Limited Fluctuation – Example

- Calculate the expected loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.
  - Scenario 1:
    Data: Observed loss ratio = 67%, Claim count = 600
    - What is the standard for full credibility?
    - Does this data have full credibility?
    - What is the expected loss ratio?
  - Answer:
    - For P=95% and k=10%, the number of claims needed is 584. Since we have 600, the data is considered fully credible.

Limited Fluctuation – Example (continued)

- Calculate the loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.
  - Scenario 2:
    Data: Observed loss ratio = 67%, Claim count = 400
    - Assuming Z = 0.72, what is the expected loss ratio?
  - Answer:
    E2 = Z*T + (1-Z)*E1
    E2 = 0.72 x 67% + (1 – 0.72) x 75%
    E2 = 69.2%

Limited Fluctuation – Example 2

- Calculate the credibility-weighted loss ratio and indicated change, given that the expected loss ratio is 75%. Use the square root rule and when P= 90% and k = 2.5%.

Year	Loss Ratio	Claim Count
2006	67%	530
2007	77%	610
2008	79%	630
2009	77%	620
2010	86%	690

	Loss Ratio	Claim Count	Credibility	Cred-Wght Loss Ratio	Indicated Rate Chg
'08-'10	81%	1,940	67%	79.0%	5.3%
'06-'10	77%	3,080	84%	76.7%	2.3%

79.0% = 81% x (0.67) + 75% x (1 - 0.67)

67%= √(1940/4326)

5.3% = 79.0%/75.0%

Limited Fluctuation – Example 3

- Given a current territory factor of 1.08, determine the indicated territory factor with 5 years of data. Use the square root rule and the limited fluctuation formula for pure premium. Assume a Poisson frequency distribution and severity coefficient of variation of 1.5.

Year	Territory Exposure	Territory Claim Count	Territory Loss Ratio	Statewide Loss Ratio
2006	3,000	330	125%	78%
2007	3,020	420	153%	83%
2008	3,030	630	269%	85%
2009	3,020	210	122%	79%
2010	3,050	190	108%	72%
'06-'10	15,120	1,780	162%	80%

Limited Fluctuation – Example 3 (continued)

$$N = (z_p / k)^2 * \quad (Var(N)/E(N) \quad + \quad Var(S)/E(S)^2)$$

- Remember, with a Poisson distribution, Var(N) = E(N), so the second term is 1. The third term is the square of the coefficient of variation, which is 1.5^2. Now we just need to select the confidence levels.

- If we want to be within 5% of the true value 90% of the time, the value for $(z_p / k)^2$ is 1,082. Plugging into the formula:

$$N_{claims} = 1{,}082 * \ (1 \ + \ 1.5^2) \ = \ 3{,}516.5$$

- Assuming the 5-year statewide frequency is 0.2:

$$N_{exposures} = 3{,}516.5 / 0.2 \quad = 17{,}582.5$$

Limited Fluctuation – Example 3 (continued)

- To show the impact of our selection of an exposure standard instead of a claims standard.

Year	Territory Exposure	Territory Claim Count	Exposure Credibility	Claim Credibility
2006	3,000	330	41.3%	30.6%
2007	3,020	420	41.4%	34.6%
2008	3,030	630	41.5%	42.3%
2009	3,020	210	41.4%	24.4%
2010	3,050	190	41.6%	23.2%
'06-'10	15,120	1,780	92.7%	71.1%

Using a claims standard of 3,517 and an exposure standard of 17,583

Limited Fluctuation – Example 3 (continued)

- Determine what the indicated territorial factor, assuming 15% for fixed expenses.

Year	Territory Loss Ratio	Territory Credibility	Statewide Loss Ratio	Cred Wght Loss Ratio
'06-'10	162%	92.7%	80%	156.0%

156.0% = 92.7% x 162% + 7.3% x 80%

The final indicated territorial factor is (156% / 80%)*0.85 + 0.15 = **1.81**

An alternative approach would be to calculate the indicated factor prior to applying credibility, and then credibility weight the current factor with the indicated factor.

Least Squares – EVPV and VHM

- Assume we have 3 types of risk: low, medium, and high, which associated probabilities. Calculate the EVPV and VHM.

Risk	P(Claim)	P(Risk)	Variance	$Mean^2$
Low	20%	60%	0.16	0.04
Medium	30%	25%	0.21	0.09
High	40%	15%	0.24	0.16
Total	25.5%	100%	0.1845	0.0705

- EVPV: For binomial, variance = P(claim) x P(no claim)
  = (20%)(80%)(60%) + (30%)(70%)(25%) + (40%)(60%)(15%)
  = 0.1845
- VHM: $Mean^2 – (Mean)^2$
  = $0.0705 – (0.255)^2$
  = 0.0055

Least Squares – Example

- Assuming that you have the following book of business, calculate the EVPV, VHM, K, and Z. The prior estimate of the frequency is 0.517. With 4 years of observations and an observed frequency of 0.75, what is the estimated future frequency? Assume the claims are binomially distributed.

Risk	P(Claim)	P(Risk)	Variance	$Mean^2$
Low	40%	65%	0.24	0.16
Medium	70%	23%	0.21	0.49
High	80%	12%	0.16	0.64
Total	51.7%	100%	0.2235	0.2935

- EVPV: For binomial, variance = P(claim) x P(no claim)
  = (40%)(60%)(65%) + (70%)(30%)(23%) + (80%)(20%)(12%)
  = 0.2235
- VHM: $Mean^2 – (Mean)^2$
  = $0.2935 – (0.517)^2$
  = 0.0262

Least Squares – Example (continued)

- To determine K, we use K = EVPV/VHM, which is

  K = 0.2235 / 0.0262 = 8.53
- Since we're told that we have 4 years of observations, n = 4. Therefore,

  Z = n / (n + K) → 4 / (4 + 8.53) = 0.319.
- The prior estimate of frequency is the same as the mean calculated before, 0.517, and the observed data results in a frequency of 0.75. This observed data as 31.9% credibility, so…

  E2 = Z * T + (1 – Z) * E1 → 31.9% * 0.75 + 68.1% * 0.517 = 0.5913