CAS Ratemaking and Product Management Seminar- March 2012

RR-2: Risk LOAD/COST OF CAPITAL: HOW REINSURERS CONSIDER THESE IN REINSURANCE RATES FOR PROPERTY CAT COVERS

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- Writing CAT covers is risky – results may be catastrophic to your bottom line.
- Examples are for illustrative purposes only. Do not use the results from any example in real-world applications.
- There may be a quiz at the end – so pay attention!

Agenda
-A Mix of Theory and Practice

- CAT Pricing Process Fundamentals
  - Event Loss Table
  - Random Trials
- CAT Context
- Pricing Overview
- Basic Equations
- Required Capital Paradigms
- Order Dependence and Reference Portfolios
- Risk Measures
  - Properties
  - Take your pick
  - Ranking definitions of Var and TVaR
- Conclusions

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Event Loss Table

Event Rank	Peril	Region	Annual Prob	Treaty A Loss	Treaty B Loss	Treaty C Loss	...	Total Portfolio Loss
1	EQ	CA	0.021%	300	1,200	0	...	125,000
2	EQ	CA	0.040%	0	1,000	0	...	100,000
3	HU	FLA	0.080%	0	0	3,000	...	90,000
4	EQ	CA	0.070%	900	400	0	...	80,000
5	HU	LA	0.045%	0	0	2,100	...	75,000
6	EQ	CA	0.055%	700	0	700	...	70,000
7	EQ	PNW	0.006%	0	400	500	...	60,000
8	HU	FLA	0.150%	0	550	100	...	50,000
9	EQ	PNW	0.010%	0	0	900	...	50,000
10	EQ	AK	0.025%	0	0	5,500	...	40,000
.	.	.	.	.	.	.	...	.
.	.	.	.	.	.	.	...	.
.	.	.	.	.	.	.	...	.
1998	HU	NC	2.000%	0	0	2	...	3
1999	HU	FL	4.000%	0	0	2	...	2
2000	HU	SC	3.000%	0	0	0	...	1

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Occurrence Exceeding Probability

k Event Rank	Peril	Region	p(k) Annual Prob	EP(k) Exceeding Probability	Portfolio Event Loss
1	EQ	CA	0.021%	0.021%	125,000
2	EQ	CA	0.040%	0.061%	100,000
3	HU	FLA	0.080%	0.141%	90,000
4	EQ	CA	0.070%	0.211%	80,000
5	HU	LA	0.045%	0.256%	75,000
6	EQ	CA	0.055%	0.311%	70,000
7	EQ	PNW	0.006%	0.317%	60,000
8	HU	FLA	0.150%	0.466%	50,000
9	EQ	PNW	0.010%	0.476%	50,000
10	EQ	AK	0.025%	0.501%	40,000
.	.	.	.		.
.	.	.	.		.
.	.	.	.		.
1998	HU	NC	2.000%	24.000%	3
1999	HU	FL	4.000%	27.040%	2
2000	HU	SC	3.000%	29.229%	1

$$EP(k+1) = EP(k) + (1 - EP(k)) \cdot p(k+1)$$

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Random Trials

Trial Year	Event 1	Event 2	Event 3	...	Largest Event over the Year	Total Annual Loss
1	40,000	-	-	-	40,000	40,000
2	2,100	3,500	450	- -	3,500	6,050
3	-	-	-		0	0
4	5,500	27,550	-		27,550	33,050
5	700	400	50		700	1,150
6	1,250	900	25		1,250	2,175
7	8,750	-	-		8,750	8,750
8	75	45	70,000		70,000	70,120
9	-	-	-		0	0
10	15	3,500	45		3,500	3,560
⋮	.	.	.		⋮	⋮
9998	25	-	-		25	25
9999	550	7,750	-		7,750	8,300
10000	650	-	-		650	650

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AEP and OEP PML from Ordered Trials

Trial Year Rank		Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		125,000	165,000
4		100,000	137,500
5		100,000	135,000
6		100,000	130,000
7		90,000	125,000
8		90,000	115,000
9		90,000	110,000
10		90,000	110,000
.		.	.
.		.	.
.		.	.
99		21,250	37,500
100	100/10000 = 1.0% 100 year return period AEP PML = 36,675 OEP PML = 21,000	21,000	36,675
101		21,000	35,950
.		.	.
.		.	.
.		.	.
9998		.	0
9999		.	0
10000		.	0

- PML = Probable Maximum Loss
- AEP = Annual Exceeding Probability
- OEP = Occurrence Exceeding Probability

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Context

- CAT Pricing is part of the process of writing CAT business, but not the only part.
- Pricing models give indications – the market sets the price.
- Risk Management sets limits on PMLs and TIV/Limit Aggregations by peril/zone .
  - Compliance monitoring essential
- Business bunched –lots of 1/1s. Waiting can work to reduce price if there is excess capacity or increase price if capacity gets tight.
- Selection problem is constrained optimization: Reinsurers looks to get most profitable portfolio with smallest risk. No one prices that way.

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Pricing Overview

- Emerald City Pricing: Don't look at that man behind the curtain
  - Reinsurers use the same set of models, but don't get the same answers.
  - Some adopt new versions –others wait.
  - Differences in data quality
  - Loading factors
- Non-modeled CAT events (Thai flood): Not always priced
  - Ostrich Excuse - "It was not in the model"
  - Hiding-in-Plain-Sight Swan - May not show up on risk management radar – obvious after the fact.
- Pricing Method Flavors: Different ways of translating model stats into indicated prices.
  - Can't we just all agree?

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Basic Equations

- P= E[X]+ RL(X)

  P = Indicated premium prior to expense loading

  X = CAT Loss

  RL(X) = Risk Load

- RL(X) = r_{target}*C(X)
- C(X) = Required Capital
- RORAC Approach
  - Universally used in actual CAT Treaty pricing

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What is the right way to compute Required CAT Capital?

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Required Capital Paradigms

- Standalone: $C(X) = \rho(X)$, where is $\rho(X)$ is a risk measure.
- Incremental: Let T be the existing portfolio
  $C(X|T) = \rho(T+X) - \rho(T)$,
- Real Allocation
  $C(X|T) = A(X,T) * \rho(T+X)$

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Order Dependence and Reference Portfolios

- Order Dependence – Pricing depends on the order in which accounts are priced (Mango)
  - Universe A : Zoe's CAT Treaty is priced first at $100 then Jessica's CAT Treaty is priced next at $150
  - Universe B: Jessica's CAT Treaty is priced first at $100 then Zoe's CAT Treaty is priced next at $150
- A major problem for Incremental methods
- A small problem for Allocation methods
- Not a problem for Standalone
- Reference Portfolio Cure
  - Portfolio fixed over a given period
  - How often should it be updated??

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Risk Measure: Definitions and properties

- A **risk measure**, ρ, is a monotonic function that maps a real-valued random variable, X, to a non-negative number, $\rho(X)$, such that:
- Risk Measure Basic Properties

1. *Non-negative:* $\rho(X) \geq 0$
2. *Monotonic Premium: If* $X_1 \leq X_2$, *then* $E[X_1]+\rho(X_1) \leq E[X_2]+\rho(X_2)$
- A risk measure is **pure** if it maps constants to zero: $\rho(c) = 0$

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Risk Measure: Coherence properties

1. Scalable: $\rho(\lambda X) = \lambda \cdot \rho(X)$
2. Translation Invariant: $\rho(X+ \alpha) = \rho(X)$
3. Subadditive: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

- Some academicians refuse to refer to a function as a risk measure unless it is coherent
- Most academicians uses reverse signs (X represents the value of assets instead of CAT losses)

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Risk Measures: Take Your Pick

1. *Variance: $Var(X) = E[(X-\mu)^2]$*
2. *Semivariance: $Var^+(X)$: $E[(X-\mu)^2 \mid X \geq \mu] * Prob(X \geq \mu)$*
3. *Standard Deviation: $\sigma = Var^{1/2}(X)$*
4. *Semi Standard Deviation: $\sigma^+ = Var^{+1/2}(X)$*
5. *Value at Risk: for $0 < \theta < 1$, $VaR(\theta) = \sup\{x \mid F(x) \leq \theta\}$*
6. *Tail Value at Risk: $TVaR(\theta)$ = conditional mean for all x values associated with the tail, $1-\theta$, of probability*
7. *Excess Tail Value at Risk: $XTVaR(\theta) = TVaR(\theta) - \mu$*
8. *Distortion Risk Measure: (Wang) $E^*[X] = E[X^*]$ where $F^*(x) = g(F(X))$ for g a distortion function*
9. *Excess Distortion Risk Measure: $E^*[X] - E[X]$*

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Ranking Definition of VaR and TVaR on Random Sample Data

- Let $X_1 \geq X_2 \ldots \geq X_n$ *be an ordering of n trials of X*
- *Suppose $k = (1-\theta)n$*

$$VaR(\theta) = X_k$$

$$TVaR(\theta) = \frac{1}{k}\sum_{j=1}^{k} X_j$$

- Note TVaR is not necessarily equal to the Conditional Tail Expectation (CTE) when the data is discrete.
- *$CTE(\theta) = E[X \mid X > VaR(\theta)]$*

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TVaR and CTE -not the same

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.80	26.00	28.80
VaR	*2.00*	*33.00*	*34.00*
TVaR	5.00	34.80	35.40
CTE	5.75	36.00	35.75

Loss Data by Trial

Trial	A	Ref	A+Ref
1	8.00	12.00	20.00
2	0.00	37.00	37.00
3	0.00	36.00	36.00
4	0.00	35.00	35.00
5	1.00	33.00	34.00
6	2.00	17.00	19.00
7	7.00	16.00	23.00
8	2.00	33.00	35.00
9	4.00	27.00	31.00
10	4.00	14.00	18.00

Ordered Loss Data

Rank	A	Ref	A+Ref
1	8.00	37.00	37.00
2	7.00	36.00	36.00
3	4.00	35.00	35.00
4	4.00	33.00	35.00
5	*2.00*	*33.00*	*34.00*
6	2.00	27.00	31.00
7	1.00	17.00	23.00
8	0.00	16.00	20.00
9	0.00	14.00	19.00
10	0.00	12.00	18.00

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VaR Subadditivity-Epic Fail

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.80	26.00	28.80
VaR	*2.00*	*33.00*	*37.00*
TVaR	5.00	34.80	39.40

Loss Data by Trial

Trial	A	Ref	A+Ref
1	0.00	12.00	12.00
2	0.00	37.00	37.00
3	8.00	36.00	44.00
4	7.00	35.00	42.00
5	4.00	33.00	37.00
6	2.00	17.00	19.00
7	0.00	16.00	16.00
8	4.00	33.00	37.00
9	2.00	27.00	29.00
10	1.00	14.00	15.00

Ordered Loss Data

Rank	A	Ref	A+Ref
1	8.00	37.00	44.00
2	7.00	36.00	42.00
3	4.00	35.00	37.00
4	4.00	33.00	37.00
5	*2.00*	*33.00*	*37.00*
6	2.00	27.00	29.00
7	1.00	17.00	19.00
8	0.00	16.00	16.00
9	0.00	14.00	15.00
10	0.00	12.00	12.00

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Conclusions

- Target return on required capital is the basis for reinsurer pricing indications.
- Debate is over required capital
- A profusion of methods and approaches
- Tail focus and portfolio dependence are key areas of disagreement.

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