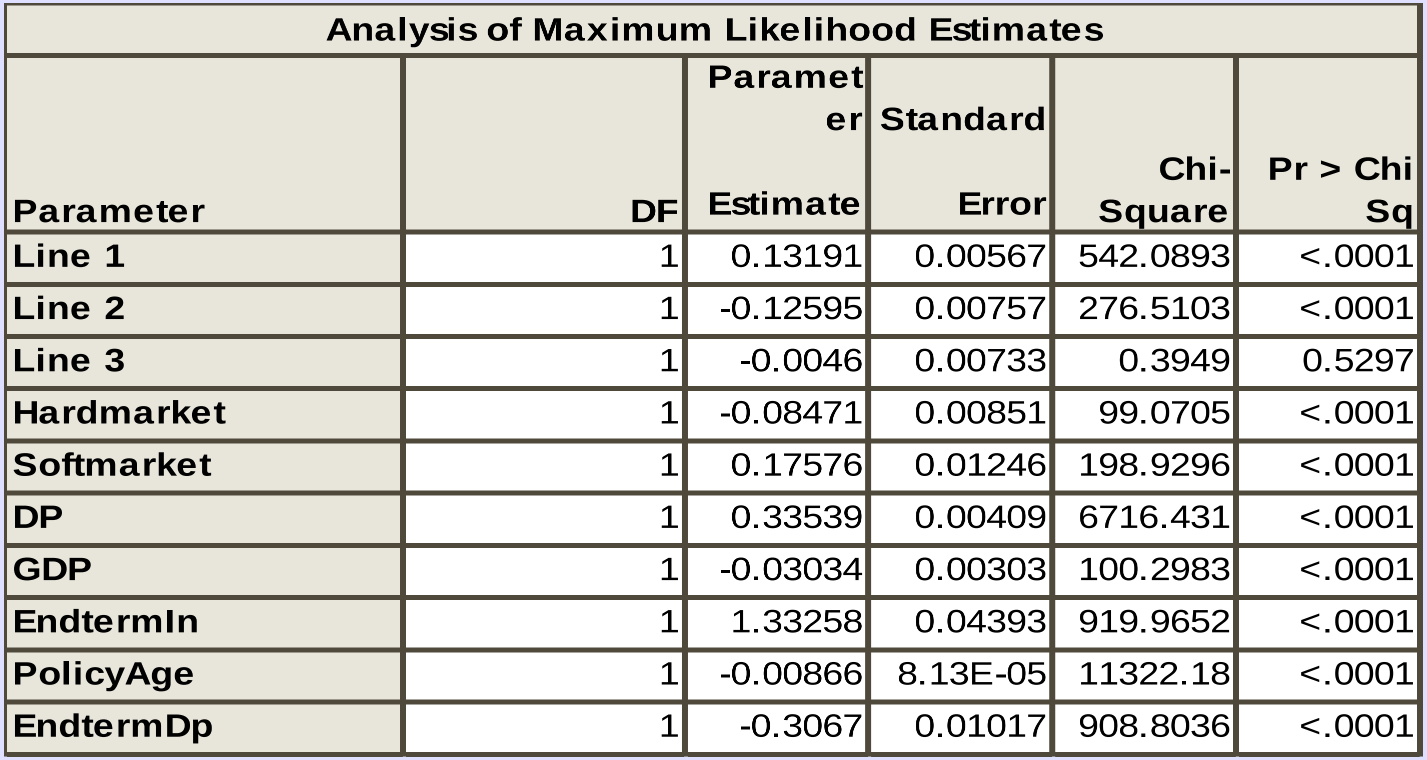
# Monthly View

**Monthly Snapshot**

| | Active | Withdraw | Percent |
|---|---|---|---|
| Endterm | 16,939 | 2,086 | 12.32% |
| Others | 182,161 | 1,609 | 0.88% |
| Total | 199,099 | 3,695 | 1.86% |

| BaseMonth | nonRenewed | Renewed | Midterm Canceled | Total | Mid-term Stayed | nonRenewPer | Midterm_cancelPer |
|---|---|---|---|---|---|---|---|
| 200503 | 2,086 | 14,852 | 1,609 | 199,099 | 180,552 | 12.32% | 0.88% |
| 200506 | 2,089 | 14,789 | 1,609 | 200,793 | 182,305 | 12.38% | 0.87% |
| 200509 | 1,750 | 12,879 | 1,502 | 201,314 | 185,183 | 11.96% | 0.80% |
| 200512 | 1,565 | 11,330 | 1,602 | 201,192 | 186,694 | 12.13% | 0.85% |
| 200603 | 2,228 | 15,292 | 1,775 | 201,657 | 182,362 | 12.72% | 0.96% |
| 200606 | 2,083 | 14,805 | 1,455 | 201,820 | 183,477 | 12.33% | 0.79% |
| 200609 | 1,797 | 13,096 | 1,684 | 201,698 | 185,120 | 12.07% | 0.90% |
| 200612 | 1,584 | 11,437 | 1,584 | 201,145 | 186,541 | 12.16% | 0.84% |
| 200703 | 2,284 | 15,597 | 1,634 | 202,562 | 183,047 | 12.77% | 0.88% |
| 200706 | 1,910 | 14,997 | 1,483 | 203,966 | 185,576 | 11.30% | 0.79% |
| 200709 | 1,725 | 13,237 | 1,690 | 204,830 | 188,178 | 11.53% | 0.89% |
| 200712 | 1,615 | 11,578 | 1,939 | 204,858 | 189,727 | 12.24% | 1.01% |
| 200803 | 2,174 | 15,955 | 1,763 | 206,118 | 186,226 | 11.99% | 0.94% |
| 200806 | 2,055 | 15,038 | 1,687 | 208,880 | 190,100 | 12.02% | 0.88% |
| 200809 | 1,895 | 13,291 | 1,750 | 210,140 | 193,205 | 12.48% | 0.90% |
| 200812 | 1,568 | 11,547 | 2,573 | 210,703 | 195,015 | 11.95% | 1.30% |
| 200903 | 2,328 | 16,087 | 2,111 | 212,861 | 192,334 | 12.64% | 1.09% |
| 200906 | 2,313 | 15,371 | 1,989 | 214,614 | 194,942 | 13.08% | 1.01% |
| 200909 | 2,168 | 13,759 | 1,910 | 214,595 | 196,758 | 13.61% | 0.96% |
| 200912 | 1,847 | 11,836 | 2,297 | 212,302 | 196,322 | 13.50% | 1.16% |

# Parameter Estimates Using PHREG



Analysis of Maximum Likelihood Estimates

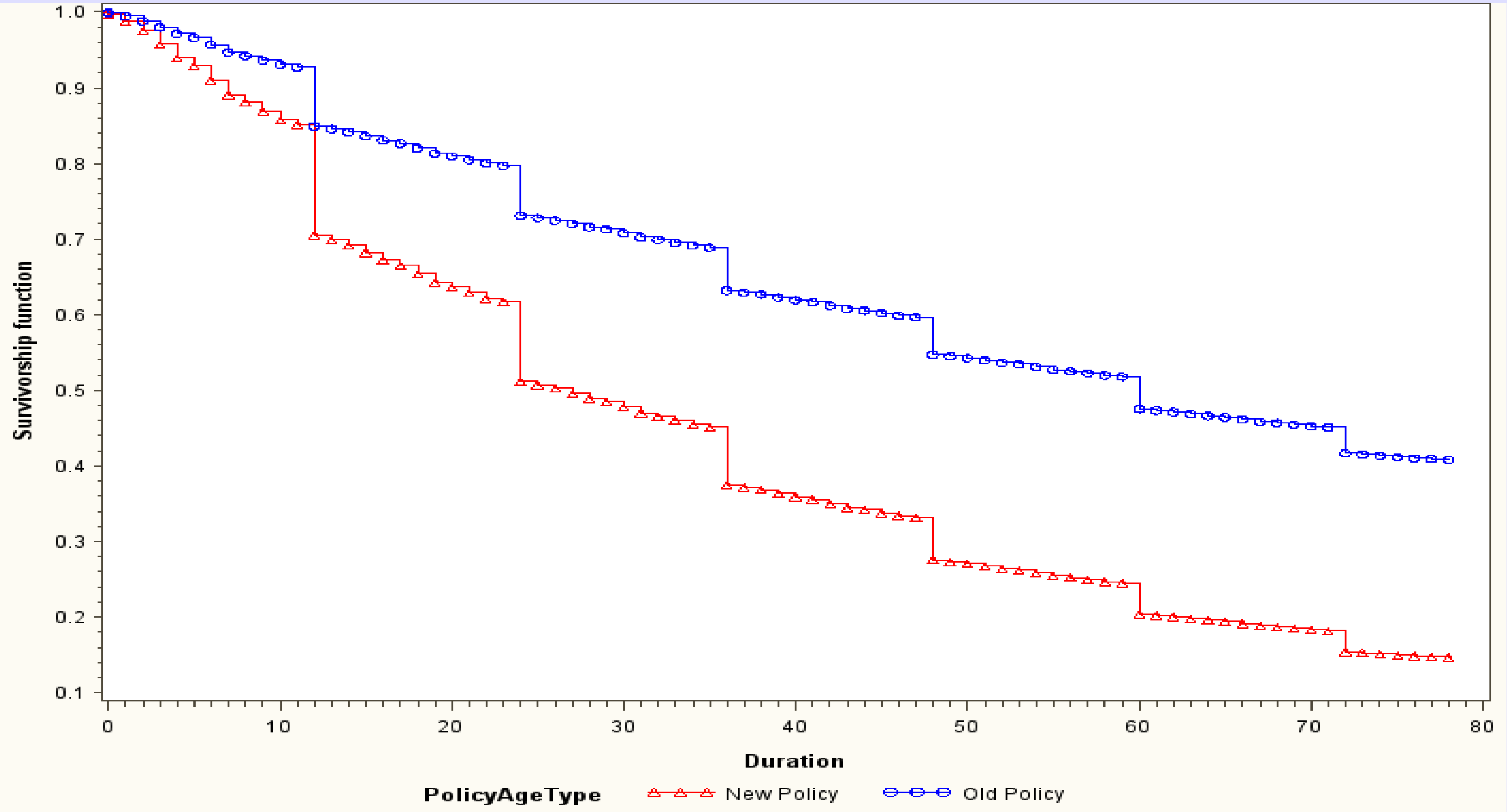
| Parameter | DF | Parameter Estimate | Standard Error | Chi-Square | Pr > ChiSq |
|---|---|---|---|---|---|
| Line 1 | 1 | 0.13191 | 0.00567 | 542.0893 | <.0001 |
| Line 2 | 1 | -0.12595 | 0.00757 | 276.5103 | <.0001 |
| Line 3 | 1 | -0.0046 | 0.00733 | 0.3949 | 0.5297 |
| Hardmarket | 1 | -0.08471 | 0.00851 | 99.0705 | <.0001 |
| Softmarket | 1 | 0.17576 | 0.01246 | 198.9296 | <.0001 |
| DP | 1 | 0.33539 | 0.00409 | 6716.431 | <.0001 |
| GDP | 1 | -0.03034 | 0.00303 | 100.2983 | <.0001 |
| EndtermIn | 1 | 1.33258 | 0.04393 | 919.9652 | <.0001 |
| PolicyAge | 1 | -0.00866 | 8.13E-05 | 11322.18 | <.0001 |
| EndtermDp | 1 | -0.3067 | 0.01017 | 908.8036 | <.0001 |

There are about 20 variables plus several interaction terms in the models. Only selected variables are reported.

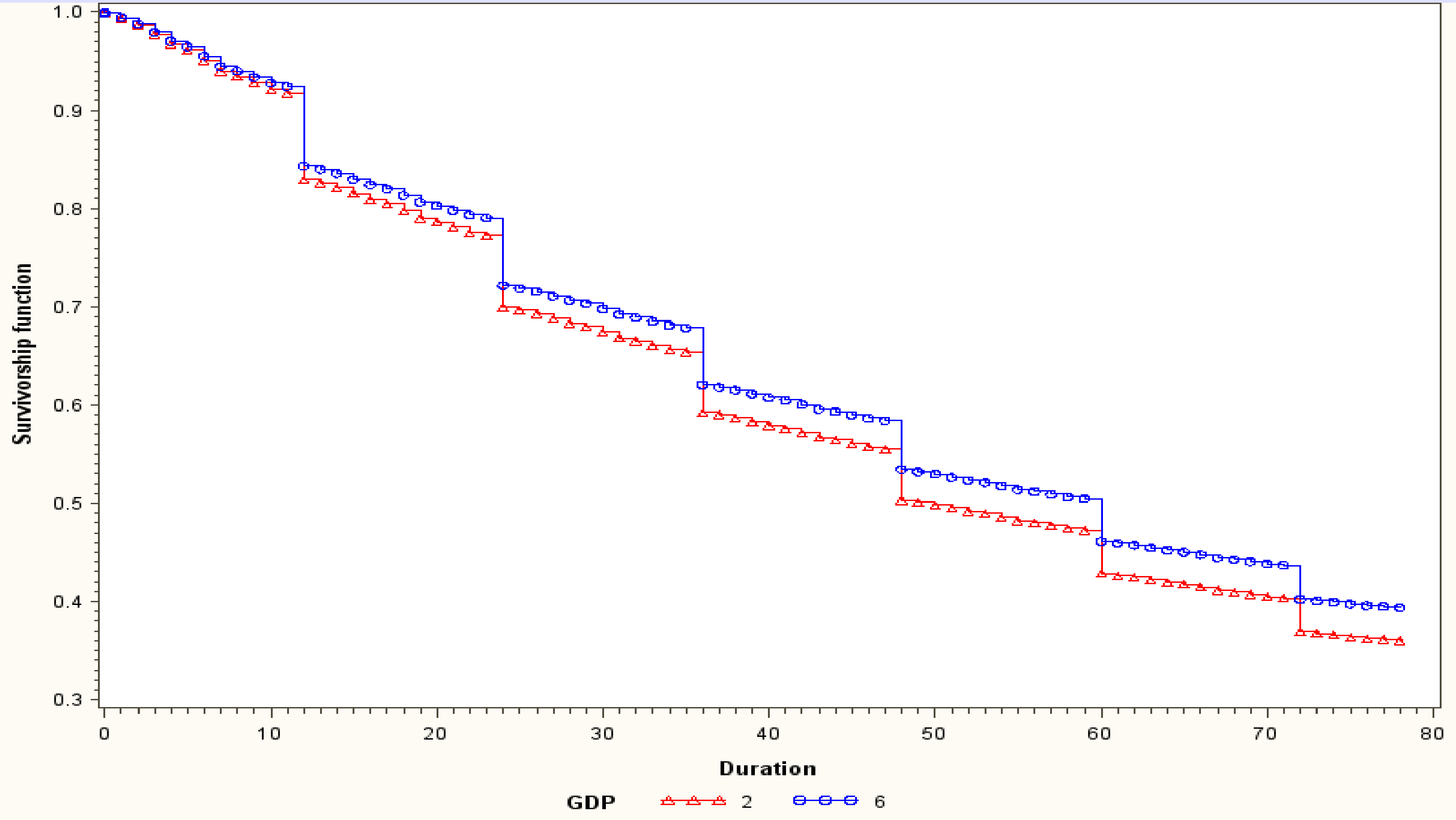
# Parameter Estimates Using Logistic

| Analysis of Maximum Likelihood Estimates | | | | | |
|---|---|---|---|---|---|
| **Parameter** | **DF** | **Estimate** | **Standard Error** | **Wald Chi-Square** | **Pr > Chi Sq** |
| **Intercept** | 1 | -1.6028 | 0.0804 | 397.1769 | <.0001 |
| **Line 1** | 1 | 0.0816 | 0.0145 | 31.513 | <.0001 |
| **Line 2** | 1 | -0.1732 | 0.0143 | 146.9164 | <.0001 |
| **Line 3** | 1 | -0.0539 | 0.0164 | 10.7724 | 0.001 |
| **Softmarket** | 1 | 0.1281 | 0.0267 | 22.9947 | <.0001 |
| **DP** | 1 | 0.4227 | 0.0118 | 1290.378 | <.0001 |
| **GDP** | 1 | -0.0311 | 0.00486 | 40.8976 | <.0001 |
| **PolicyAge** | 1 | -0.00717 | 0.000094 | 5866.395 | <.0001 |

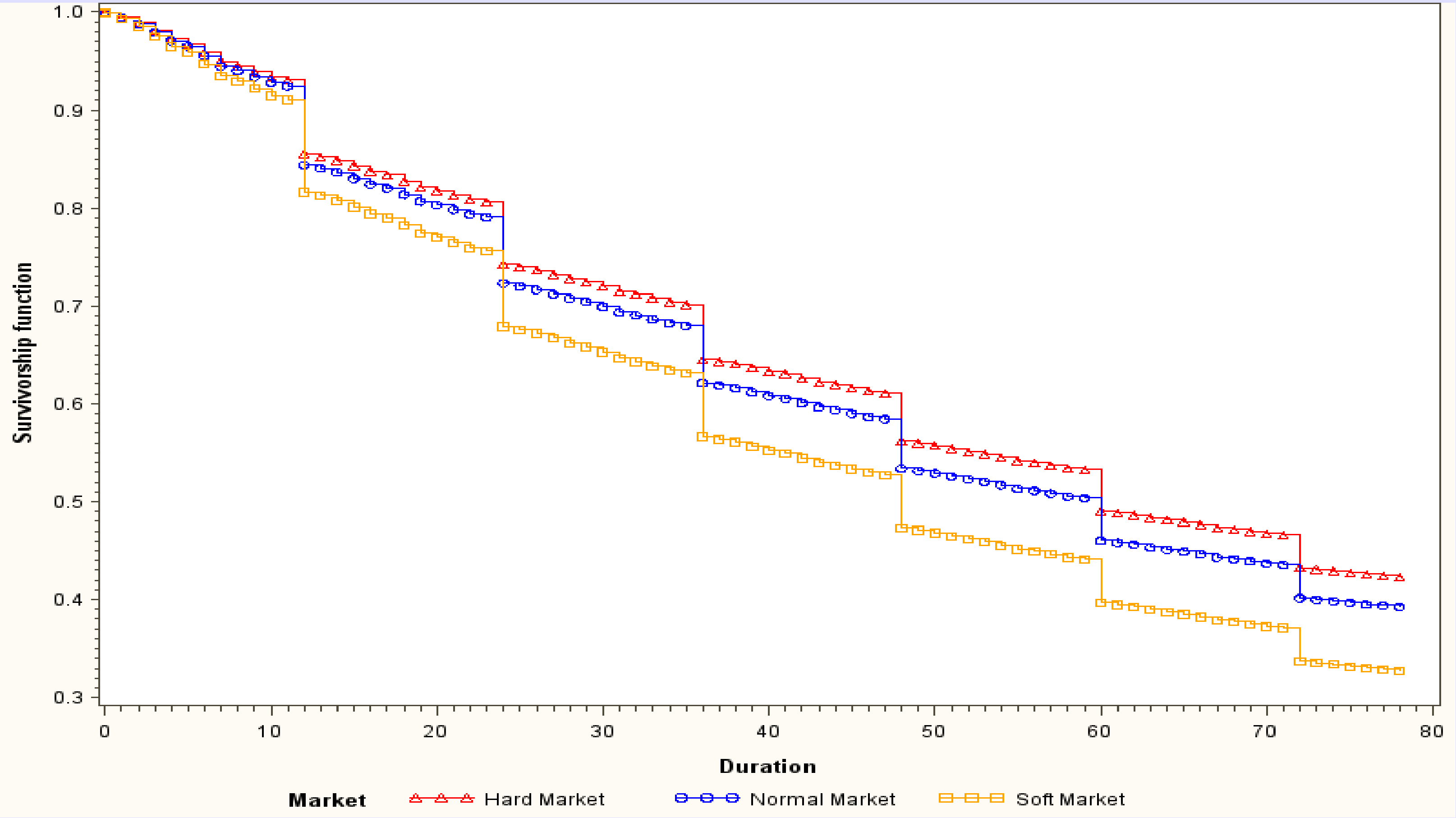
# Survival Curve for Policy Age



# Survival Curve for GDP Change (Percent)



# Survival Curve for Market Condition



# Conclusions

- Survival analysis addresses not only whether a policy will leave, but also when it will leave.
- Provide a dynamic insight by utilizing panel data and improve the static view derived from snapshot data.
- Analyze mid-term cancellation and end-term nonrenewal sequentially and simultaneously.
- Able to measure the impacts of time-variant macroeconomic variables on attrition.

# Q & A

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