



GLM III - The Matrix Reloaded
Duncan Anderson

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Agenda

- Testing the link function
- "Quadrant Saddles"
- "Emergent Interactions"
- Case deleted deviance
- Man (with GLM) vs machine
- For example - UK bodily injury
- GLMs - getting better with age?

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Formularization of GLMs

$$E[Y_i] = \mu_i = g^{-1}(\sum X_{ij} \cdot \beta_j + \xi_i)$$

$$\text{Var}[Y_i] = \phi \cdot V(\mu_i) / \omega_i$$

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Formularization of GLMs

$$E[Y_i] = \mu_i = g^{-1}(\sum X_{ij} \cdot \beta_j + \xi_i)$$

↑
Link function

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Link function

Eg if $\sum X_{ij} \cdot \beta_j =$
 $\beta_1 + \beta_2$ if male + β_3 if small car + β_4 if big car

$g(x) = x \Rightarrow E[Y_i] = \beta_1 + \beta_2 + \beta_3 + \beta_4$

$g(x) = \ln(x) \Rightarrow E[Y_i] = e^{\beta_1 + \beta_2 + \beta_3 + \beta_4}$
 $= e^{\beta_1} \cdot e^{\beta_2} \cdot e^{\beta_3} \cdot e^{\beta_4}$
 $= A \cdot B \cdot C \cdot D$

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Box-Cox link function test

$$E[Y_i] = \mu_i = g^{-1}(\sum X_{ij} \cdot \beta_j + \xi_i) \quad \text{Var}[Y_i] = \phi \cdot V(\mu_i) / \omega_i$$

Box-Cox link function defined as:

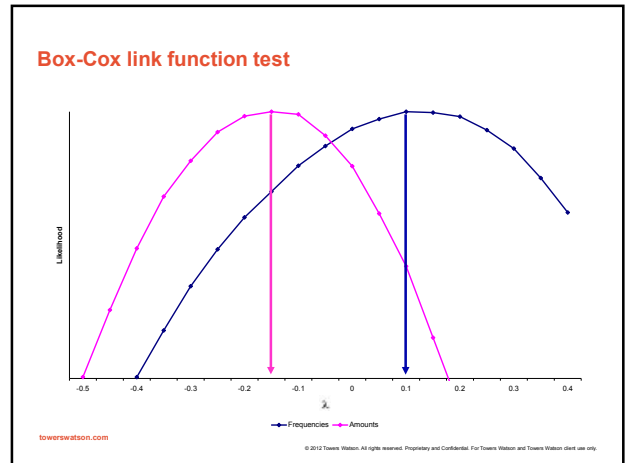
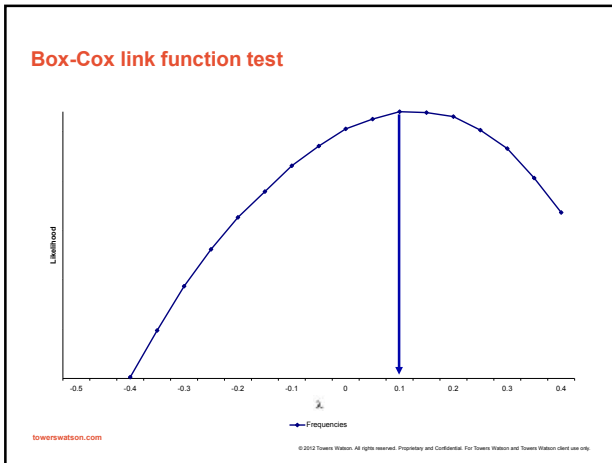
$g(x) = (x^\lambda - 1) / \lambda$ for $\lambda \neq 0$; $\ln(x)$ for $\lambda = 0$

$\lambda = 1 \Rightarrow g(x) = (x - 1) \Rightarrow$ additive (with a base level shift)
 $\lambda \rightarrow 0 \Rightarrow g(x) \rightarrow \ln(x) \Rightarrow$ multiplicative (via l'Hôpital)
 $\lambda = -1 \Rightarrow g(x) = 1 - 1/x \Rightarrow$ inverse (with a base level shift)

Test a range of values of λ and see which maximizes likelihood

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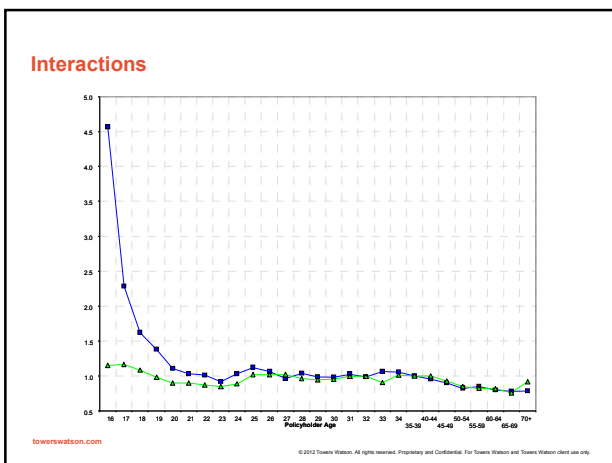
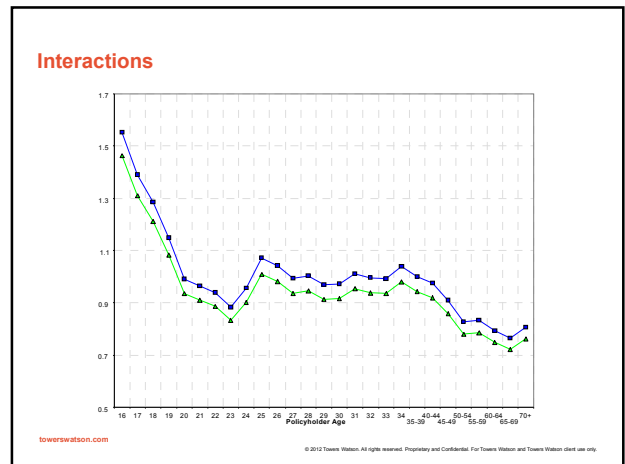
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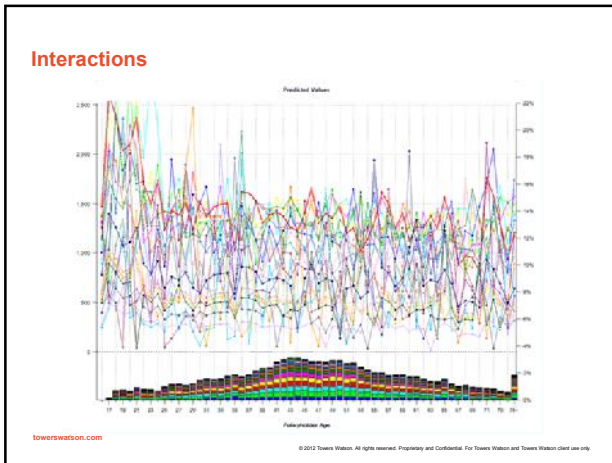
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Why are interactions present?

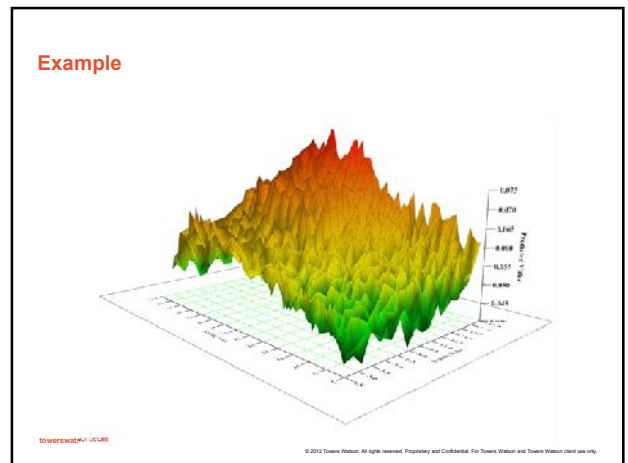
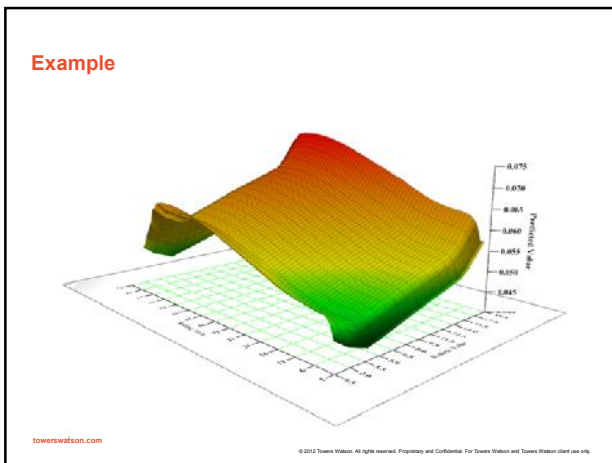
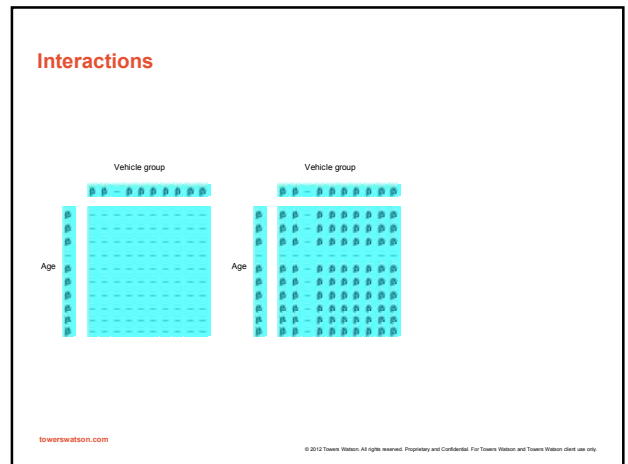
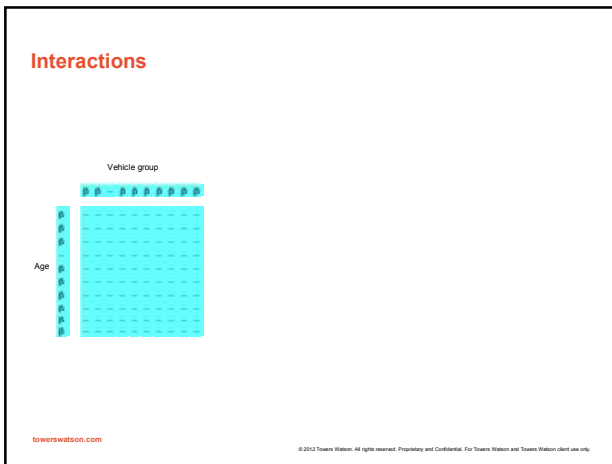
- Because that's how the factors behave
- Because the multiplicative model can go wrong at the edges
 - $1.5 * 1.4 * 1.7 * 1.5 * 1.8 * 1.5 * 1.8 = 26!$

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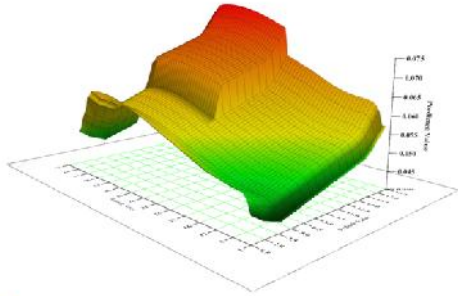


Interactions

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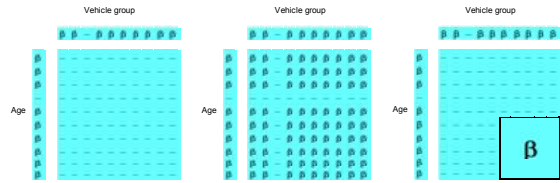
Example



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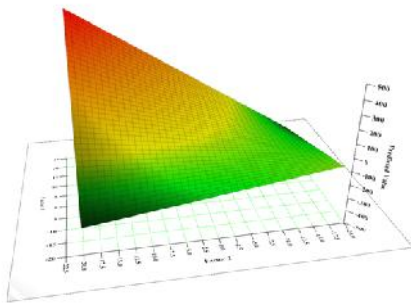
Interactions



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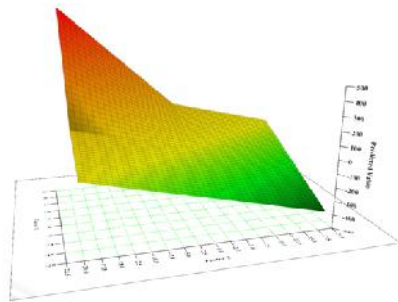
Saddles



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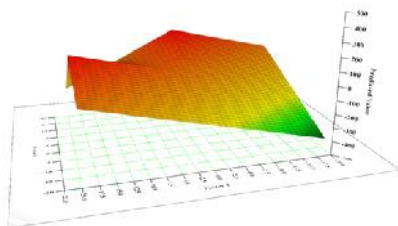
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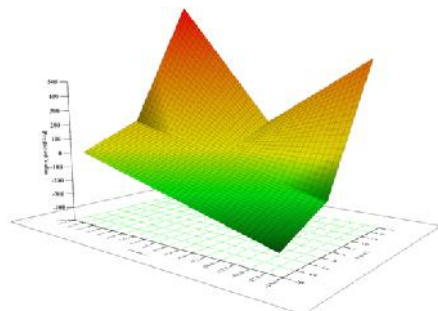
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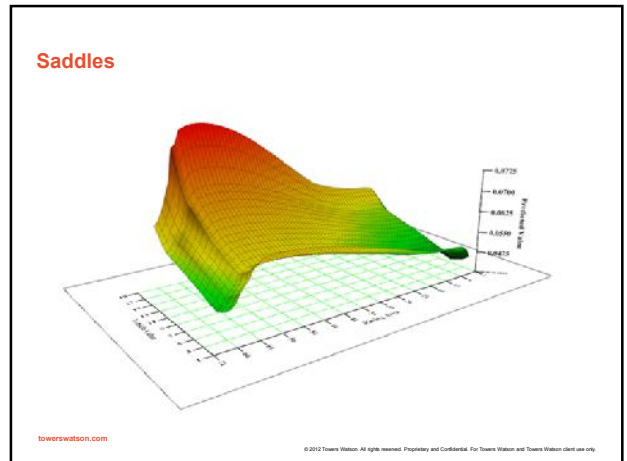
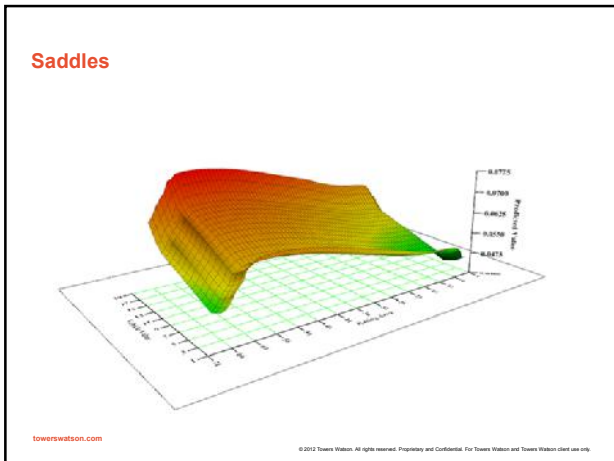
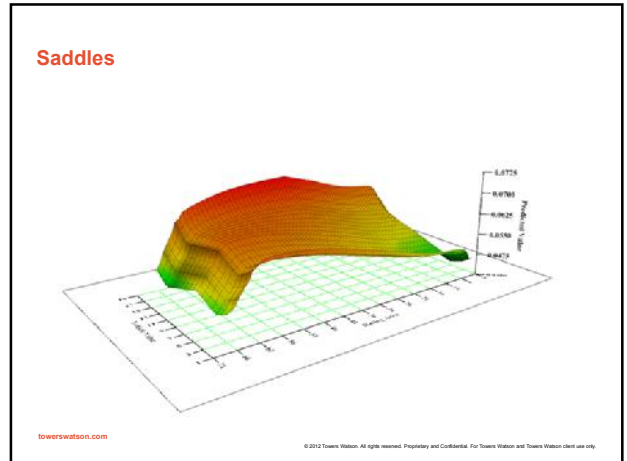
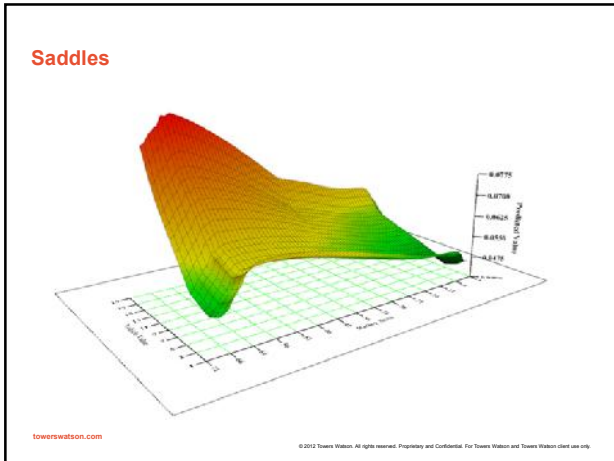
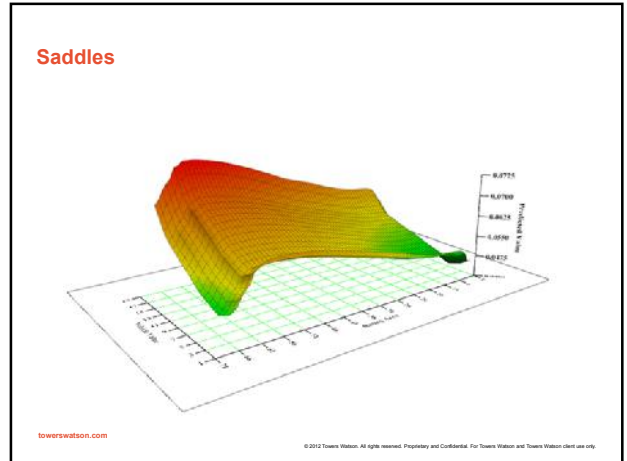
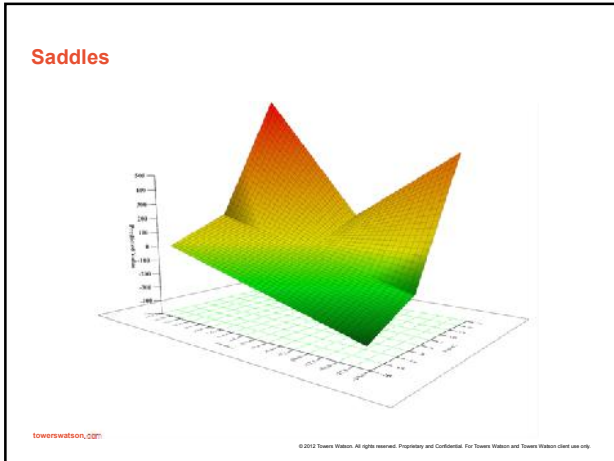
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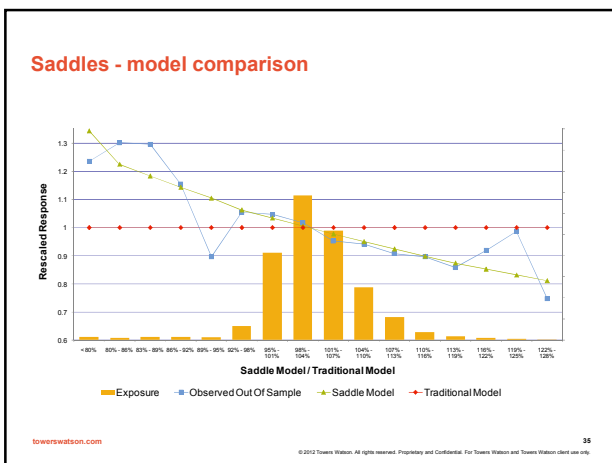
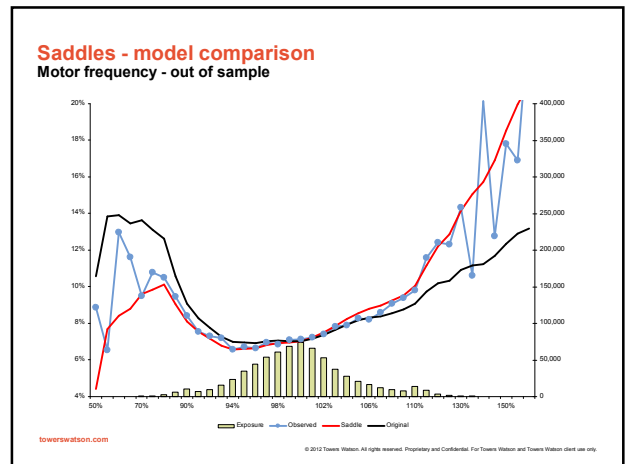
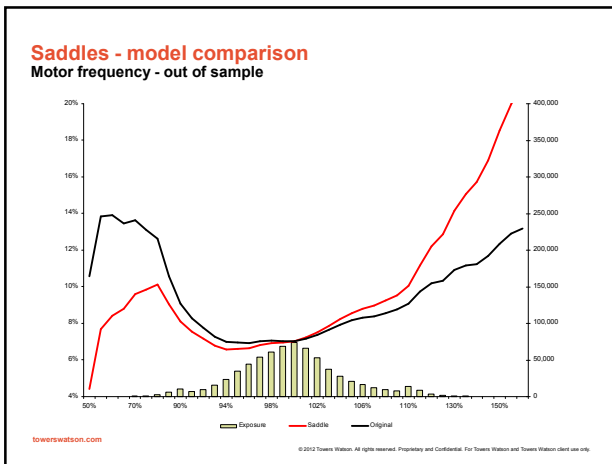
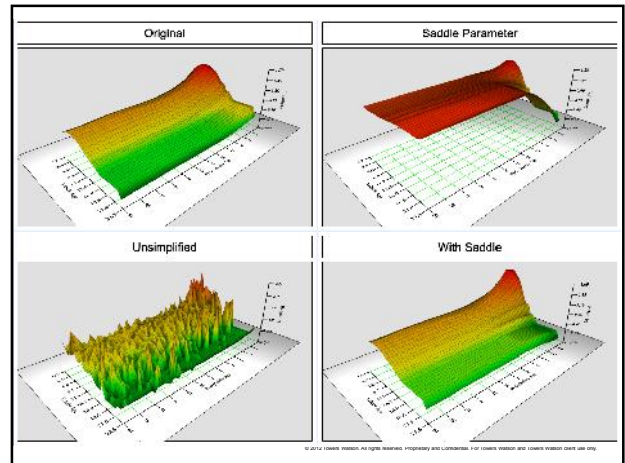
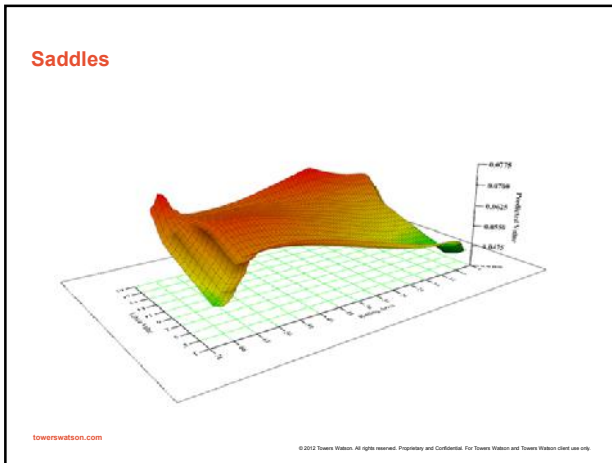
Saddles



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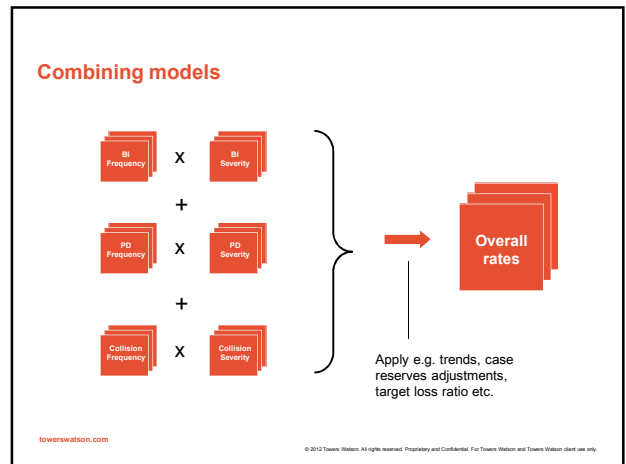
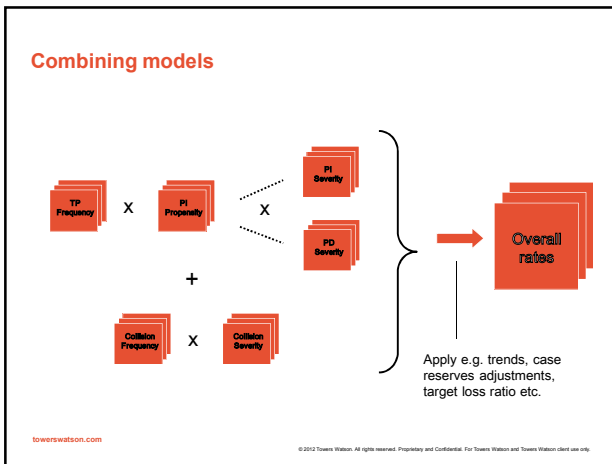
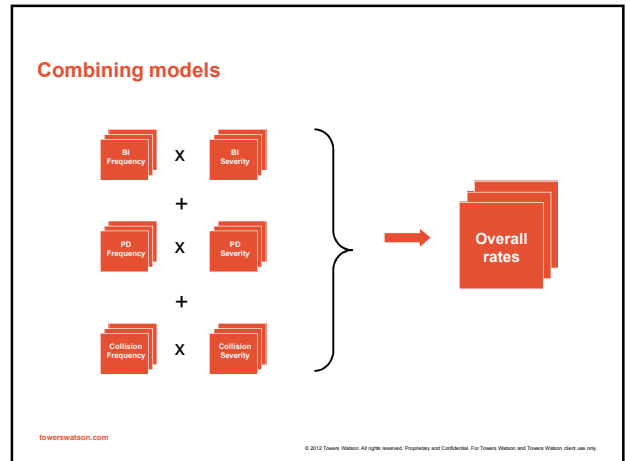
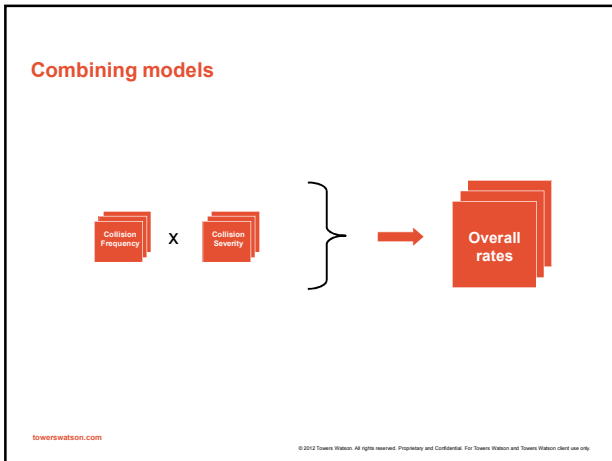


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- ### Combining models
- Take models
 - Take relevant mix of business
 - eg current in force policies
 - For each record calculate expected frequencies and severities according to the models
 - For each record, calculate expected total cost of claims "C"
 - Fit a GLM to "C" using all available factors
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Combining models

		PD Numbers	PD Amounts	BI Numbers	BI Amounts
Intercept		32%	\$1000	12%	\$4860
Sex	Male	1.000	1.000	1.000	1.000
	Female	0.750	1.200	0.667	0.900
Area	Town	1.000	1.000	1.000	1.000
	Country	1.250	0.700	0.750	0.833

Policy	Sex	Area	NUM1	AMT1	NUM2	AMT2	CC1	CC2	RISKPREM
...
82155654	M	T	32%	1000	12%	4860	320	583.20	903.20
82168746	F	T	24%	1200	8%	4374	288	349.92	637.92
82179481	M	C	40%	700	9%	4050	280	364.50	644.50
82186845	F	C	30%	840	6%	3645	252	218.70	470.70
...

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Except...

Policy	Sex	Area	NUM1	AMT1	NUM2	AMT2	CC1	CC2	RISKPREM
...
82155654	M	T	32%	1000	12%	4860	320	583.20	903.20
82168746	F	T	24%	1200	8%	4374	288	349.92	637.92
82179481	M	C	40%	700	9%	4050	280	364.50	644.50
82186845	F	C	30%	840	6%	3645	252	218.70	470.70
...

- The global risk premium is not multiplicative
- In the town, women have a modelled claim cost 29% lower than men
 - $637.92/903.20=0.706$
- In the country, women have a modelled claim cost 27% lower than men
 - $470.07/644.50=0.730$

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To solve...

Policy	Sex	Area	NUM1	AMT1	NUM2	AMT2	CC1	CC2	RISKPREM
...
82155654	M	T	32%	1000	12%	4860	320	583.20	903.20
82168746	F	T	24%	1200	8%	4374	288	349.92	637.92
82179481	M	C	40%	700	9%	4050	280	364.50	644.50
82186845	F	C	30%	840	6%	3645	252	218.70	470.70
...

- We can capture this result exactly with an interaction

Total risk premium			
Intercept		\$903.20	
Area	Sex	Male	Female
	Town	1.000	0.706
Country		0.714	0.521

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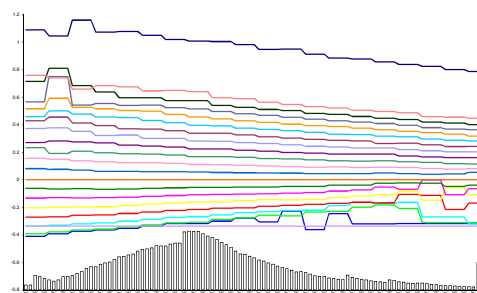
"Emergent" interactions

- In the above example the interaction "emerged" from the risk premium step
- Emergent interactions are *not* risk insights, there is no subtle risk effect we have just discovered
 - The different behaviour is by peril, and the rating factors are just bad proxies for the peril effects
- Emergent interactions are corrections to fix problems we have introduced
- Best solution is by peril pricing
 - Reflects true behaviour
 - Underlying models simple to understand and implement
- If not, check for emergent interactions in the risk premium

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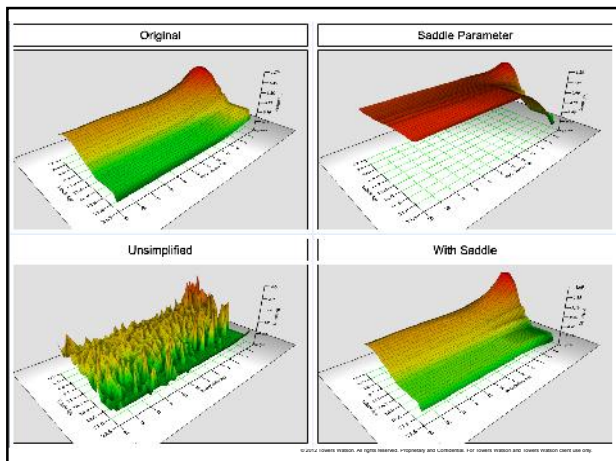
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Example "emergent" interaction



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"Case Deleted Deviance"

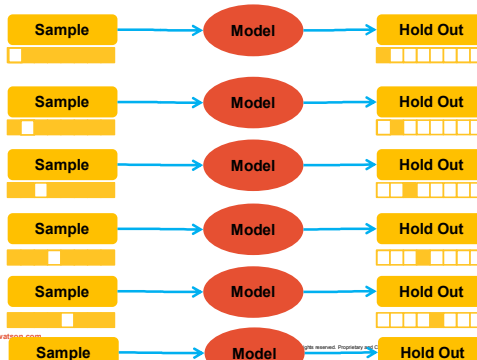
- Tony Lovick & Peter Lee
- Sessional Meeting of the Institute and Faculty of Actuaries 28 March 2012
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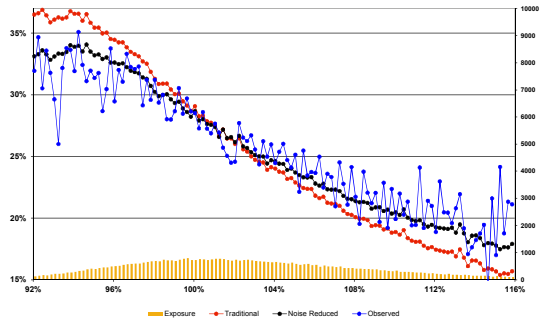
Motivation for methods Cross validation



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"Case Deleted Deviance"



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Machine vs man



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Machine vs man



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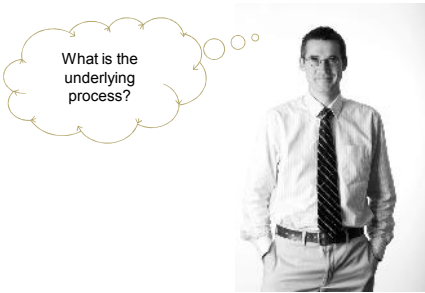
Machine vs man



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Machine vs man

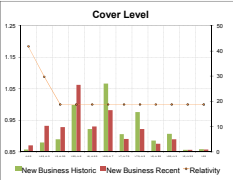


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Machine vs man

Underwriting




Claims



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Machine vs man



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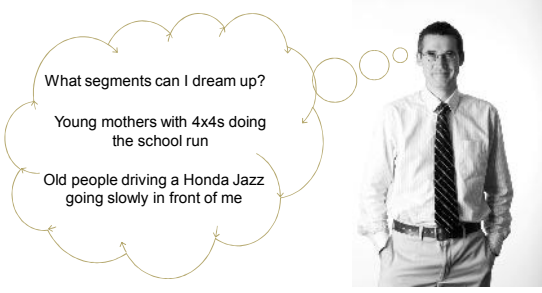
Drivers of elasticity

(Contents deleted from handout)

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Machine vs man



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Machine vs man

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Machine vs man

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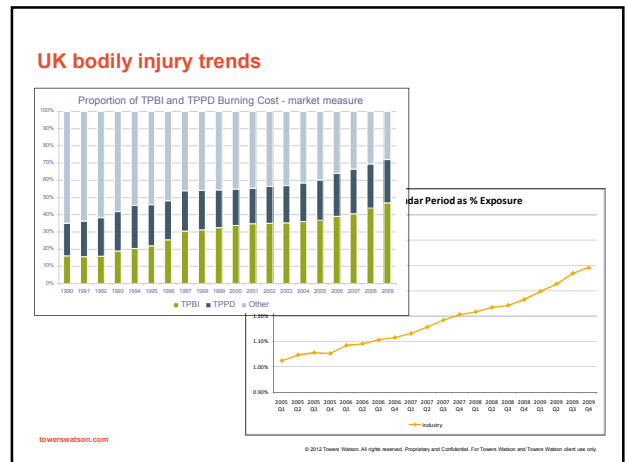
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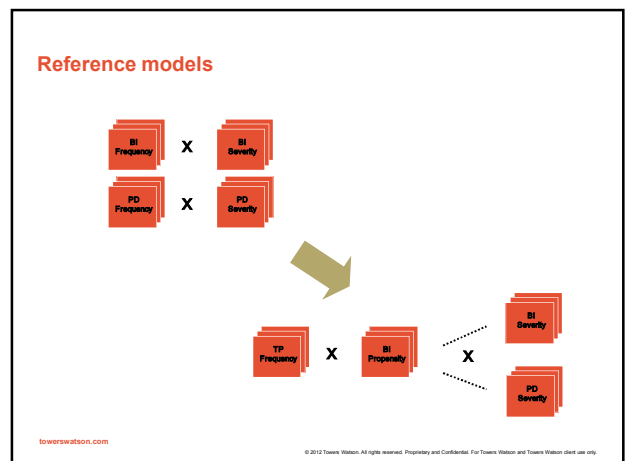


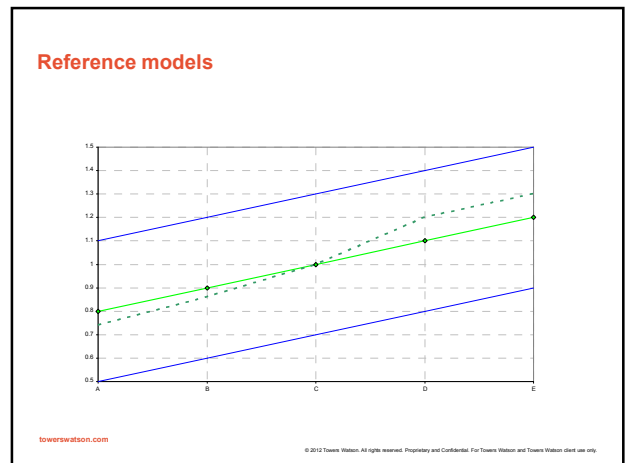
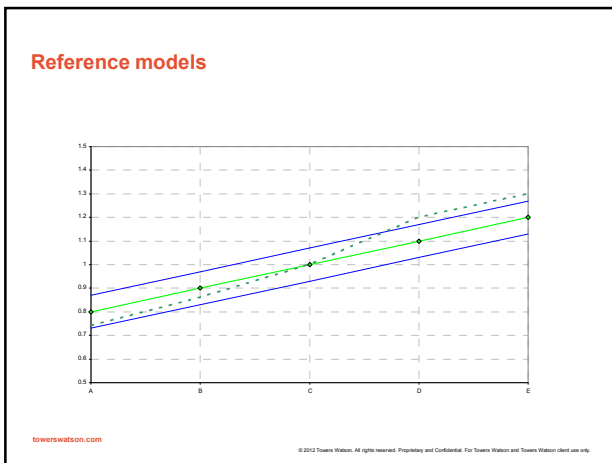
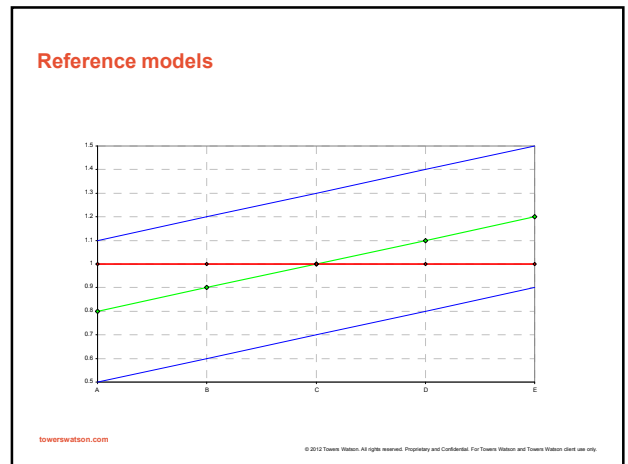
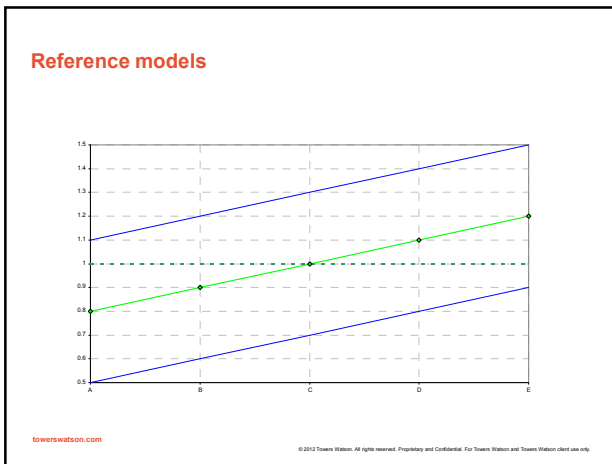
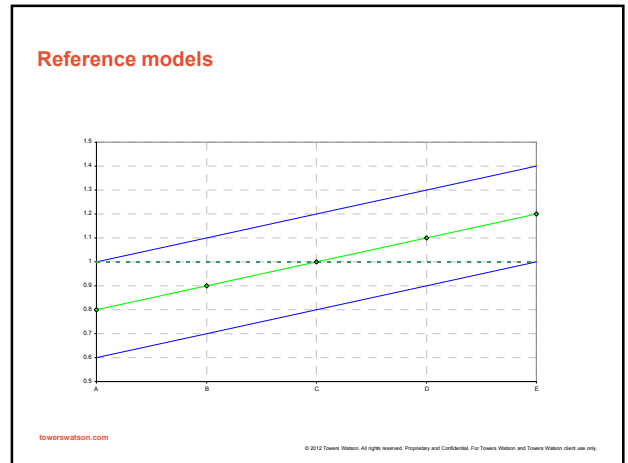
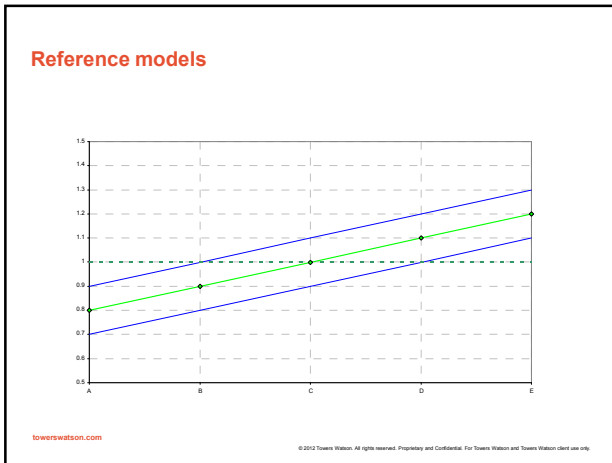
Amplification of the BI signal using PD experience

- BI/PD proportion model:
 - BI frequency = BI/PD proportion * PD frequency
- Offset PD relativities onto BI data as starting point
- Use PD model structure
- Use PD model as a guide in free fitting BI

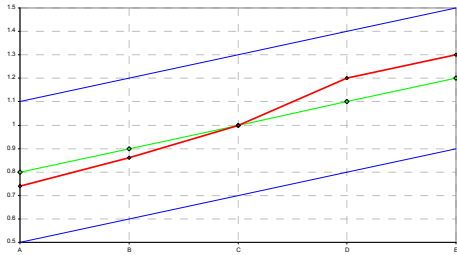
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Reference models



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Reference models

$$E[Y_i] = \mu_i = g^{-1}(\sum X_{ij} \cdot \beta_j + \xi_i)$$

↑
Offset term

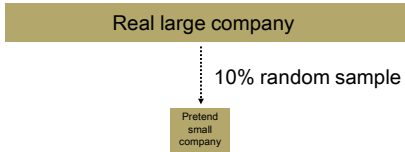
- When modeling BI set PD fitted values to be offset term
- GLM will seek effects over and above assumed PD effect

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Experiment

(1) GLM on BI claims on all the data - the "correct" answer

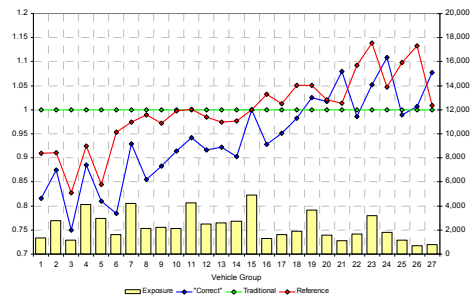


- (2) Traditional GLM on BI claims on the "small company"
- (3) Propensity reference model on BI claims of PD claims

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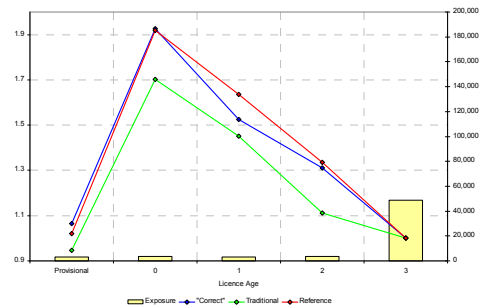
Example result



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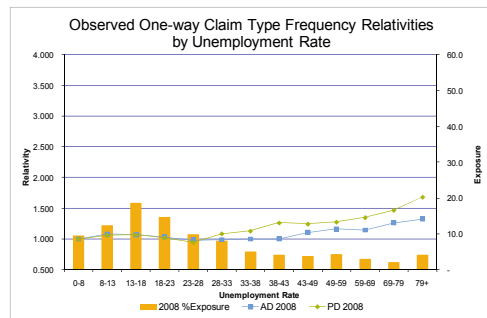
Example result



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BI models - "insurance" and "compensation" risk

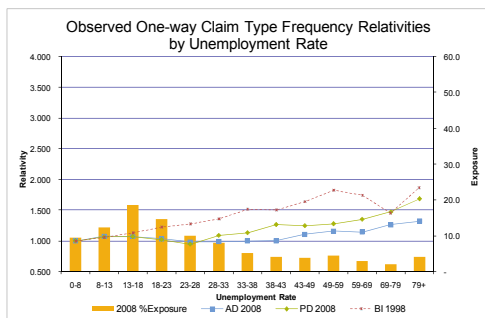


- The nature of the insured risk has changed - "compensation" risk has increased

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BI models - "insurance" and "compensation" risk

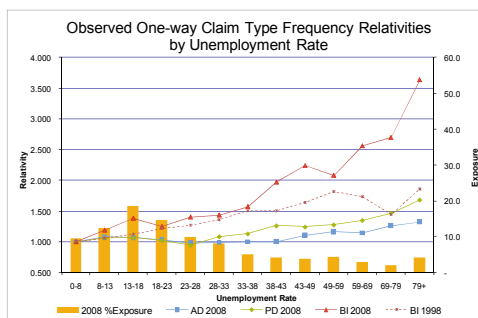


The nature of the insured risk has changed - "compensation" risk has increased

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BI models - "insurance" and "compensation" risk

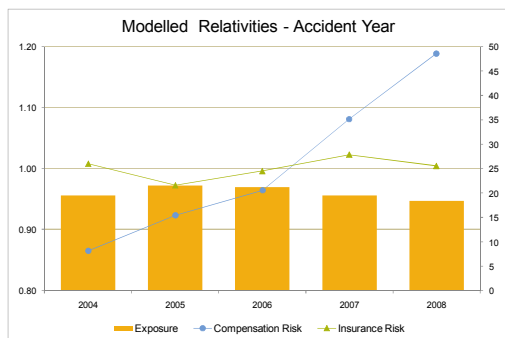


The nature of the insured risk has changed - "compensation" risk has increased

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BI models - "insurance" and "compensation" risk



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(Contents deleted from handout)

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Agenda

- Testing the link function
- "Quadrant Saddles"
- "Emergent Interactions"
- Case deleted deviance
- Man (with GLM) vs machine
- For example - UK bodily injury
- GLMs - getting better with age?

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Are there really problems with GLMs?

Problem	Comments	Solution
Interactions <ul style="list-style-type: none"> • Hard to find • Error/link • Hard to understand • Can't look for them all 	Interactions <ul style="list-style-type: none"> • Yes, first time • Need good visualization... • ...and a way to search for many 	"Quadrant Saddles" <ul style="list-style-type: none"> • Automatically finds interactions • Simplifies them to aid understanding
Inflexible <ul style="list-style-type: none"> • Error/link • Non-linearity 	Inflexible <ul style="list-style-type: none"> • Output driven • Level dependent 	Flexibility present <ul style="list-style-type: none"> • By-peril pricing deals with non-linearity
Over-fitting <ul style="list-style-type: none"> • Hard to spot • Hard to fix 	Over-fitting <ul style="list-style-type: none"> • Time/holdout sample • It is hard to fix automatically 	Emerging methods <ul style="list-style-type: none"> • Noise Reduction GLM - uses cross validation in familiar surroundings

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GLM III - The Matrix Reloaded
Duncan Anderson

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