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WC-5 Just How Credible Is That Employer?
Exploring GLMs and Multilevel Modeling for NCCI's Excess Loss Factor Methodology

CAS RPM Seminar

Philadelphia, PA

March 21, 2012

Presented by Chris Laws

Overview

- ELF Primer
- Motivation
- Alternatives to Current Approach
- Preliminary Approaches
- Results
- Next Steps
- Conclusion

Introduction

- NCCI is updating the methodology it uses to calculate Excess Loss Factors (ELFs)
- NCCI produces ELFs by state and hazard group
- ELFs are separated into the same two major components
  - Excess Ratio Curves
  - Severities and Loss Weights
- This presentation focuses on the improvements to the methodology used to arrive at the Severities and Loss Weights

What Are ELFs?

- An Excess Loss Factor (ELF) is the ratio of the expected portion of losses greater than a particular loss limit to standard premium
  - For example, given a loss limit of $200,000 and an associated ELF of 10%, the expected losses over the deductible or retention of $200,000 per occurrence is equal to 10% of standard premium
  - An ELF is the product of the Excess Ratio at a particular loss limit and the ratio of expected ground up losses to standard premium
    - $ELF = Excess\ Ratio\ \times Expected\ Loss\ Ratio$
- Let $R(y)$ be the Excess Ratio for the loss random variable Y with density function f at loss limit y
  - $R(y)$ is defined as the ratio of expected losses in excess of y to expected ground up losses
  - $R(y) = \frac{\int_y^\infty (t-y)f(t)dt}{E[Y]}$

ELF Primer

- The heart of NCCI's ELF calculation is the Excess Ratio Curve
- Underlying curves are only updated once every 5 to 10 years
- However ELFs are generally updated annually
- There are two design features in NCCI's ELF methodology which allow our ELFs to be responsive on an annual timescale while holding the underlying curves constant
  1. The curves are normalized to the average cost per case and are thus unitless
  2. Different curves are created for each of the following injury types:
     - Medical Only,
     - Temporary Total,
     - Permanent Partial,
     - Permanent Total, and
     - Fatal

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ELF Primer

Entry Ratios

- An entry ratio is defined as the ratio of a particular loss amount to the mean
  - If the mean loss is $250,000 an entry ratio of 2.0 would correspond to a loss of $500,000
- NCCI calculates and stores the excess ratio curves underlying the ELF calculation in terms of entry ratios
  - The implicit assumption is that losses of all sizes (within a category described by a single underlying curve) share a common severity trend
- When calculating excess ratios corresponding to the loss amounts needed for ELFs
  - The dollar amounts are normalized by the average cost per case (i.e. severity) to produce entry ratios
  - The entry ratio is then used to find the excess ratio corresponding to the dollar amount
- The result is
  - ELFs are responsive to annual severity trends
  - Underlying curves are comparable between states
  - Annual updates of ELFs require a sound severity estimate for each underlying state, hazard group, and injury type combination

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ELF Primer

Curves by Injury Type

- Final ELFs are intended to represent the loss experience for the entire state, hazard group combination
  - Hazard groups are industry classifications which range from
    - A (the least hazardous) to
    - G (the most hazardous)
- NCCI calculates excess ratios for injury types and averages these excess ratios together using loss weights
  - Injury Types are assumed to
    - Represent homogeneous losses
    - Separate heterogeneous losses
  - Medical Only claims have
    - A low average severity
  - Permanent Total claims have
    - A thick tail
    - A high average severity
- Some changes in shape at the state hazard group level can be captured
  - By changes in loss weights at the injury type level
  - By relative changes in severity at the injury type level
- Annual updates of ELFs require sound loss weight estimates

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ELF Primer

From Injury Types to Claim Groups

- NCCI is switching from injury types to claim groups as shown below

Current Grouping	Proposed Grouping
Fatal	Fatal
Permanent Total	Permanent Total
Permanent Partial	"Likely to Develop"* Permanent Partial & Temporary Total
Temporary Total	"Not Likely to Develop"* Permanent Partial & Temporary Total
Medical Only	Medical Only

*Claim groupings are differentiated based upon combinations of the injury type, claim status (open or closed) and the injured part of body. The various combinations are mapped to determine "Likely to Develop" or "Not Likely to Develop" claims.

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Motivation

- The most important ingredient in the annual ELF update is the severities and loss weights for each combination of state, hazard group, and claim group
- Such partitioning can result in extremely small sample sizes
  - Over 20 percent of NCCI states have zero Permanent Total claims for hazard group A (the least hazardous group) over a 5 year period
- Empirical statistics derived from such small samples generally have little resemblance to the true underlying data generating process
  - The heavy tail of the loss distribution for some claim groups only exacerbates the problem
- The smallest sample sizes are seen in the claim groups with thicker tails and thus a disproportionate impact on ELFs
  - As such, when deriving loss weights and severities, one needs a method to introduce a measure of stability balanced with responsiveness to the data

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Current Approach

Tempering for Large Fluctuations

- The current approach uses tempering to stabilize the effect of large fluctuations in empirical severities and loss weights
- Methods for tempering include
  - Removing development from large losses when calculating severities and loss weights (manually done as needed)
  - Taking weighted averages of indicated severities and/or loss weights with prior values (manually done as needed)
  - Averaging calculated excess ratio with prior trended excess ratios (done automatically as part of the ELF calculation)
- If we can reduce the amount of tempering required
  - We can streamline the production process
  - Produce more objective ELFs

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Alternative
Generalized Linear Model

- GLMs are one approach currently in use by the insurance industry to address problems similar to the one at hand
- GLMs extend least squares regression by allowing for
  – The assumption that observations follow a "non-normal" distribution
  – The assumption of a multiplicative (as opposed to additive) relation
- A large set of ready made tools exist for GLMs
  – Diagnostic and goodness-of-fit tests
  – Model fitting software and algorithms
- GLMs have interpretable parameters
  – Can be used to gain insight
  – Can be used to describe the approach to less technical audiences

GLMs also allow for other than multiplicative relations, but those are not of interest for this particular application

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Alternative
Multilevel Models\Random Effects

- GLMs do not automatically reduce the uncertainty surrounding parameters estimated in small samples
  – Multilevel models are models in which the parameters are themselves modeled; such models can serve to mitigate the problem of parameter uncertainty
- Multilevel models are similar in concept to Bühlmann's credibility
  – Both concepts rest on the ability to discern the variance within a group from the variance between groups in order to determine the appropriate level of credibility individual groups should receive
  – All else being equal
    - Larger groups receive more credibility
    - Low between group variation points toward decreased individual group credibility
- While actuaries generally speak of the concept of credibility, multilevel modelers generally speak of the concept of "shrinkage"

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Alternative

Multilevel Models\Random Effects and Bühlmann's Credibility

- Following Gelman and Hill (2007), let y be a normally distributed variable:

  $y_i \sim N\left(\alpha_{j[i]}, \sigma_y{}^2\right)$ (where j indicates a category (state, etc.) and i indicates the observation)

- A multilevel model would assume that the parameter α_j that governs the process in category j is a draw from a distribution common for all levels of this category:

  $$\alpha_j \sim N(\mu_\alpha, \sigma_\alpha{}^2) \;\; for\; j = 1, \ldots, m$$

- It can be shown that the multilevel estimator for α_j reads:

  $$\hat{\alpha}_j = \omega_j \mu_\alpha + \left(1 - \omega_j\right)\bar{y}_j, \quad \omega_j = 1 - \frac{\sigma_\alpha{}^2}{\sigma_\alpha{}^2 + \frac{\sigma_y{}^2}{n_j}}$$

Gelman, Andrew, and Jennifer Hill, Data Analysis Using Regression and Multilevel/Hierarchical Models, Cambridge (MA): Cambridge University Press, 2007

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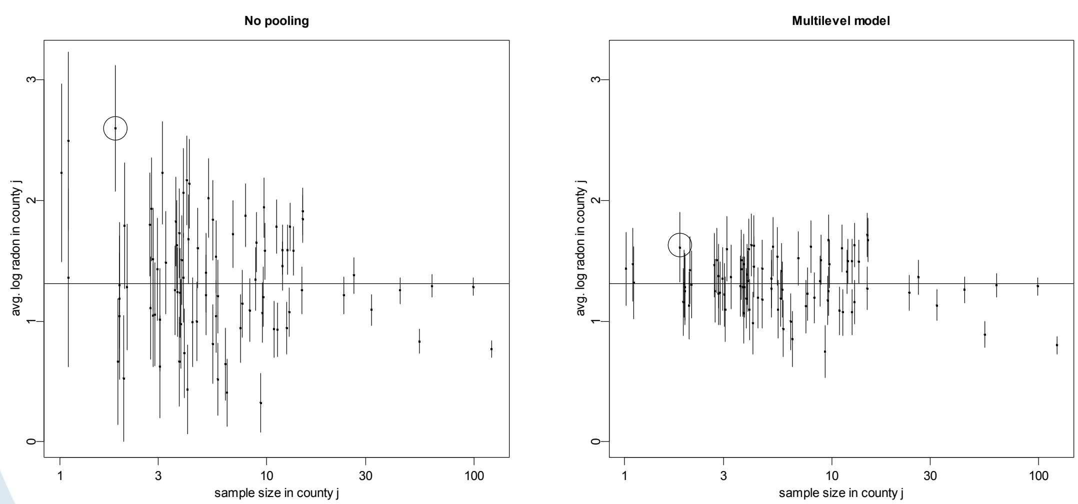
Alternative

Multilevel Models\Random Effects: Radon Example



The chart on the following slide displays a canonical example of multilevel modeling taken from Gelman and Hill (2007). The aim is to estimate radon level by county from several samples within each county. Many samples are taken in some counties. Few samples are taken in others. The chart on the left displays the sample mean. The chart on the right illustrates multilevel modeling.

In both charts:

The x-axis shows (on the log scale) the (jittered) number of observations in each county.
The y-axis measures the estimated county radon level.
Each bands represent a one standard deviation interval from the mean.
The highlighted county has the highest sample mean.

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Preliminary Severity Approach
The Model

A multilevel generalized linear model is used to model loss severity

- The model is linear on the log-scale and all covariates are categorical in nature
  - Thus, one can think of the model in the terms of rating factors, where the model attempts to estimate a “base” rate and multiplicative “factors” for the categories of interest
  - The model estimates such rating factors for state, hazard group, and claim group
  - The model allows for state specific claim group factors (interaction between state and claim group) and takes into account the correlation between these factors
    - Accounts for differences in state benefits by claim group

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Preliminary Severity Approach
The Model

Severities are assumed to follow a Gamma distribution

- This simplifies model specification since the arithmetic mean of independent Gamma distributed losses is also Gamma distributed
- Given the state, hazard group, and claim group of a claim, individual losses
  - Are assumed to be independent and to follow a Gamma distribution, thus
  - Their empirical arithmetic average will follow Gamma distribution with the same underlying mean and a variance scaled by $^1/_N$
    - A decrease in variance corresponds to an increase in credibility

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Possible Approaches For Loss Weights

- Using a GLM to estimate loss weights is not as straight forward as it is for estimating severities
- Three possible ways to estimate loss weights using GLMs are
  - **Option 1**
    Model loss weights directly
  - **Option 2**
    Model total losses and compute the necessary loss weights from the indicated total losses
  - **Option 3**
    Model claim counts and compute the necessary loss weights from the product of the indicated claim counts and indicated severities from a separate model

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Possible Approaches For Loss Weights

Pros and Cons

Options	Pros	Cons
1	• Models values of interest directly • Accounts for correlation between claim counts and severities	• Difficult to model due to uncommon distributions and support space • Difficult to interpret parameters
2	• Parameters have a more intuitive interpretation • Distributions are commonly used • Accounts for correlation between claim counts and severities	• Cannot handle observed $0 losses without sophisticated techniques
3	• Parameters have a more intuitive interpretation • Distributions are commonly used • Handles observed $0 total losses	• Does not account for correlation between claim counts and severities – such correlation should be mild as severities and claim counts refer to the aggregation of many risks

Option 3 was selected as the best option to pursue.

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Preliminary Loss Weight Approach

The Model

Claim counts are assumed to follow a Negative Binomial distribution

- Assume Y_{gshr} represents total claim counts for claim group g, state s, hazard group h, and report r and $\mu_{gshr} = E(Y_{gshr})$, then
  - The Negative Binomial is parameterized such that $Var(Y_{gshr}) = \mu_{gshr} + \frac{\mu^2_{gshr}}{\theta_g}$
  - θ_g is a parameter estimated from the observed data which varies by claim group
  - For $\theta_g = \infty$, the model reduces to a Poisson distribution
- The expected number of claims for each claim group, state, hazard group, and report combination is estimated as $\mu_{gshr} = \delta_{shr} \cdot e^{\gamma_g+\xi_s+\eta_h+\rho_r+\varepsilon_{gsh}}$
  - δ_{shr} represents the (unadjusted) payroll and serves as a proxy for exposure
  - ε_{gsh} represents an error term for each claim group, state, and hazard group combination
    - Credibility is introduced on the estimated error terms by assuming that $\varepsilon_{gsh} \sim t(0, \sigma_g, 4)$, where t represents the t distribution and σ_g is estimated from the data
    - All else begin equal, the larger the estimated relative variation for observed claim counts within a claim group g, state s, and hazard group h, the closer ε_{gsh} will be to zero
  - γ_g, ξ_s, η_h, and ρ_r are parameters to be estimated

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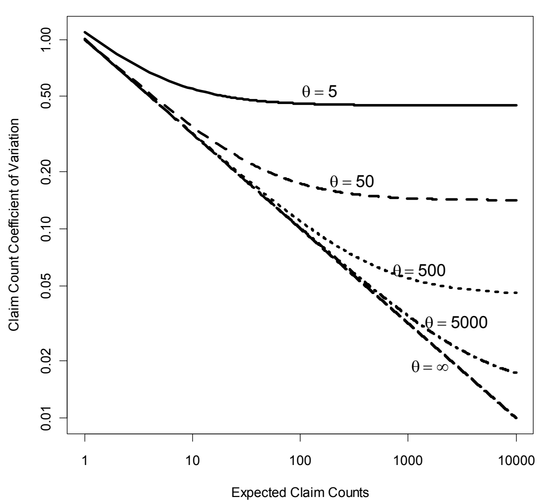
Preliminary Loss Weight Approach

The Negative Binomial Distribution



The above chart displays the resulting relation between the expected claim counts and the implied standard deviation for select values of $\boldsymbol{\theta}$. As $\boldsymbol{\theta}$ approaches ∞ the Negative Binomial converges to a Poisson distribution.

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Summary of Preliminary Approaches

- The severity model requires as input
  - Observed severities (medical plus indemnity) by state, hazard group, and claim group
    - Developed
    - Trended
    - On-leveled
  - Observed claim counts by state, hazard group, and claim group
    - Developed
- The claim count model requires as input
  - Observed claim counts by state, hazard group, claim group, and report
    - Developed
  - Observed payroll by state, hazard group, and report
    - Simple trending is currently handled by the model
- The estimated claim counts will then be combined with estimated severities from the severity model to produce the required loss weights

Implementation

- R is used in the pre and post-estimation process
- The model is estimated in JAGS
- R, http://www.r-project.org/
  - Open source software environment for statistical computing and graphics
  - Implementation of the S language, which was developed at Bell Laboratories
- JAGS – Just Another Gibbs Sampler, http://sourceforge.net/projects/mcmc-jags/files/
  - Open source program for the statistical analysis of Bayesian hierarchical models by Markov Chain Monte Carlo simulation
  - Called from R using the package rjags, http://cran.r-project.org/web/packages/rjags/index.html

Evaluations Performed

- Both models were evaluated for
  - Model Fit: The closeness of the indicated values to observed values
  - Sensitivity: The impact of random fluctuations on indicated values
- Residual Plots were examined and Goodness of Fit Test were preformed to evaluate the “Model Fit”
- To evaluate the sensitivity of the severity model, a bootstrap analysis was performed
- To evaluate the sensitivity of the total claim count model, a “remove-one report” analysis was performed
- The implemented sensitivity evaluations also guard against over-fitting
  - If the model over-fits, the indicated values will follow the random functions

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Evaluation of the Severity Model

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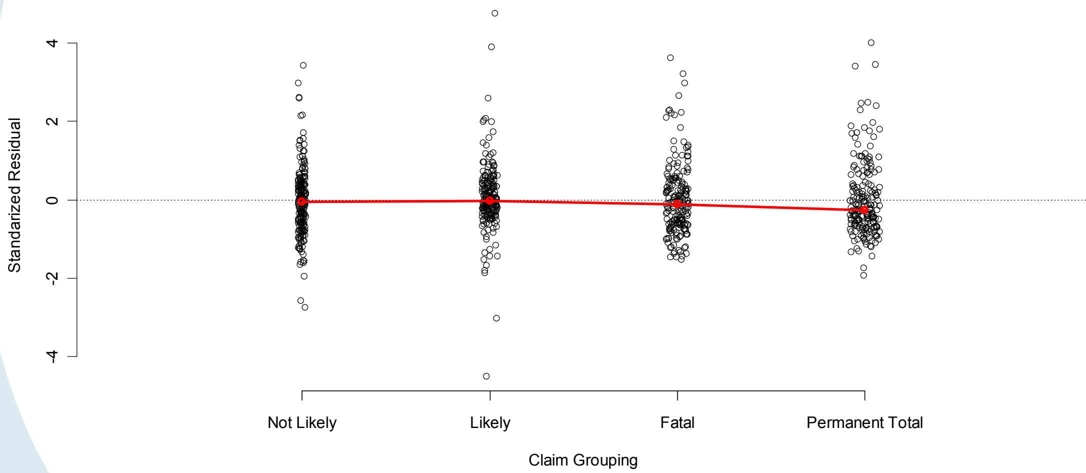
Severity Model Fit

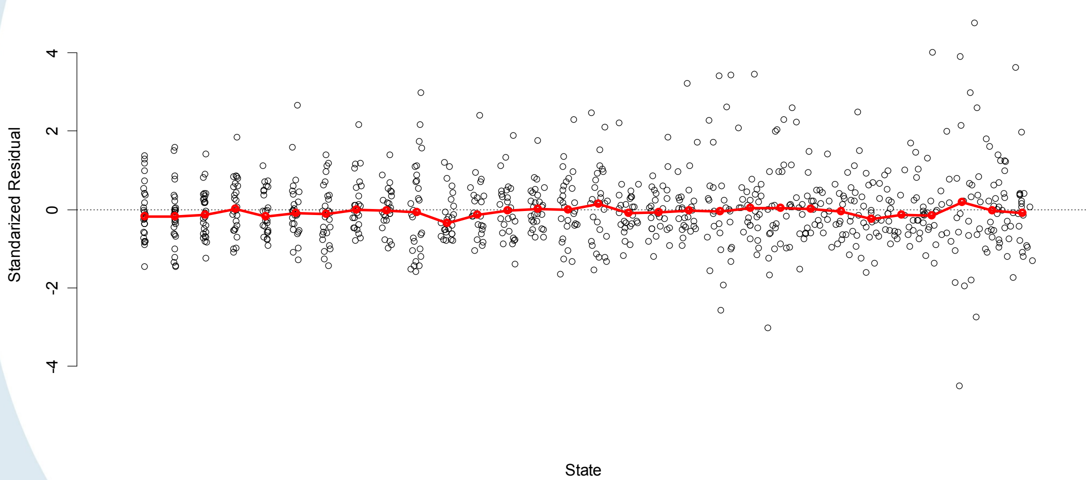
Standardized Residual Charts: By Claim Group



Each point represents an observed state, hazard group, and claim group combination.

Severity Model Fit

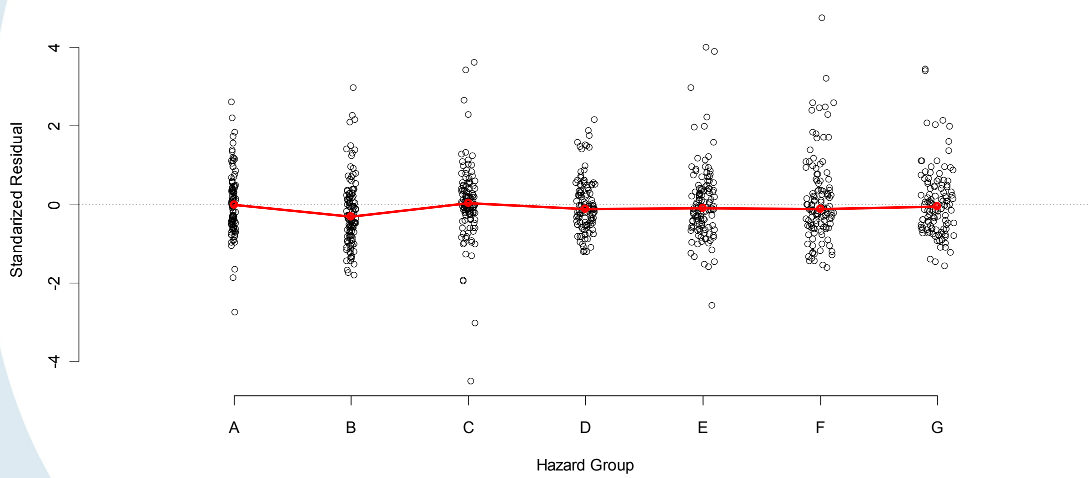
Standardized Residual Charts: By State



Each point represents an observed state, hazard group, and claim group combination.

Severity Model Fit

Standardized Residual Charts: By Hazard Group



Each point represents an observed state, hazard group, and claim group combination.

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Bootstrapping

- Simple bootstrapped samples are generated by resampling, with replacement, from the observed dataset
  - The theoretical motivation is to generate data from a process similar to the true underlying process with the assumption that the empirical distribution is such a process
  - These samples can then be used to evaluate, among other things, the volatility of an estimator
- For example, suppose one observes loss of:
  $15k, $12k, $2k, $10k, $7k and $5k with a mean of $8.5k
  - One randomly generated bootstrapped sample might be:
    $15k, $2k, $2k, $5k, $7k and $5k with a mean of $6k
  - Another sample might be:
    $15k, $15k, $2k, $10k, $7k and $7k with a mean of $9.3k
- Claim characteristics (such as state and hazard group) are maintained throughout the sampling process
  - Categories with more claims in the empirical sample are likely to have more claims in any given bootstrapped sample

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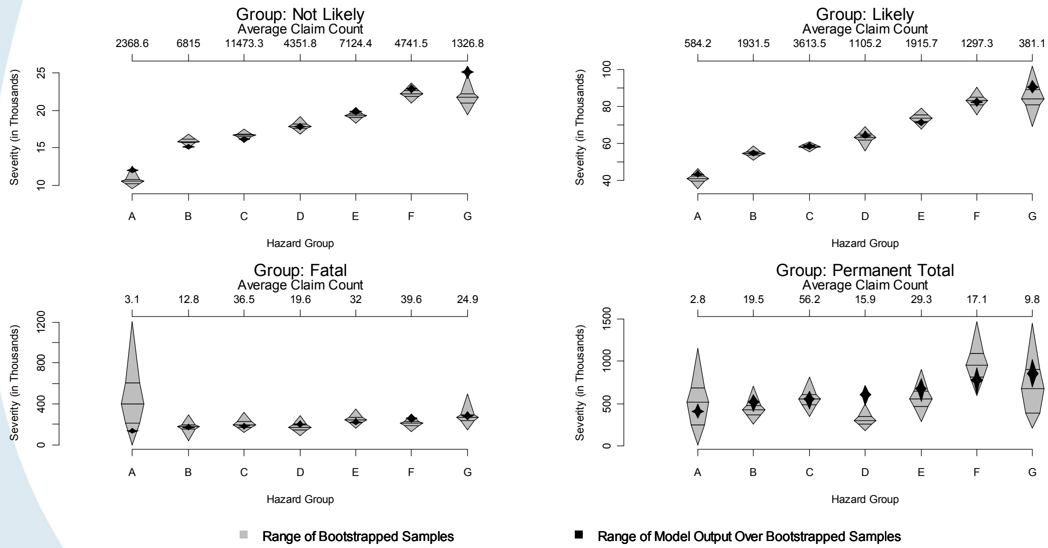
Bootstrapped Results

Large State



Black polygons represent the range of model estimated severities over 100 bootstrapped samples
Gray polygons represent the range of empirical severities over 100 bootstrapped samples
The top and bottom of each polygon represents the max and min
The widest point on the polygon represents the median
The top and bottom “notches” represent the 75th and 25th percentiles

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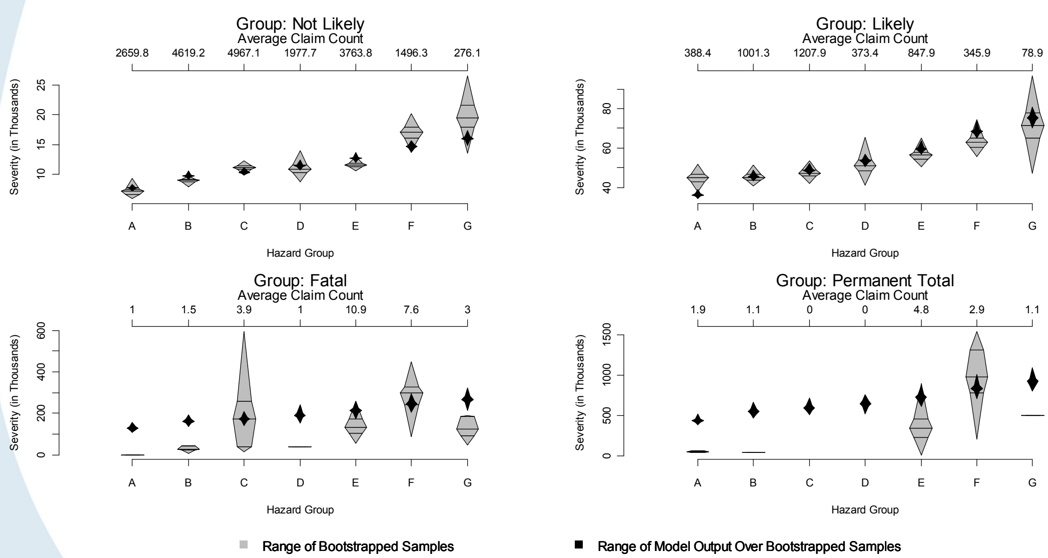
Bootstrapped Results

Small State



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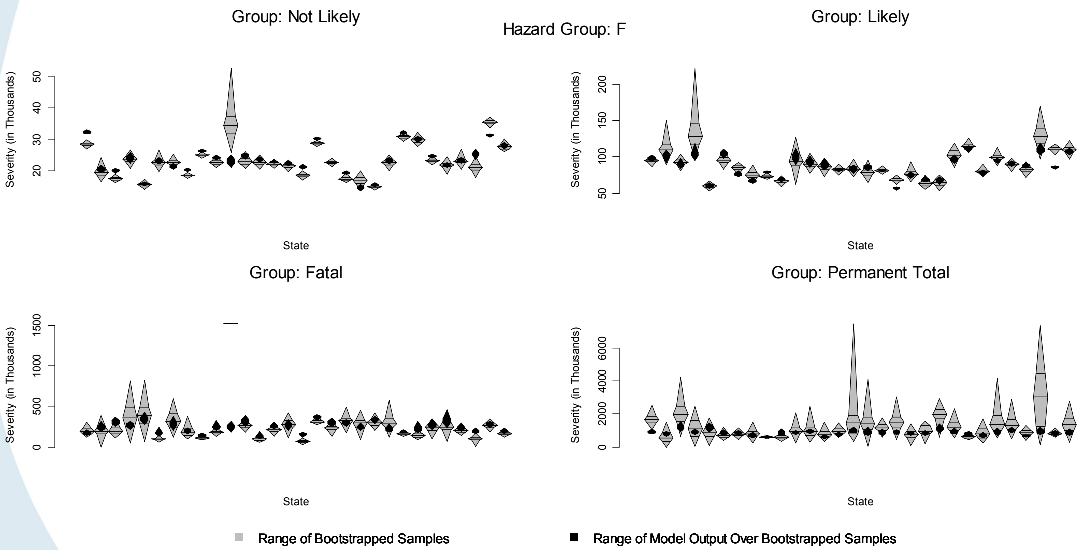
Bootstrapped Results Across States

Hazard Group F



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The widest point on the polygon represents the median
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Evaluation of the Claim Count Model

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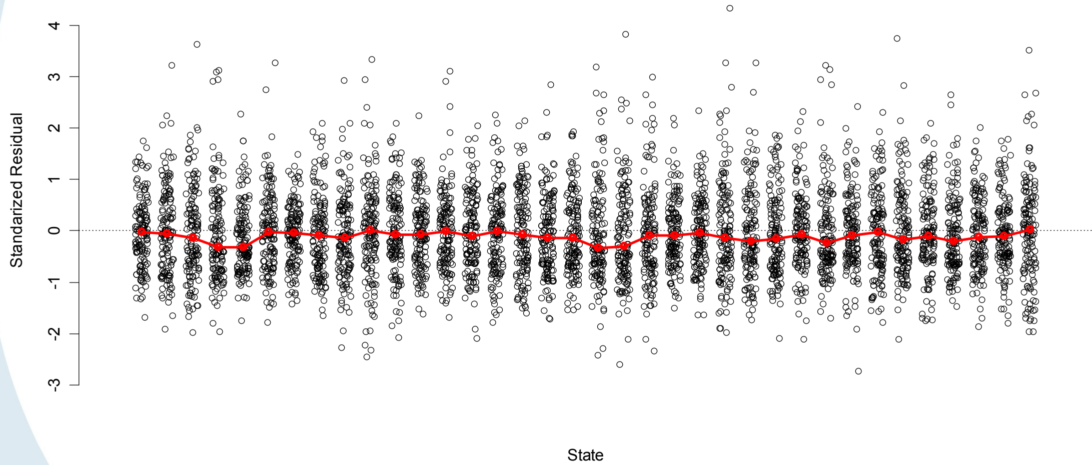
Claim Count Model Fit

Standardized Residual Charts: By State



Each point represents an observed state, hazard group, claim group, and report combination.
The red line indicates the median residual.

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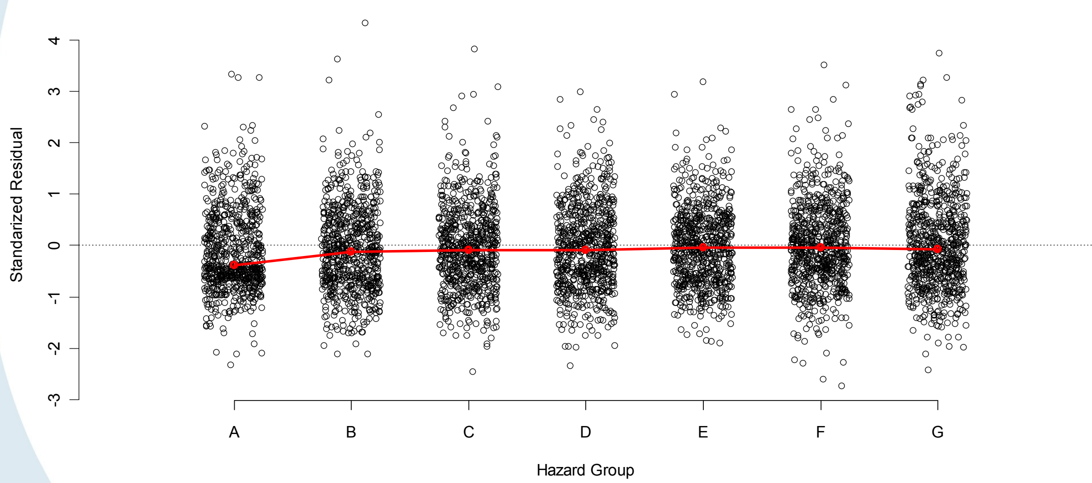
Claim Count Model Fit

Standardized Residual Charts: By Hazard Group



Each point represents an observed state, hazard group, claim group, and report combination.
The red line indicates the median residual.

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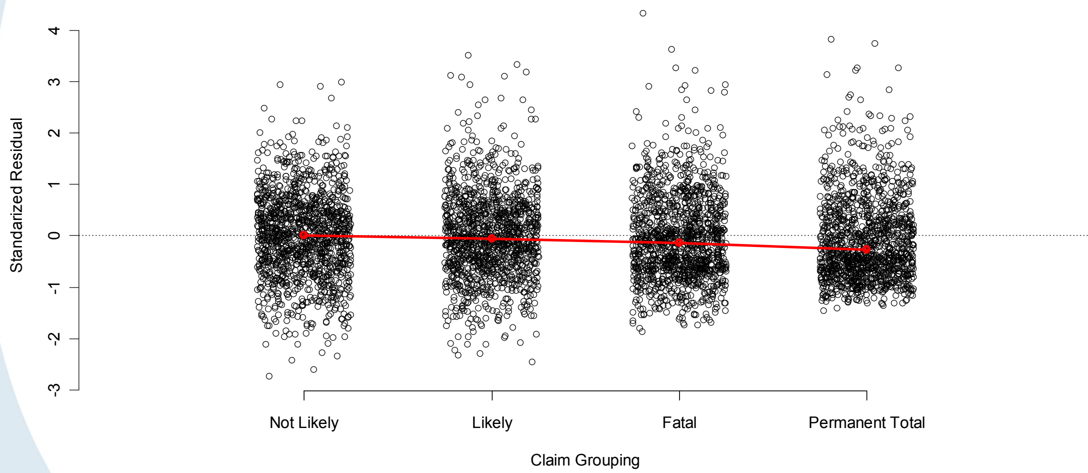
Claim Count Model Fit

Standardized Residual Charts: By Claim Group



Each point represents an observed state, hazard group, claim group, and report combination.
The red line indicates the median residual.

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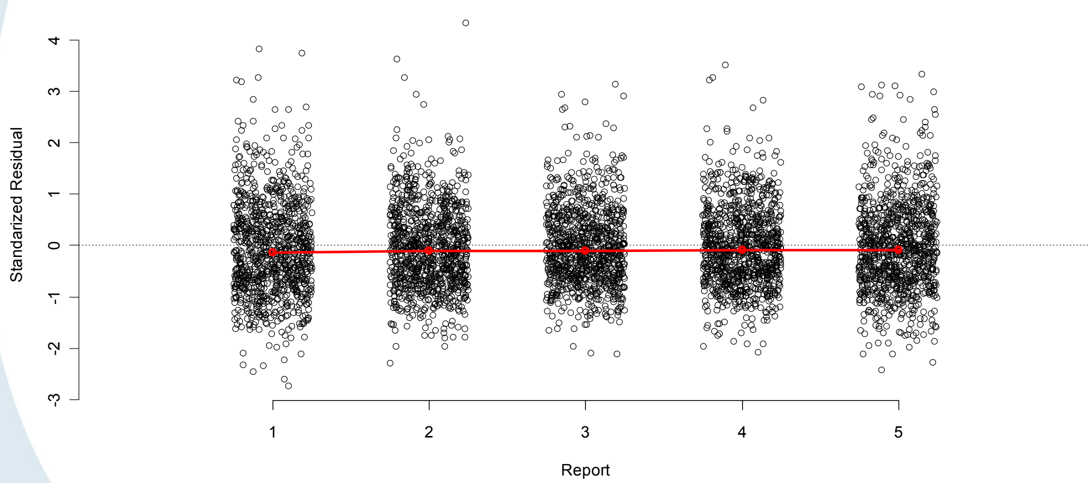
Claim Count Model Fit

Standardized Residual Charts: By Report



Each point represents an observed state, hazard group, claim group, and report combination.
The red line indicates the median residual.
Reports represent policy periods developed to 5th report.

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Remove-One Report

- To assess the influence of statistical noise in the annual update, the model is estimated for the 5 sets of 4 reports created by removing, in turn, each report from the 5 reports included in the full dataset
- The range of the 5 predicted values is then compared
  - to the 5 observed values and
  - to the range of the empirical mean calculated on the 5 sets of 4
- For example,
  - suppose that we have observed claim counts of 0, 1, 5, 7, and 10
  - The 5 sets of 4 would then be
    - 0, 1, 5, and 7; with an arithmetic mean of 3.25
    - 0, 1, 5, and 10; with an arithmetic mean of 4
    - 0, 1, 7, and 10; with an arithmetic mean of 4.5
    - 0, 5, 7, and 10; with an arithmetic mean of 5.5
    - 1, 5, 7, and 10; with an arithmetic mean of 5.75

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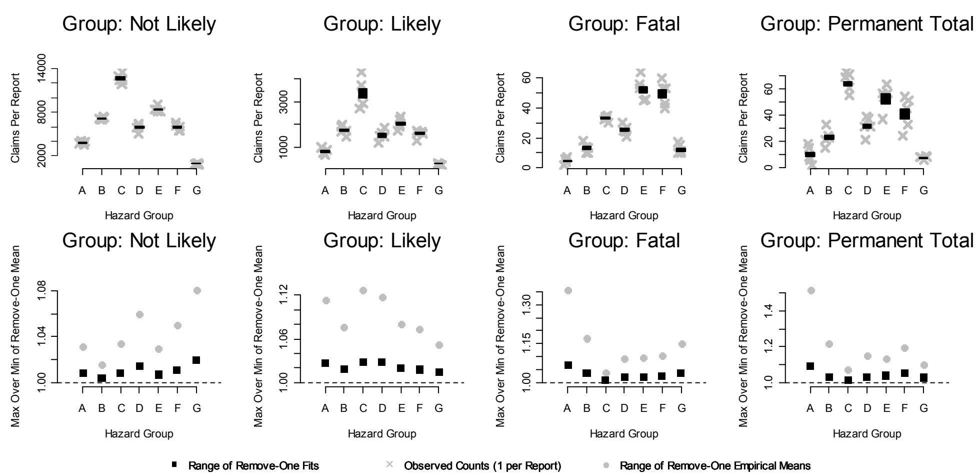
Remove-One Results

Large State



Gray "X-s" in the first row represent observed claim counts – one for each report.
The height of the black rectangles in the first row represent the range of remove-one predicted values.
Gray circles in the bottom row represent the ratio of the maximum to the minimum remove-one empirical means.
The center of the black boxes in the bottom row represent the ratio of the maximum to the minimum remove-one fitted means.

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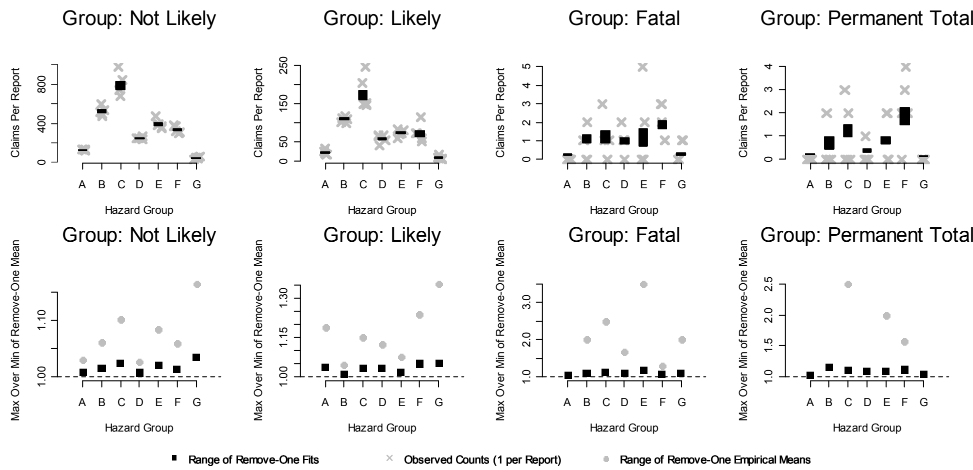
Remove-One Results

Small State



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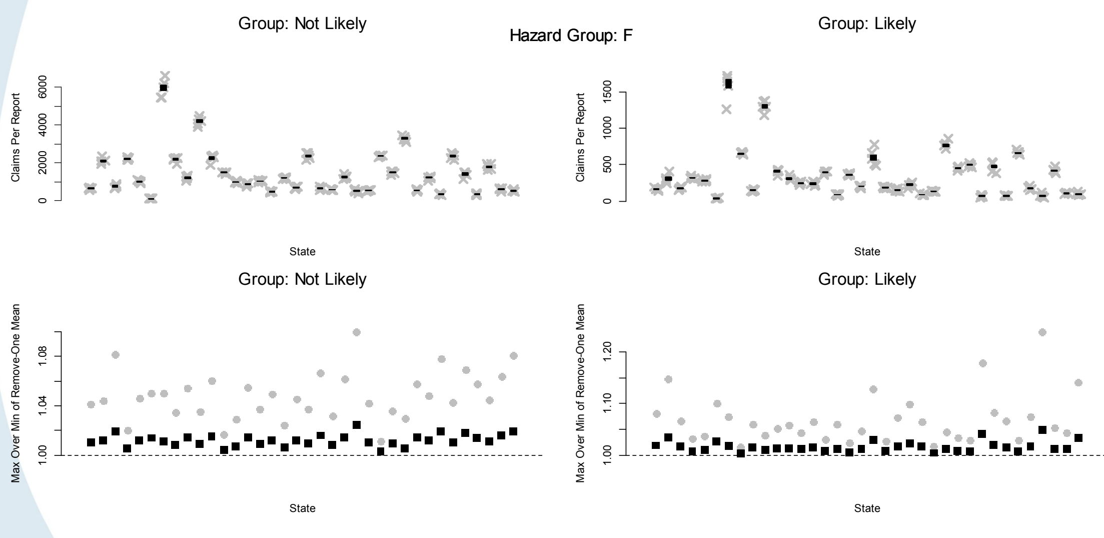
Remove-One Results Across States

Hazard Group F



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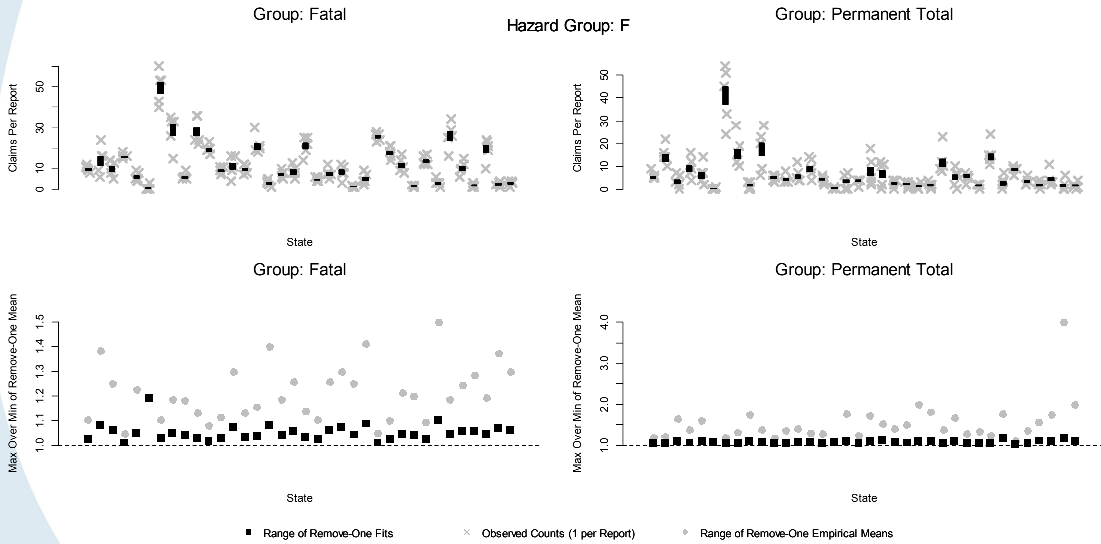
Remove-One Results Across States

Hazard Group F



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Next Steps

- Endogenous model improvements
  - The claim count model uses reports and has an error term for each state, hazard group, and claim group combination
    - As such, it is more flexible than the severity model
    - We are currently exploring incorporating such flexibility into the severity model
  - We are seeking final structural form for both models
- Implementation
  - Simple tempering of the data prior to model estimation, e.g. remove development from large claims
  - Integration with production process
  - Determining the appropriate spread of values across hazard groups

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Conclusion

- This presentation introduces a new approach to calculating severities and loss weights by state, hazard group, and claim group for the ELF methodology
- The approach uses commonly employed techniques to introduce a measure of stability
- The proposed approach offers the opportunity for
  - Increased automation
  - A decreased need for manual tempering
  - Allows for a more streamlined ELF calculation

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Questions?

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