STATE AUTO® Insurance Companies

Concentration Risk Measures and De-concentration Optimization

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Agenda

- Introduction
- Risk measures
- Concentration risk measures (CRM)
- Capital and PML allocation
- Optimal de-concentration: a case study
- Q&A

1. Introduction

- Bad loss ratios on property lines, especially homeowners
  - Worst performance line of business
  - Lost money in 8 of last 10 years
- Increasing losses from wind-hail perils
  - Soaring catastrophe loss ratios in recent years
  - Experienced 35 of the 37 catastrophe events identified by Property Claim Services (PCS) in 2008

1. Introduction

Strategies to Mitigate Catastrophe Risk

- Rate Increase
- Predictive Models
- Higher all-peril and wind-hail deductibles
- ITV and building inspection
- Cat reinsurance and aggregate reinsurance
- Risk De-concentration

1. Introduction

Concentration Risk: Traditional Approach

- A marketing type of method
- The insurer’s exposures or TIV (total insurance value) in a region
- Total exposures or TIV
- If a region’s exposure percentage is significantly higher than average, then overconcentration, vice versa
- Not directly related to risk appetites

1. Introduction

Typical risk appetites for P&C insurers

- X% chance of GAAP ROE below –YY% on an annual basis
- X% risk of falling below YYY BCAR (financial downgrade)
- X% risk of falling below authorized control level RBC (government takeover)
- Cat loss PML for a 1-in-XXX year event, net of reinsurance, won't deplete beginning of year surplus by more than YY%

2. Risk Measures

- Variance and standard deviation
  - Not downside risk measures
  - Desirable swings are also treated as risk
- VaR (Value-at-Risk), TVaR, XTVaR
  - VaR: predetermined percentile point
  - TVaR: expected value when loss>VAR
  - XTVaR: TVaR-mean

2. Risk Measures

- Lower partial moment and downside variance

$$LPM(L|T,k)=\int_{T}^{\infty}(L-T)^{k}dF(L)$$

  - T is the maximum acceptable losses, benchmark for "downside"
  - k is the risk perception parameter to large losses, the higher the K, the stronger risk aversion to large losses
  - When k=1 and T is the 99th percentile of loss, LPM is equal to 0.01*VaR
  - When K=2 and T is the mean, LPM is semi-variance
  - When K=2 and T is the target, LPM is downside variance

2. Risk Measures

- EPD expected policyholder deficit
  - EPD=probability of default * average loss from default
- Cost of default option
  - An insurer will not pay claims once the capital is exhausted
  - A put option that transfers default risk to policyholders
- PML (probable maximum loss per event) and AAL (average annual Loss)

3. Concentration Risk Measures

- Marginal Risk Reduction (MRR)

$$\frac{dPML}{dprem_i}, \quad \frac{dPML}{d\exp_i}, \quad \frac{dPML}{dTIV_i}$$

  - If premium in a region is reduced by 10K, how much will PML decrease?
  - Direct measure of risk reduction by deconcentration
  - Deconcetration strategy: reduce exposure with highest MRR
  - PML can be replaced by any other risk measures

$$\frac{dLPM}{dprem_i}, \quad \frac{dVariance}{dprem_i}$$

3. Concentration Risk Measures

➢Risk Reduction Elasticity (RRE)

$$\frac{dPML/PML}{d\,\mathrm{Prem}_i/\mathrm{Prem}_i},\quad \frac{dPML/PML}{d\exp_i/\exp_i},\quad \frac{dPML/PML}{dTIV_i/TIV_i}$$

- ➢If premium in a region is reduced by 1%, by what percentage will PML decrease?
- ➢Direct measure of percentage risk reduction by deconcentration
- ➢Deconcetration strategy: reduce exposure with highest RRE

3. Concentration Risk Measures

- Balanced Marginal Risk Reduction (BMRR)

$$\frac{d'PML}{dprem_i}, \quad \frac{d'PML}{d\exp_i}, \quad \frac{d'PML}{dTIV_i}$$

  - If premium in a region is reduced by 10K, and other regions increase 10K proportionally, how much will PML decrease?
  - Direct measure of risk reduction by deconcentration if the overall premium remains the same.
  - Deconcetration strategy: reduce exposure with largest positive BMRR; increase exposure with largest negative BMRR.

3. Concentration Risk Measures

- Balanced Risk Reduction Elasticity (BRRE)

$$\frac{d' PML / PML}{d \, Prem_i / Prem_i}, \quad \frac{d' PML / PML}{d \exp_i / \exp_i}, \quad \frac{d' PML / PML}{dTIV_i / TIV_i}$$

  - If premium in a region is reduced by 1% and other regions increases the premium proportionally, by what percentage will PML decrease?
  - Direct measure of percentage risk reduction by deconcentration if premium remains the same
  - Deconcetration strategy: reduce exposure with largest positive BRRE, increase exposure with largest negative BRRE

3. Concentration Risk Measures

- Co-Measure
  - Kreps R., 2005, “Riskness Leverage Models”, CAS Proceedings, Vol XCII, 31-60.
  - If risk is defined as R(x), then Co-measure is

$$R(x) = E(f(x) \mid condition)$$

$$CoR(x_i) = E(f(x_i) \mid condition)$$

  - For example, the co-measure for XTVaR is

$$XTVaR(x_q) = E(x - m \mid x > x_q)$$

$$CoXTaR(x_{q,i}) = E(x_i - m_i) \mid x > x_q)$$

3. Concentration Risk Measures

- A hypothetical case

Region	Premium	Cat Loss Distribution			
		1%	1%	1%	97%
1	100	50	100	0	0
2	100	70	0	80	0
Total	200	120	100	80	0

3. Concentration Risk Measures

Region	Premium	Cat Loss Distribution			
		1%	1%	1%	97%
1	100	50	100	0	0
2	100	70	0	80	0
Total	200	120	100	80	0

- Marginal Risk Reduction: If region1 premium reduces by 1 dollar, 99% VaR is 119.5 (49.5+70). PML reduces 0.5 dollar. MRR1=0.5.

$$\frac{dPML}{dprem1} = 0.5 \qquad \frac{dPML}{dprem2} = 0.7$$

- Risk Reduction Elasticity: If region1 premium reduces by 1%, 99% VaR is 119.5. RRE1=(0.5/120)/1%=0.417.

$$\frac{dPML / PML}{d\,\mathrm{Pr}\,em1 / \mathrm{Pr}\,em1} = 0.417 \qquad \frac{dPML / PML}{d\,\mathrm{Pr}\,em2 / \mathrm{Pr}\,em2} = 0.583$$

3. Concentration Risk Measures

Region	Premium	Cat Loss Distribution			
		1%	1%	1%	97%
1	100	50	100	0	0
2	100	70	0	80	0
Total	200	120	100	80	0

- Balanced Marginal Risk Reduction: If region1 premium reduces 1 dollar, and region2 premium increases 1 dollar, 99% VaR is 122.2 (49.5+70.7), BMRR1=-0.2

$$\frac{d' PML}{dprem1} = -0.2 \qquad \frac{d' PML}{dprem2} = 0.2$$

- Balanced Risk Reduction Elasticity

$$\frac{d' PML / PML}{d\,\mathrm{Pr}em1 / \mathrm{Pr}em1} = -0.167 \qquad \frac{d' PML / PML}{d\,\mathrm{Pr}em2 / \mathrm{Pr}em2} = 0.167$$

- Co-Measure: $Co - PML1 = 50 \qquad Co - PML2 = 70$

3. Concentration Risk Measures

- De-concentration Optimization using MRR and RRE, assuming premium reduction

1. Reduce one unit premium in the region with highest MRR/RRE, that is, Region 2
2. Repeat 1 till achieving target premium reduction in certain regions.

3. Concentration Risk Measures

- De-concentration Optimization using BMRR and BRRE. Premium decreased in one region balanced by proportional increases from other regions

1. Reduce one unit premium in the region with highest BMRR/BRRE
2. Proportionally distribute the premium to rest of regions
3. Repeat 1-2 till optimal equilibrium (or target premium reduction in certain regions). The region with highest concentration risk may change in each iteration
4. In this example, the equilibrium is region 1 premium 116.7, and region 2 premium 83.3

3. Concentration Risk Measures

- De-concentration Optimization using BMRR and BRRE. Premium decreased in one region balanced by selective growth of other regions (or new regions)
  1. Reduce one unit premium in the region with highest BMRR/BRRE
  2. Increase one unit premium in the region with largest negative BMRR/BRRE
  3. Repeat 1-2 till optimal equilibrium (or target premium reduction in certain regions).

3. Concentration Risk Measures

- The concentration risk measures can be extended to asset management and non-insurance industries
  - How much is PML (the worst loss 1 in 100 years) of equities or a specific stock?
  - How much does a specific line of business contribute to a company's PML?
  - If we switch 10 Million investment from stocks to municipal bonds, how much will it reduce PML of overall investment?

4. Capital and PML Allocation

Capital Allocation

- Insurers need to hold sufficient capital to pay for worst catastrophe losses, say 1:100 year PML
- Management need to know the capital constraints on geographic expansion.
- Actuaries need to know the underwriting margins in cat-prone areas in order to achieve a target return on capital.

4. Capital and PML Allocation

Capital Allocation

- Capital supports "even for a 99th percentile loss", but not "only for a 99th percentile loss"
- People are not just afraid of extreme large losses. They also dislike small losses.
- Capital allocation should consider the whole loss distribution, not just extreme right tail events. It should allocate disproportionate capital to severe losses.

4. Capital and PML Allocation

Capital Allocation Principles

- Add-up to company capital
- The larger the correlation, the higher the capital allocated to a region
- The larger the regional volatility, the higher capital allocated to a region

4. Capital and PML Allocation

Capital Allocation: Bodoff Method

- Allocate capital to all losses
- Allocate capital separately on each layer and perform the allocation across all layers
- Allocate disproportionate capital to extreme losses
- De-concentration strategy: reduce exposures from the region that consumes the highest capital
- Bodoff N. M. 2009, “Capital Allocation by Percentile Layers,” *Variance*, Vol.3:1, 13-30

4. Capital and PML Allocation

Capital Allocation: Bodoff Method

$$\int_{y=0}^{y=PML} \int_{x=y}^{x=\infty} \frac{f(x)}{1-F(y)} dxdy$$

x: loss amount

y: capital

F() and f(): the cumulative and density distribution functions of loss

4. Capital and PML Allocation

Capital Allocation: Bodoff Method

Previous Example

Layer	Capital	Expected Loss		Capital Allocation	
		Region1	Region2	Region1	Region2
0-50	50	0.71	0.79	23.6	26.4
50-70	20	0.28	0.32	9.4	10.6
70-80	10	0.14	0.16	4.7	5.3
80-100	20	0.28	0.12	14.2	5.8
100-120	20	0.08	0.12	8.3	11.7
Total	120	1.50	1.50	60.3	59.7

4. Capital and PML Allocation

Capital Allocation: Myers-Read Method

- Allocation depends on the marginal contribution to default value (put option)
- Marginal Default values add up to the total default value of the company
- Can be simplified assuming zero correlation between investment and loss
- Myers, Stewart C., and Read Jr., James A., “Capital Allocation for Insurance Companies,” *Journal of Risk and Insurance*, vol. 68, No. 4 (2001), pp. 545-580.

4. Capital and PML Allocation

Capital Allocation: Myers-Read Method

- Original version

$$c_i = c + \frac{(1+c)n(y)}{N(y)v}\left[(v_{i,L} - v_L^2) - (v_{i,A} - v_{A,L})\right]$$

$$y = \log(1+c)/v - v/2$$

c is capital per unit of loss

v is the standard deviation of log(loss)
N(y) is the cumulative standard normal probability.

4. Capital and PML Allocation

Capital Allocation: Myers-Read Method

- Butsic's simplified version
- Butsic, Robert J. (1999) "Capital Allocation for Property-Liability Insurers: A Catastrophe Reinsurance Application," *CAS Forum*, Spring

$$c_i = c + (\beta_i - 1)Z$$

$$\beta_i = \rho_{i,L}\sigma_i / \sigma_L$$

$$Z \cong (1+c)\frac{n(y)}{N(y)}\frac{\sigma_L{}^2}{\sigma}$$

$$\sigma_L^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{i,j} = \exp(v_L^2) - 1$$

σ_L is the CV of total losses

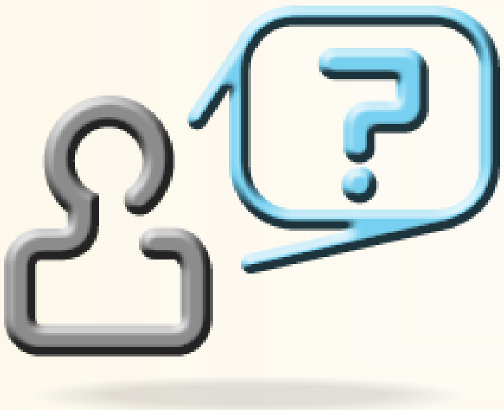
4. Capital and PML Allocation

PML Allocation

- To maintain certain A. M Best Rating, 1 from 100 years PML should not deploy x% of surplus.
- If a company targets 1 billion PML, how much PML each region/state/county/zip should target?
- Maximize profit/exposure/TIV by selecting optimal regional exposures subject to a companywide PML constraint
- Sum of allocated PML > company PML
- The larger the correlation, the lower the PML allocated to a region

5. A Case Study

- The case study will be shown in the RPM seminar presentation.



Q & A

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