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Pricing Risk in Cat Covers

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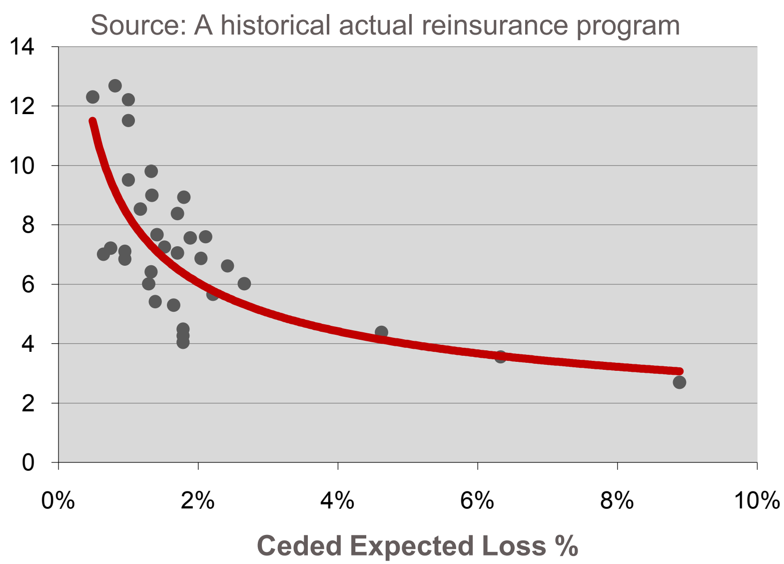
Principles for Cost of Risk

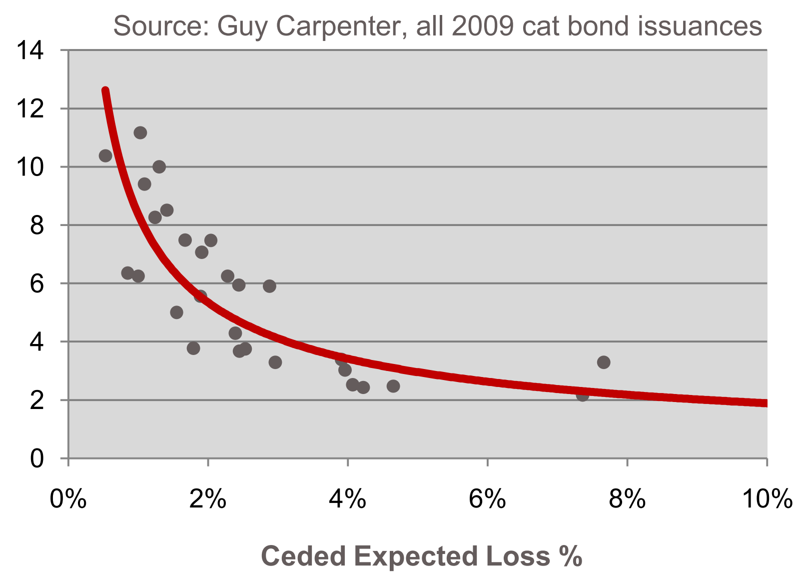
- Not proportional to mean
- Ratio of cost of risk to expected value increases for low frequency, high severity deals
- Ratio can get very high – no natural limits
  - Minimum rate on line for instance
- Risk costs seem to increase faster than quadratic
- Dividing deal up does not change total risk cost
- ∃ some cost for any risk, even if small
- Unfavorable deviations are costlier than favorable in an asymmetric portfolio
- Risk within a diversified portfolio measured by contribution to portfolio risk

Tail Loading1: Cat Ceded

Multiples (Price divided by Expected Loss) paid in:

Sample company cat reinsurance layers and in… Cat Bond securitization market





Tail risk is expensive in capital markets

Lines here are just fitted curves

Tail Loading 2: US Bonds

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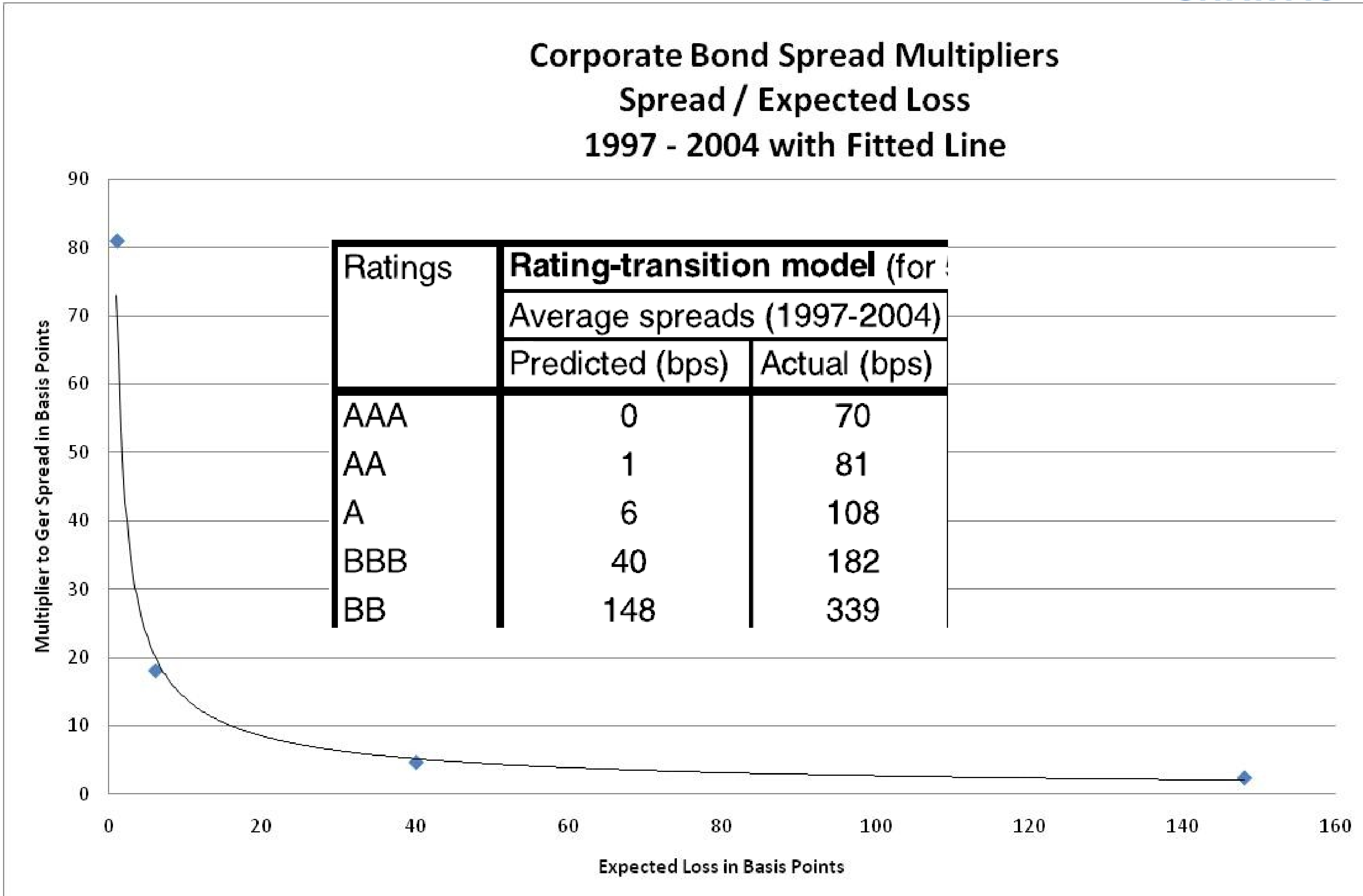
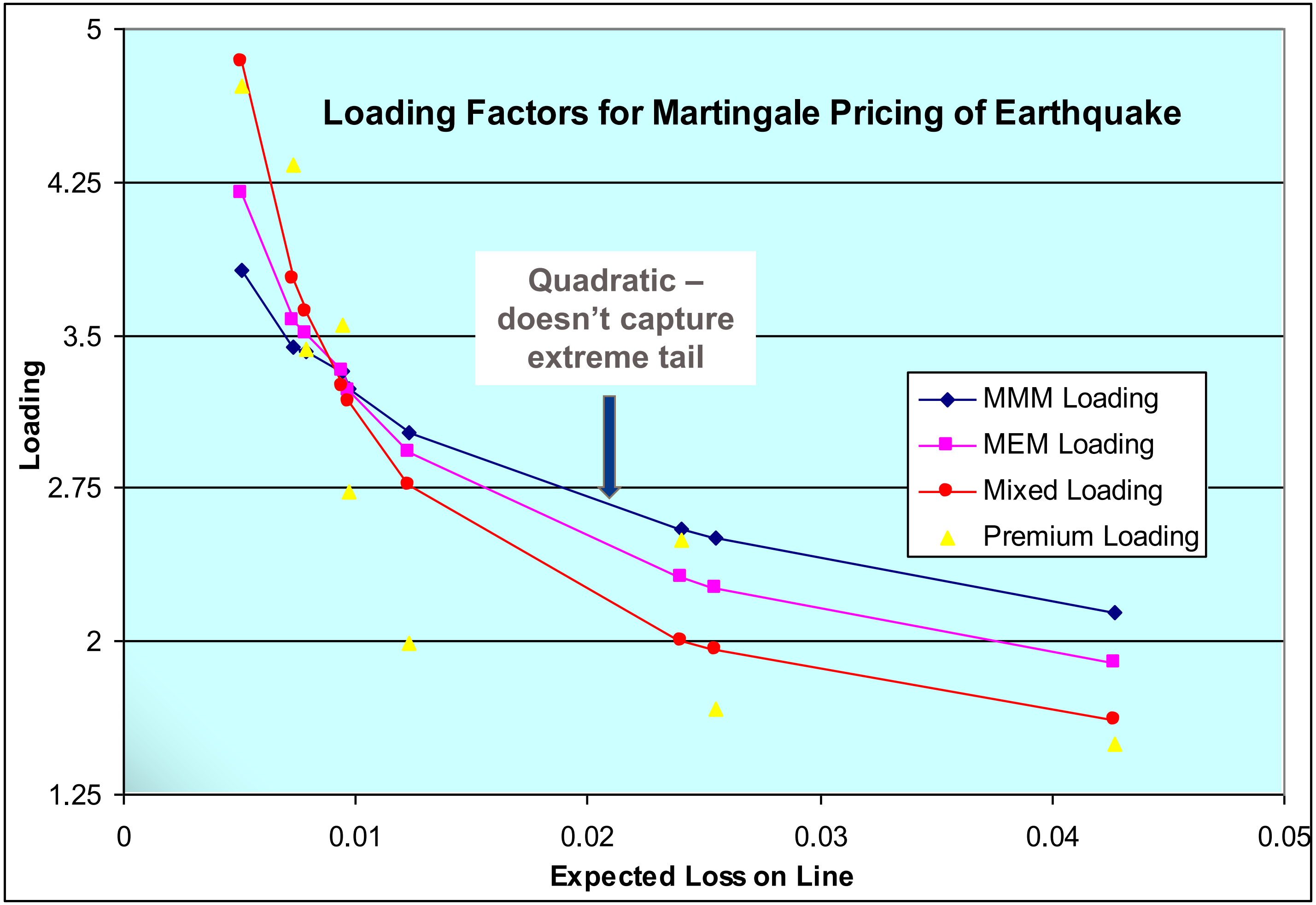


Ratings	Rating-transition model (for	
	Average spreads (1997-2004)	
	Predicted (bps)	Actual (bps)
AAA	0	70
AA	1	81
A	6	108
BBB	40	182
BB	148	339

Bond spreads get large as a multiple of expected credit losses

Tail Loading 3: Japan EQ



Yellow dots are actual programs. Lines represent attempts at modeling prices.

Contribution to Portfolio Risk

•Marginal impact of business unit on company risk measure is decrease in overall risk measure due to ceding a small increment of the unit by a quota share treaty

•It is incremental last-in marginal, and = derivative of the risk measure wrt the volume of the business unit

•If risk measure scales to changes in units, like dollars to yen, then these marginal impacts add up to the whole risk measure

•Called marginal decomposition in Venter G, Major J and Kreps R (2006) "Marginal Decomposition of Risk Measures" ASTIN Bulletin 36, #2: **http://www.actuaries.org/LIBRARY/ASTIN/vol36no2/375.pdf**

•Called Euler method in Patrik G, Bernegger S, and Rüegg M (1999) "The Use of Risk Adjusted Capital to Support Business Decision-making" CAS Forum Spring: **http://casact.org/pubs/forum/99spforum/99spf243.pdf**

•Holds for tail measures, standard deviation, but not variance. Comeasures are special case.

• Marginal $r(X_j) = \lim_{\varepsilon \to 0}[\rho(Y+\varepsilon X_j) - \rho(Y)]/\varepsilon$.

•Take derivative of numerator and denominator wrt ε.

•L'Hopital's rule then gives $r(X_j) = \rho'(Y+\varepsilon X_j)|_0$.

•Consider $\rho(Y) = \mathrm{Std}(Y)$

• $\rho(Y+\varepsilon X_j) = [\mathrm{Var}(Y)+2\varepsilon \mathrm{Cov}(X_j,Y)+\varepsilon^2 \mathrm{Var}(X_j)]^{½}$ so $\rho'(Y+\varepsilon X_j)|_0 =$

• $[\mathrm{Var}(Y)+2\varepsilon \mathrm{Cov}(X_j,Y)+\varepsilon^2 \mathrm{Var}(X_j)]^{-½}\,[\mathrm{Cov}(X_j,Y) + \varepsilon \mathrm{Var}(X_j)]|_0$

• $r(X_j) = \mathrm{Cov}(X_j,Y)/\mathrm{Std}(Y) = \mathrm{Corr}(X_j,Y)*\mathrm{Std}(Y)$

Pricing for Risk

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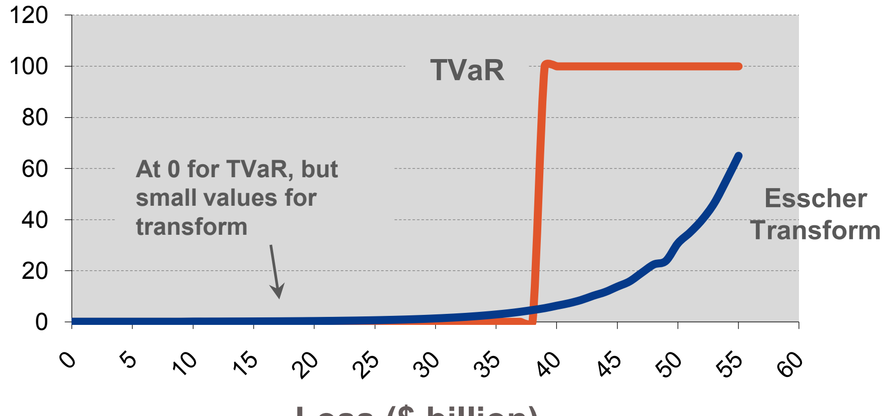
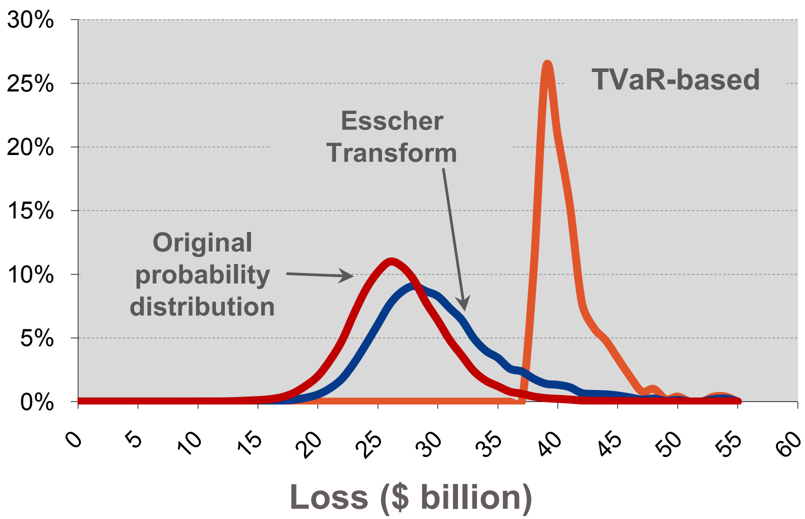
- Survey of reinsurers by Guy Carpenter 5 or 10 years ago found risk element of pricing mostly using standard deviation but higher percentage for layers in tail
- Financial pricing theory says price should be mean under transformed probability distribution.
  - CAPM is special case of transformed distribution pricing
- Corresponding risk measures are the distortion measures, which are also means under transformed probability distribution
- Euler allocation of distortion measure is mean of the unit under the transformed probabilities for the whole company
  - Same transform has to be used for all contracts for pricing to be arbitrage-free
- Wang and Esscher = MEMM are popular transforms
- Transforms and standard deviation both use whole distribution but allocation of tail measures ignores some risk

Comparison of Methods

Weights applied to losses output from a simulation



Probabilities applied to loss scenarios



Steps to apply weights:

1. Simulate 50,000 years of possible outcomes view of actual probabilities of losses.
2. Apply weights to each outcome and take weighted average. Greater weight is applied to larger losses to charge for risk.

Weights based on standard deviation would be lower than Esscher transform in tail but slightly higher for smaller losses.

The weights can be normalized so that they add up to 1.0. The result is similar to a probability distribution that is adjusted in comparison to the original distribution to skew towards larger losses as a way to give more weight to them.

Possible Transforms

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- $G^*(x) = Q_k[\Phi^{-1}(G(x)) + \lambda]$ where Q_k is the t-distribution with k dof – Wang transform
- Can use normal on outside, especially when amount in [] < 0
  - > $k \in [5,6]$ fit prices of cat bonds and various grades of commercial bonds
  - > k can be non-integer with beta distribution
- Compound Poisson martingale transform
  - > Requires function $\phi(x)$, with $\phi(x) > -1$ for $x>0$
  - > $\lambda^* = \lambda[1+E\phi(X)]$ for frequency
  - > $g^*(x) = g(x)[1+\phi(x)]/[1+ E\phi(X)]$ for severity
- Minimum martingale measure with $0<s<1$ (MMM)
- $\lambda^* = \lambda/(1 - s)$
- $g^*(y) = [1 - s + sy/EY]g(y)$
- Claim sizes above the mean get increased probability and below the mean get decreased
- No claim size probability decreases more than the frequency increases
- Thus no layers have prices below expected losses
- s selected to give desired ground-up profit load

Minimum Entropy Measure = MEMM

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- Also called Esscher
- Has parameter c
- $\lambda^* = \lambda E e^{Y/c}$ – only works if this exponential moment exists
- $g^*(y) = g(y)e^{y/c}/Ee^{Y/c}$
- For small claims $g(y) > g^*(y) > g(y)/Ee^{Y/c}$ so probability never decreases more than frequency probability increases
- If you set c = percentile of Y, can keep that percentile as distribution inflates or has other scale changes

Hypothetical Example

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- λ=2500, $g(y) = 0.00012/(1+y/10{,}000)^{2.2}$, policy limit 10M
- To get a load of 20%, take the MMM s = 0.45%
- $\lambda^* = \lambda/(1 - s) = 2511$
- $g^*(y) = [1-s+sy/EY]g(y) = (.9955+y/187{,}215)g(y)$
- Probability at 10M goes to 0.055% from 0.025%
- 4M x 1M gets load of 62.3%, 5M x 5M gets 112.8%
- For MEM these are 50.8% and 209.1%, as more weight is in the far tail
- 89% of the risk load is above $1M for MEM; 73% for MMM

Testing with Pricing Data

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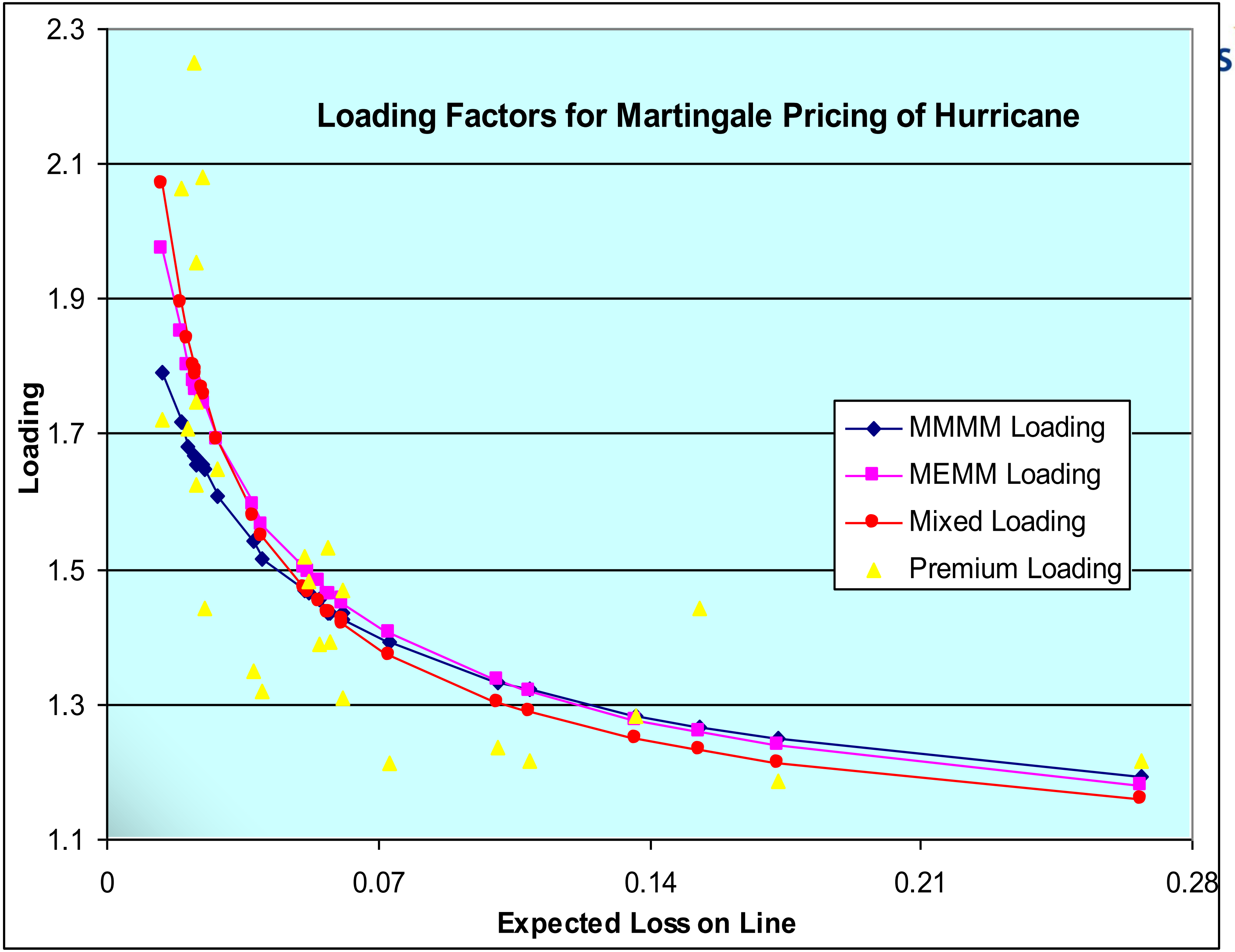
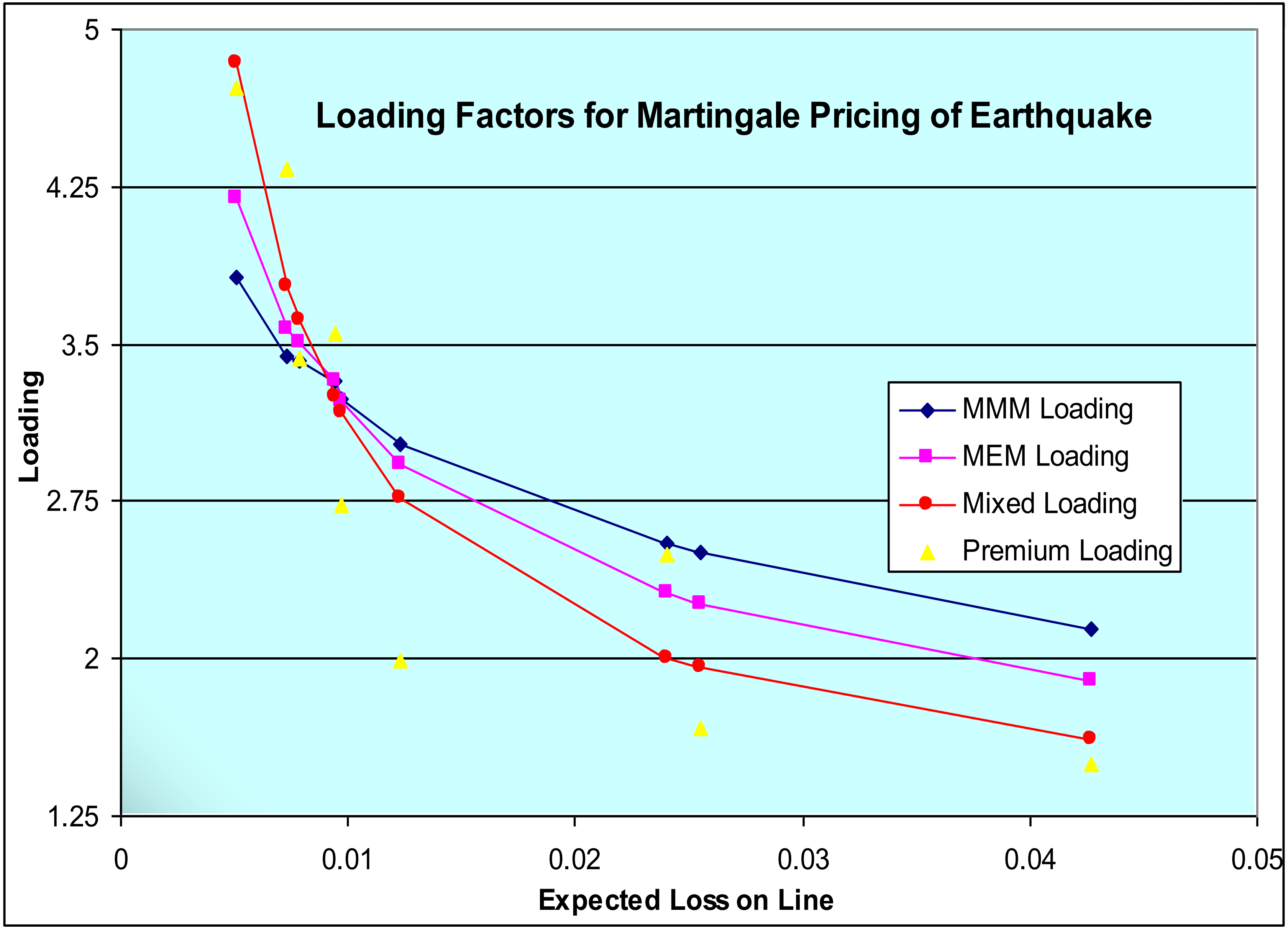
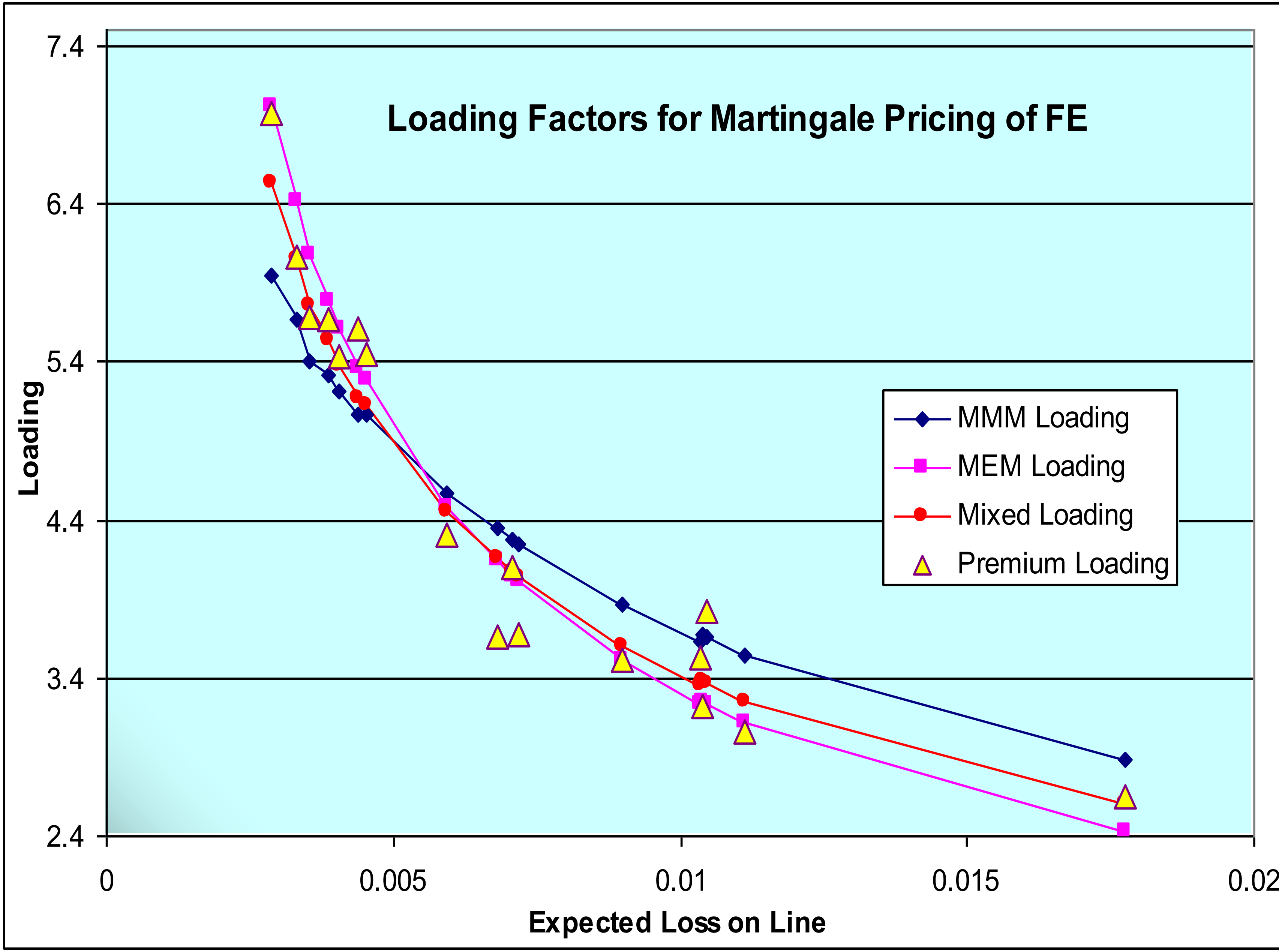
- Had prices and cat model losses for a group of reinsurance treaties
- Fit MMM, MEM and a mixture of them to this data with transforms based on industry loss distribution = distribution of sum across companies
- Had separate treaties and modeled losses for three perils: H, E, and FE
- Mixture always fit best, but not usually much better than MEM alone, which was better than MMM
- Fit by minimizing expected squared relative errors



Loading Factors for Martingale Pricing of Hurricane
2.3
2.1
1.9
1.7
1.5
1.3
1.1
Loading
0
0.07
0.14
0.21
0.28
Expected Loss on Line
MMMM Loading
MEMM Loading
Mixed Loading
Premium Loading



Loading Factors for Martingale Pricing of Earthquake
5
4.25
3.5
2.75
2
1.25
Loading
0
0.01
0.02
0.03
0.04
0.05
Expected Loss on Line
MMM Loading
MEM Loading
Mixed Loading
Premium Loading


Loading Factors for Martingale Pricing of FE
Loading
7.4
6.4
5.4
4.4
3.4
2.4
0
0.005
0.01
0.015
0.02
Expected Loss on Line
MMM Loading
MEM Loading
Mixed Loading
Premium Loading

In conclusion

•Price all risk, not just tail

•Euler allocation is in line with economic principle of pricing in proportion to marginal cost

•If you must allocate capital, price first by transform then allocate so that everyone gets the same expected return

•Will give same answer as capital consumption if you get options prices by transforms:

> Business unit has option to put any losses it has to the company up to 100% of surplus
> Company has call option on all the profits of the business unit
> You want value of call to be at least value of put
> Merton Perold version of capital consumption, with twist of Froot