Limited Fluctuation – Example 1

- Calculate the expected loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.

  - Scenario 1:
    Data: Observed loss ratio = 67%, Claim count = 400
    - What is the standard for full credibility?
    - Does this data have full credibility?
    - What is the expected loss ratio?

  - Answer:
    - For P=95% and k=10%, the number of claims needed is 384. Since we have 400, the data is considered fully credible.
    - Remember, **E2 = Z*T + (1-Z)*E1**

      E2 = 1 x 67% + (1 – 1) x 75%

      E2 = 67%

Limited Fluctuation – Example 1 (continued)

- Calculate the loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.

  - Scenario 2:
    Data: Observed loss ratio = 67%, Claim count = 200
    - Assuming Z = 0.72, what is the expected loss ratio?

  - Answer:

    E2 = Z*T + (1-Z)*E1

    E2 = 0.72 x 67% + (1 – 0.72) x 75%

    E2 = 69.2%

Limited Fluctuation – Example 1 (Revisited)

- Calculate the loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.
  - Scenario 2:
    Data: Observed loss ratio = 67%, Claim count = 200
  - Answer:
    E2 = Z*T + (1-Z)*E1
    E2 = $\sqrt{200/384}$ x 67% + (1 – $\sqrt{200/384}$) x 75%
    E2 = 69.2%

Limited Fluctuation – Example 2

- For the 3 and 5-year periods, calculate the credibility (using the square root rule), credibility-weighted loss ratio and indicated change, given that the expected loss ratio is 75%. Assume P= 90% and k = 2.5%.

Year	Loss Ratio	Claim Count
2007	67%	530
2008	77%	610
2009	79%	630
2010	77%	620
2011	86%	690

	Loss Ratio	Claim Count	Credibility	Cred-Wght Loss Ratio	Indicated Rate Chg
'09-'11	81%	1,940	67%	79.0%	5.3%
'07-'11	77%	3,080	84%	76.7%	2.3%

79.0% = 81% x (0.67) + 75% x (1 - 0.67)

67%= $\sqrt{1940/4326}$

5.3% = 79.0%/75.0%

Limited Fluctuation – Example 3

- Given a current territory factor of 1.08, determine the indicated territory factor with 5 years of data. The frequency distribution is Poisson and the severity coefficient of variation of 1.5. Use the square root rule and the limited fluctuation formula for pure premium. Assume that you want to be within 5% of the true value 90% of the time. The statewide frequency is 0.20 and fixed expenses are 15%.

Year	Territory Exposure	Territory Claim Count	Territory Loss Ratio	Statewide Loss Ratio
2006	3,000	330	125%	78%
2007	3,020	420	153%	83%
2008	3,030	630	269%	85%
2009	3,020	210	122%	79%
2010	3,050	190	108%	72%
'06-'10	15,120	1,780	162%	80%

Limited Fluctuation – Example 3 (continued)

$$N = (z_p / k)^2 * \quad (Var(N)/E(N) \quad + \quad Var(S)/E(S)^2)$$

- If we want to be within 5% of the true value 90% of the time, $(z_p / k)^2$ is 1,082.
- Remember, with a Poisson distribution, Var(N) = E(N), the second term is 1. The third term is the square of the coefficient of variation, which is 1.5^2.

$$N_{claims} = 1{,}082 * \ (1 \ + \ 1.5^2) \ = \ 3{,}516.5$$

- Given the 5-year statewide frequency of 0.2:

$$N_{exposures} = 3{,}516.5 / 0.2 \ = 17{,}582.5$$

Limited Fluctuation – Example 3 (continued)

- To show the impact of our selection of an exposure standard instead of a claims standard.

Year	Territory Exposure	Territory Claim Count	Exposure Credibility	Claim Credibility
2006	3,000	330	41.3%	30.6%
2007	3,020	420	41.4%	34.6%
2008	3,030	630	41.5%	42.3%
2009	3,020	210	41.4%	24.4%
2010	3,050	190	41.6%	23.2%
'06-'10	15,120	1,780	92.7%	71.1%

Using a claims standard of 3,517 and an exposure standard of 17,583

Limited Fluctuation – Example 3 (continued)

- Determine what the indicated territorial factor, assuming 15% for fixed expenses.

Year	Territory Loss Ratio	Territory Credibility	Statewide Loss Ratio	Cred Wght Loss Ratio
'06-'10	162%	92.7%	80%	156.0%

156.0% = 92.7% x 162% + 7.3% x 80%

The final indicated territorial factor is (156% / 80%)*0.85 + 0.15 = **1.81**

An alternative approach would be to calculate the indicated factor prior to applying credibility, and then credibility weight the current factor with the indicated factor.

Least Squares – Example

- Assuming that you have the following book of business, calculate the EVPV, VHM, K, and Z. The prior estimate of the frequency is 0.517. With 4 years of observations and an observed frequency of 0.75, what is the estimated future frequency? Assume the claims are binomially distributed.

Risk	P(Claim)	P(Risk)	Variance	Mean^2
Low	40%	65%	0.24	0.16
Medium	70%	23%	0.21	0.49
High	80%	12%	0.16	0.64
Total	51.7%	100%	0.2235	0.2935

- EVPV: For binomial, variance = P(claim) x P(no claim)

  = (40%)(60%)(65%) + (70%)(30%)(23%) + (80%)(20%)(12%)

  = 0.2235

- VHM: Mean^2 – (Mean)^2

  = 0.2935 – $(0.517)^2$

  = 0.0262

Least Squares – Example (continued)

- To determine K, we use K = EVPV/VHM, which is

  K = 0.2235 / 0.0262 = 8.53

- Since we're told that we have 4 years of observations, n = 4. Therefore,

  Z = n / (n + K) → 4 / (4 + 8.53) = 0.319.

- The prior estimate of frequency is the same as the mean calculated before, 0.517, and the observed data results in a frequency of 0.75. This observed data as 31.9% credibility, so…

  E2 = Z * T + (1 – Z) * E1 → 31.9% * 0.75 + 68.1% * 0.517 = 0.5913