CAS Ratemaking and Product Management Seminar- March 2012 RR-2: Risk LOAD/COST OF CAPITAL: HOW REINSURERS CONSIDER THESE IN REINSURANCE RATES FOR PROPERTY CAT COVERS

Ira Robbin, PhD

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- Writing CAT covers is risky results may be catastrophic to your bottom line.
- Examples are for illustrative purposes only. Do not use the results from any example in real-world applications.
- There may be a quiz at the end so pay attention!

#### Agenda -A Mix of Theory and Practice

- CAT Pricing Process Fundamentals

  Event Loss Table
  Random Trials

  CAT Context
  Pricing Overview
  Basic Equations
  Required Capital Paradigms
  Order Dependence and Reference Portfolios
  Risk Measures

  Properties
  Take your pick
  Ranking definitions of Var and TVaR

  Conclusions

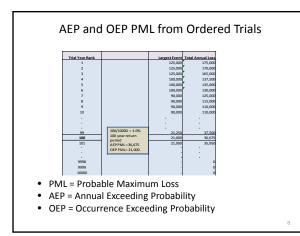
	ble	s Ta	Los	vent	E		
Tota	Trank	Trank D	Tarahi	A			
Portfoli	Treaty C	Treaty B	Treaty A	Annual			Event
Los	 Loss	Loss	Loss	Prob	Region	Peril	Rank
125,00	 0	1,200	300	0.021%	CA	EQ	1
100,00	 0	1,000	0	0.040%	CA	EQ	2
90,00	 3,000	0	0	0.080%	FLA	HU	3
80,00	 0	400	900	0.070%	CA	EQ	4
75,00	 2,100	0	0	0.045%	LA	HU	5
70,00	 700	0	700	0.055%	CA	EQ	6
60,00	 500	400	0	0.006%	PNW	EQ	7
50,00	 100	550	0	0.150%	FLA	HU	8
50,00	 900	0	0	0.010%	PNW	EQ	9
40,00	 5,500	0	0	0.025%	AK	EQ	10
•			•		•		
	 •	•	•	•			
	 2	0	0	2.000%	NC	HU	1998
	 2	0	0	4.000%	FL	HU	1999
	 0	0	0	3.000%	SC	HU	2000

				EP(k)	
k			p(k)	Exceeding	Portfolio
Event Rank	Peril	Region	Annual Prob	Probability	Event Loss
1	EQ	CA	0.021%	0.021%	125,00
2	EQ	CA	0.040%	0.061%	100,00
3	HU	FLA	0.080%	0.141%	90,00
4	EQ.	CA	0.070%	0.211%	80,00
5	HU	LA	0.045%	0.256%	75,00
6	EQ	CA	0.055%	0.311%	70,00
7	EQ.	PNW	0.006%	0.317%	60,00
8	HU	FLA	0.150%	0.466%	50,00
9	EQ	PNW	0.010%	0.476%	50,00
10	EQ	AK	0.025%	0.501%	40,00
		•			
		•			
1998	HU	NC	2.000%	24.000%	
1999	HU	FL	4.000%	27.040%	
2000	HU	SC	3.000%	29.229%	



		R	ando	om	Tria	als	
Trial						Largest Event	Total Annual
Year	Event 1	Event 2	Event 3			over the Year	Loss
1	40,000	-	-	-		40,000	40,000
2	2,100	3,500	450	-	-	3,500	6,050
3	-	-	-			0	0
4	5,500	27,550	-			27,550	33,050
5	700	400	50			700	1,150
6	1,250	900	25			1,250	2,175
7	8,750	-	-			8,750	8,750
8	75	45	70,000			70,000	70,120
9	-	-	-			0	0
10	15	3,500	45			3,500	3,560
:	•	•	•			. :	
9998	25	-	-			25	25
9999	550	7,750	-			7,750	8,300
10000	650	-	-			650	650





# Context

- CAT Pricing is part of the process of writing CAT business, but not the only part.
  Pricing models give indications the market sets the
- price. Risk Management sets limits on PMLs and TIV/Limit Aggregations by peril/zone . ٠
- Aggregations by perin/20ne.
  Compliance monitoring essential
  Business bunched –lots of 1/1s. Waiting can work to reduce price if there is excess capacity or increase price if capacity gets tight.
  Selection problem is constrained optimization: Reinsurers looks to get most profitable portfolio with smallest risk. No one prices that way.

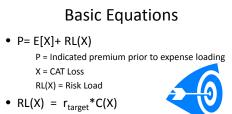
## **Pricing Overview**

- Emerald City Pricing: Don't look at that man behind the curtain ٠
  - Reinsurers use the same set of models, but don't get the same answers.
- answers.
  Some adopt new versions -others wait.
  Differences in data quality

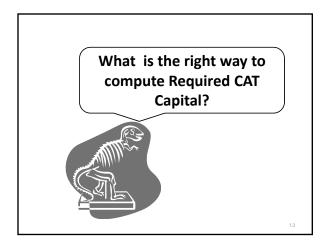
  Loading factors

  Non-modeled CAT events (Thai flood): Not always priced

  Ostrich Excuse "It was not in the model"
  Hiding-in-Plain-Sight Swan May not show up on risk management radar obvious after the fact.
- Pricing Method Flavors: Different ways of translating model stats into indicated prices. – Can't we just all agree?



- C(X) = Required Capital
- RORAC Approach
  - Universally used in actual CAT Treaty pricing



# **Required Capital Paradigms**

- Standalone:  $C(X) = \rho(X)$ , where is  $\rho(X)$  is a risk measure.
- Incremental: Let T be the existing portfolio  $C(X|T) = \rho (T+X) - \rho (T),$
- Real Allocation

 $C(X|T) = A(X,T) * \rho (T+X)$ 



#### Order Dependence and Reference Portfolios

- Order Dependence Pricing depends on the order in which accounts are priced (Mango)
  - Universe A : Zoe's CAT Treaty is priced first at \$100 then Jessica's CAT Treaty is priced next at \$150
  - Universe B: Jessica's CAT Treaty is priced first at \$100 then Zoe's CAT Treaty is priced next at \$150
- A major problem for Incremental methods
- A small problem for Allocation methods
- Not a problem for Standalone
- Reference Portfolio Cure
  - Portfolio fixed over a given period
  - How often should it be updated??

# Risk Measure: Definitions and properties

- A risk measure, ρ, is a monotonic function that maps a real-valued random variable, X, to a nonnegative number, ρ (X), such that:
- <u>Risk Measure Basic Properties</u>
- 1. Non-negative:  $\rho(X) \ge 0$
- 2. Monotonic Premium: If  $X_1 \le X_2$ , then  $E[X_1] + \rho(X_1) \le E[X_2] + \rho(X_2)$
- A risk measure is <u>pure</u> if it maps constants to zero: ρ(c) = 0

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# Risk Measure: Coherence properties



- 1. Scalable:  $\rho(\lambda X) = \lambda \cdot \rho(X)$
- 2. Translation Invariant:  $\rho(X + \alpha) = \rho(X)$
- 3. Subadditive:  $\rho(X_1 + X_2) \le \rho(X_1) + \rho(X_2)$
- Some academicians refuse to refer to a function as a risk measure unless it is coherent
- Most academicians uses reverse signs ( X represents the value of assets instead of CAT losses)

# Risk Measures: Take Your Pick



- 1. Variance:  $Var(X) = E[(X \mu)^2]$
- 2. Semivariance:  $Var^{*}(X)$ :  $E[(X-\mu)^{2} | X \ge \mu]^{*}Prob(X \ge \mu)$
- 3. Standard Deviation:  $\sigma = Var^{\frac{1}{2}}(X)$
- 4. Semi Standard Deviation:  $\sigma^+ = Var^{+\frac{1}{2}}(X)$
- 5. Value at Risk: for  $0 < \theta < 1$ , VaR( $\theta$ ) = sup{x |  $F(x) \le \theta$ }
- 6. Tail Value at Risk: TVaR( $\theta$ ) = conditional mean for all x values associated with the tail, 1- $\theta$ , of probability
- 7. Excess Tail Value at Risk: XTVaR( $\theta$ ) = TVaR( $\theta$ )  $\mu$ 8. Distortion Risk Measure: (Wang) E\*[X] = E[X\*] where
- $F^{*}(x) = g(F(X)) \text{ for } g \text{ a distortion function}$
- 9. Excess Distortion Risk Measure: E\*[X] –E[X]

# Ranking Definition of VaR and TVaR on Random Sample Data

• Let  $X_1 \ge X_2 \dots \ge X_n$  be an ordering of n trials of X

• Suppose  $k = (1 - \theta)n$ 

$$VaR(\theta) = X_{k}$$
$$TVaR(\theta) = \frac{1}{k} \sum_{i=1}^{k} X_{i}$$

• Note TVaR is <u>not</u> necessarily equal to the Conditional Tail Expectation (CTE) when the data is discrete.

$$CTE(\theta) = E[X|X > VaR(\theta)]$$

		vак а	nd CT	'E -not	the sa	me	
Statistic		Value	ſ	Results	А	Ref	A+Re
Trials		10		Mean	2.80	26.00	28.8
Pct		50%		VaR	2.00	33.00	34.00
Rank		5		TVaR	5.00	34.80	35.4
				CTE	5.75	36.00	35.7
			-				
oss Data by T	rial			Ordered Lo	ss Data		
Trial	А	Ref	A+Ref	Rank	А	Ref	A+Re
1	8.00	12.00	20.00	1	8.00	37.00	37.0
2	0.00	37.00	37.00	2	7.00	36.00	36.0
3	0.00	36.00	36.00	3	4.00	35.00	35.0
4	0.00	35.00	35.00	4	4.00	33.00	35.0
5	1.00	33.00	34.00	5	2.00	33.00	34.00
5	2.00	17.00	19.00	6	2.00	27.00	31.0
6		16.00	23.00	7	1.00	17.00	23.0
	7.00	10.00				44.00	20.0
6	7.00 2.00	33.00	35.00	8	0.00	16.00	20.0
6 7			35.00 31.00	8 9	0.00	16.00 14.00	20.0



			F				
Statis		Value		Results	А	Ref	A+Re
Trial	s	10		Mean	2.80	26.00	28.80
Pct		50%		VaR	2.00	33.00	37.00
Ranl	¢.	5		TVaR	5.00	34.80	39.40
oss Data b	y Trial			Ordered Los	ss Data		
Trial	А	Ref	A+Ref	Rank	А	Ref	A+Re
1	0.00	12.00	12.00	1	8.00	37.00	44.00
2	0.00	37.00	37.00	2	7.00	36.00	42.00
3	8.00	36.00	44.00	3	4.00	35.00	37.00
4	7.00	35.00	42.00	4	4.00	33.00	37.00
5	4.00	33.00	37.00	5	2.00	33.00	37.00
6	2.00	17.00	19.00	6	2.00	27.00	29.00
7	0.00	16.00	16.00	7	1.00	17.00	19.00
8	4.00	33.00	37.00	8	0.00	16.00	16.00
9	2.00	27.00	29.00	9	0.00	14.00	15.00
10	1.00	14.00	15.00	10	0.00	12.00	12.00



# Conclusions

- Target return on required capital is the basis for reinsurer pricing indications.
- Debate is over required capital
- A profusion of methods and approaches
- Tail focus and portfolio dependence are key areas of disagreement.