Actuarial Implications of Two-Price Markets

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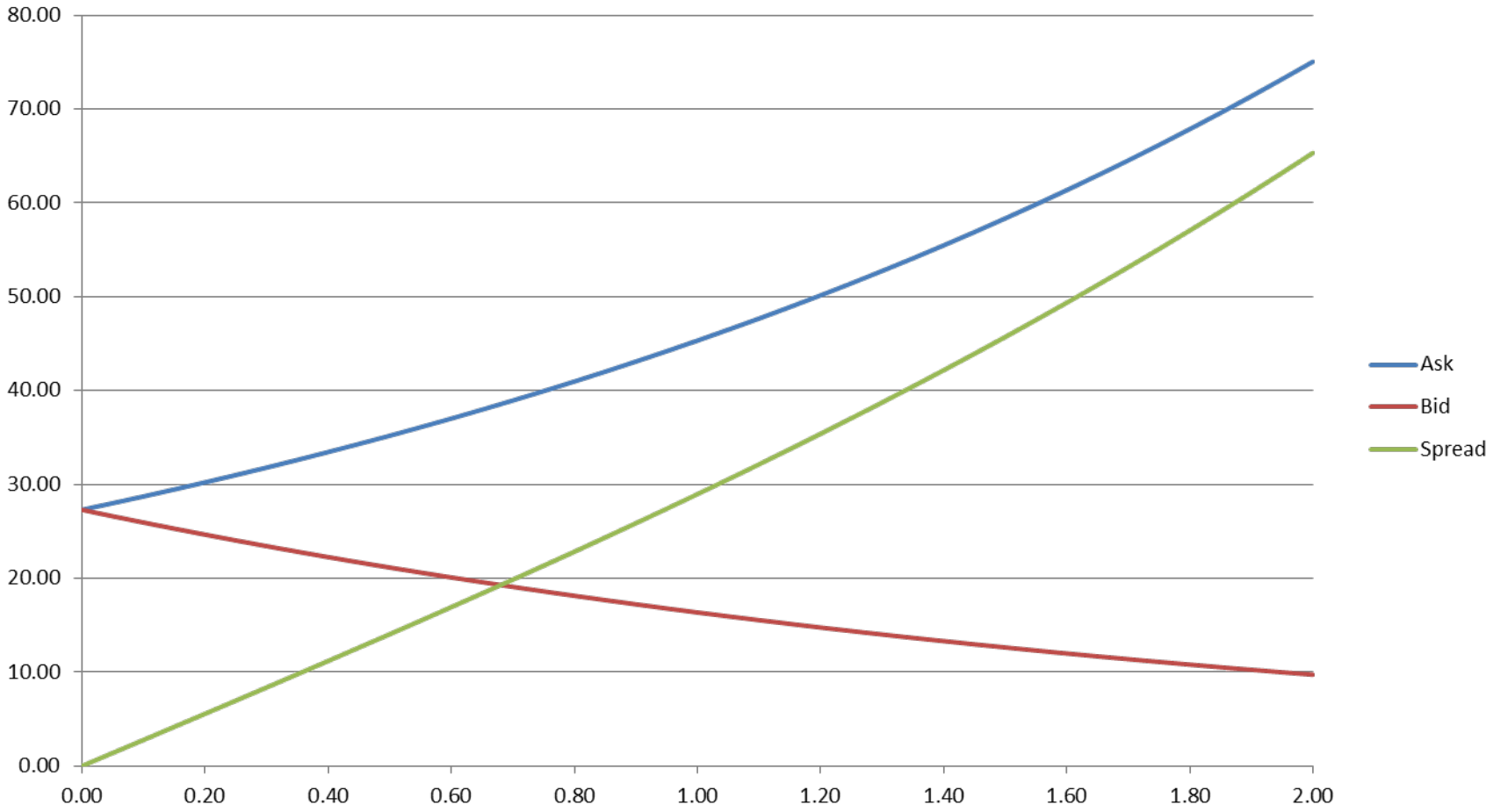
Review of Findings

- Law of one price holds in complete, liquid markets: equities, commodities, and some derivatives. Not in most markets.
- In an incomplete market, bid-ask spread measures
  - Capital needed to support the position,
  - Cost of unwinding the position,
  - Amount to minimize in hedging the position,
  - Cost of surety for the position.

More Findings

- Acceptability of a position can be mapped monotone to a probability distortion parameter, *e.g*. *minmaxvar,* Wang transform.
- When distortion is known, bid and ask prices can be modeled.
- Bid and ask are market observables. Proba-bility thresholds for VAR and TVAR are not.

L-N Variable, Wang Distortion, Ask/Bid/Spread vs. Distortion



Calibration

- Mapping acceptability to observed bid & ask.
- Depends on state of firm and market.
- Requires matching model bid/ask to market.
- Implies embedding in average market portfolio.
- Departures from market average can be hedged.

Actuarial Applications

- Valuation of assets and liabilities
- Risk margins for pricing and reserving
- Assessing capital needs
- Allocating capital costs
- Optimizing reinsurance terms
- Hedging catastrophic losses

Market-Based Valuation

- For a forward obligation, transaction price is indefinite – somewhere between bid and ask.
- Price swings seen in cycles are structural.
- Value assets at bid, liabilities at ask.
- Hold differences in actual transactions in reserve, and run off as obligation matures.

Insurance Risk Margins

- The modeled ask is a fully risk-loaded price for the obligation.
- Unless demand is slack (e.g. bottom of a cycle, visibly impaired credit), insurer can command such a price.
- Suggestion: Bid and ask mark the range of the underwriting cycle.
- Reserve valuation is governed by a consistent bid sequence. (Hard problem.)

Assessing Capital Needs

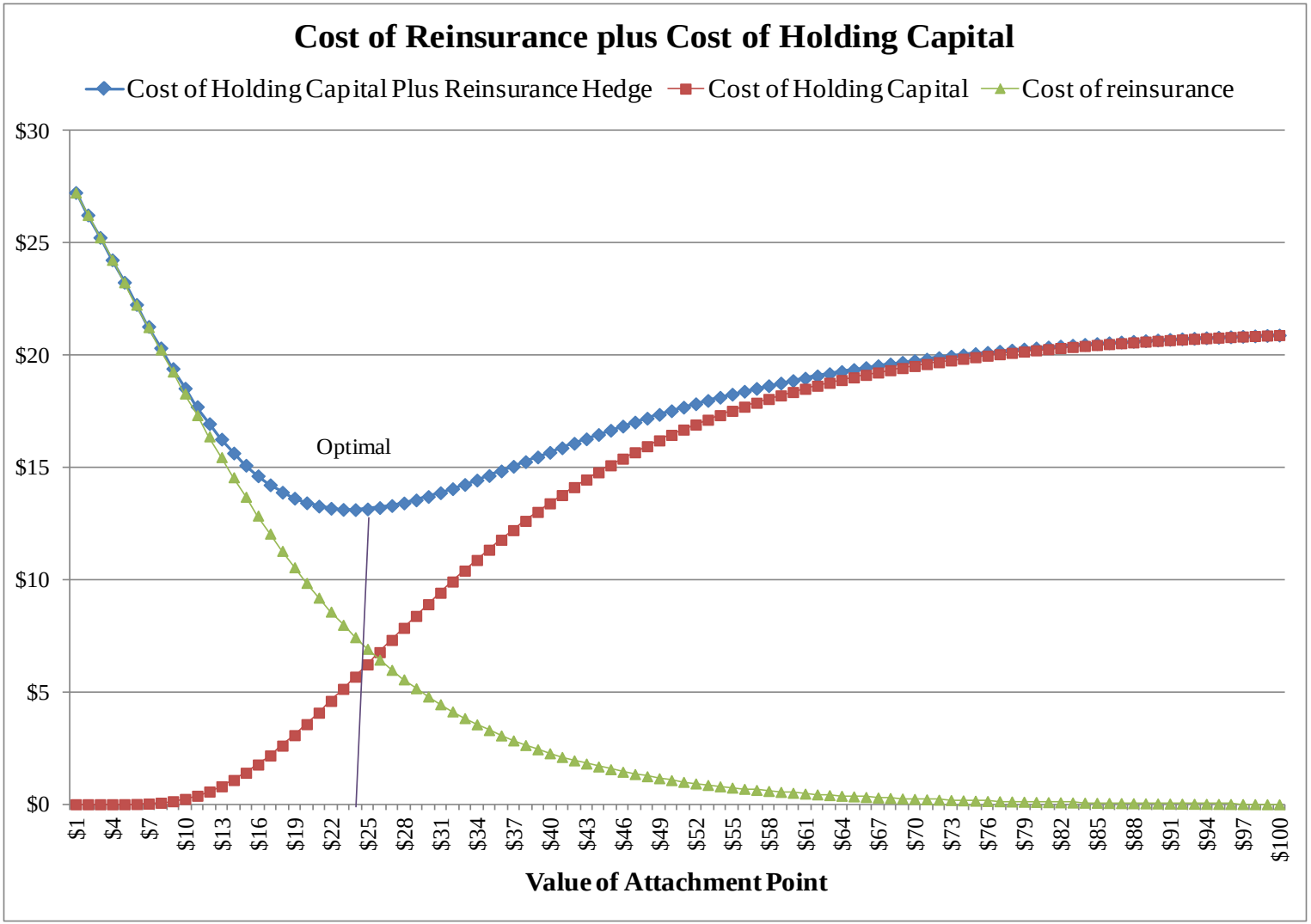
- Bid-ask spread measures capital needed to support any position in a market context.
- Can be evaluated at total portfolio level to estimate needed capital, assess adequacy of surplus.
- Places great demands on stochastic modeling.
- Level of acceptability must also be decided.

Allocating Capital Costs

- Bid-ask spread for a contract measures capital need for embedding in average market portfolio.
- Firm's actual portfolio can be replicated by hedging at no cost.
- Needed capital can be charged against the contract at a uniform rate.

Optimizing Reinsurance

- Calculate capital cost (bid-ask spread) of holding the net position.
- Add the cost of reinsuring to the net position (given) plus cost of default.
- Choose the net position that minimizes the sum.
- Minimum capital is more robust than other objectives.


Cost of Reinsurance plus Cost of Holding Capital
Cost of Holding Capital Plus Reinsurance Hedge
Cost of Holding Capital
Cost of reinsurance
$30
$25
$20
$15
$10
$5
$0
Optimal
$1
$4
$7
$10
$13
$16
$19
$22
$25
$28
$31
$34
$37
$40
$43
$46
$49
$52
$55
$58
$61
$64
$67
$70
$73
$76
$79
$82
$85
$88
$91
$94
$97
$100
Value of Attachment Point

Hedging Catastrophic Losses

- Detailed account in Section 8 of research paper.
- Devise security as stop-loss for industry.
- Optimize hedge for single firm under different criteria: 1) Variance, 2) Certainty equivalents under exponential utility, 3) Minimum capital.
- Variance too inflexible; CE can lose sensitivity; MC remains robust.