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Two-Price Markets: Break off the Traditional Theory of Risk

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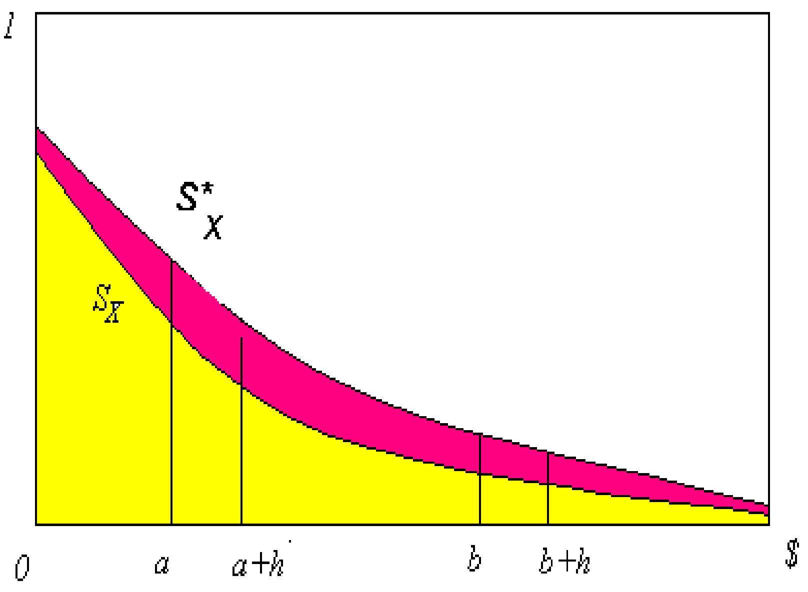
Outline

- I will give examples of probability distortions, and apply them to insurance risks
- I will discuss implications in pricing risks and capital allocations

P-measure vs. Q-measure

Mapping between

1. ***Loss Curve***
   - ***physical measure***
   - ***S(x) = 1- F(x)***
2. ***Pricing Curve***
   - ***risk-neutral measure***
   - ***S*(x) = 1- F*(x)***



Wang Transform is a probability distortion

- **Transform a <u>loss curve</u> to a <u>price curve</u>:**

  $$F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda] \quad \text{or}$$

  $$F^*(x) = \text{normsdist}(\ \text{normsinv}(F(x)) - \lambda\ )$$

  e.g. $0.97 = \Phi[\Phi^{-1}(0.99) - 0.45]$

- If F_X is normal(μ,σ), $F_X{}^*$ is normal($\mu+\lambda\sigma$, σ):
  - $E^*[X] = E[X] + \lambda\ \sigma[X]$

- If F_X is lognormal(μ, σ), $F_X{}^*$ is lognormal($\mu+\lambda\sigma$, σ)

- λ extends Sharpe ratio to skewed distributions

Benchmark Pricing Distortion: 2-factor Wang Transform

$$F^*(y) = t_5\left(\Phi^{-1}(F(y)) - 0.45\right)$$

where

- Φ is standard normal Distribution Function, and
- t_5 is Student-t Distribution Function with 5 degrees-of-freedom

Cost of Capital Distortion

- A distortion function that reflects the cost of capital

$$G(y) = 0.9 * F(y) + 0.1 * \Phi\left(\Phi^{-1}(F(y)) - \Phi^{-1}(0.995)\right)$$

Example One: Optimal Reinsurance

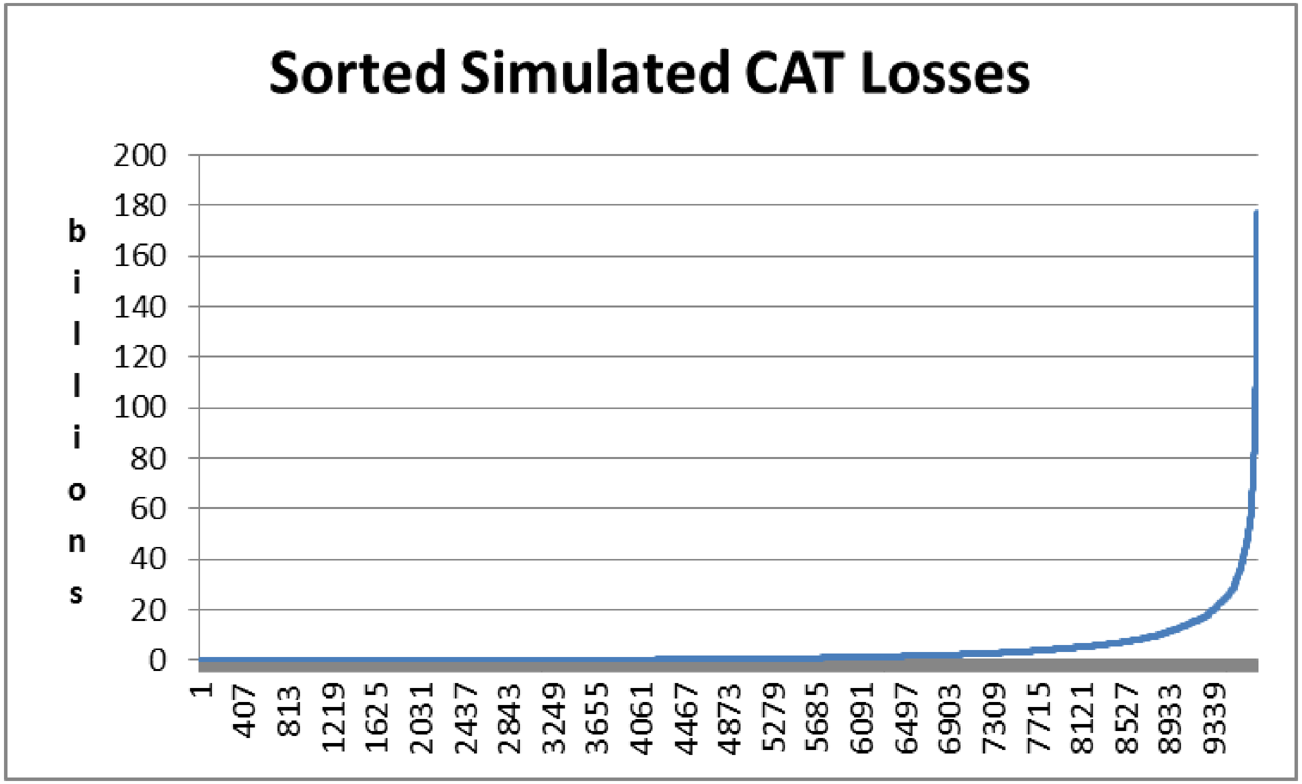
- Simulated Florida Hurricane Losses

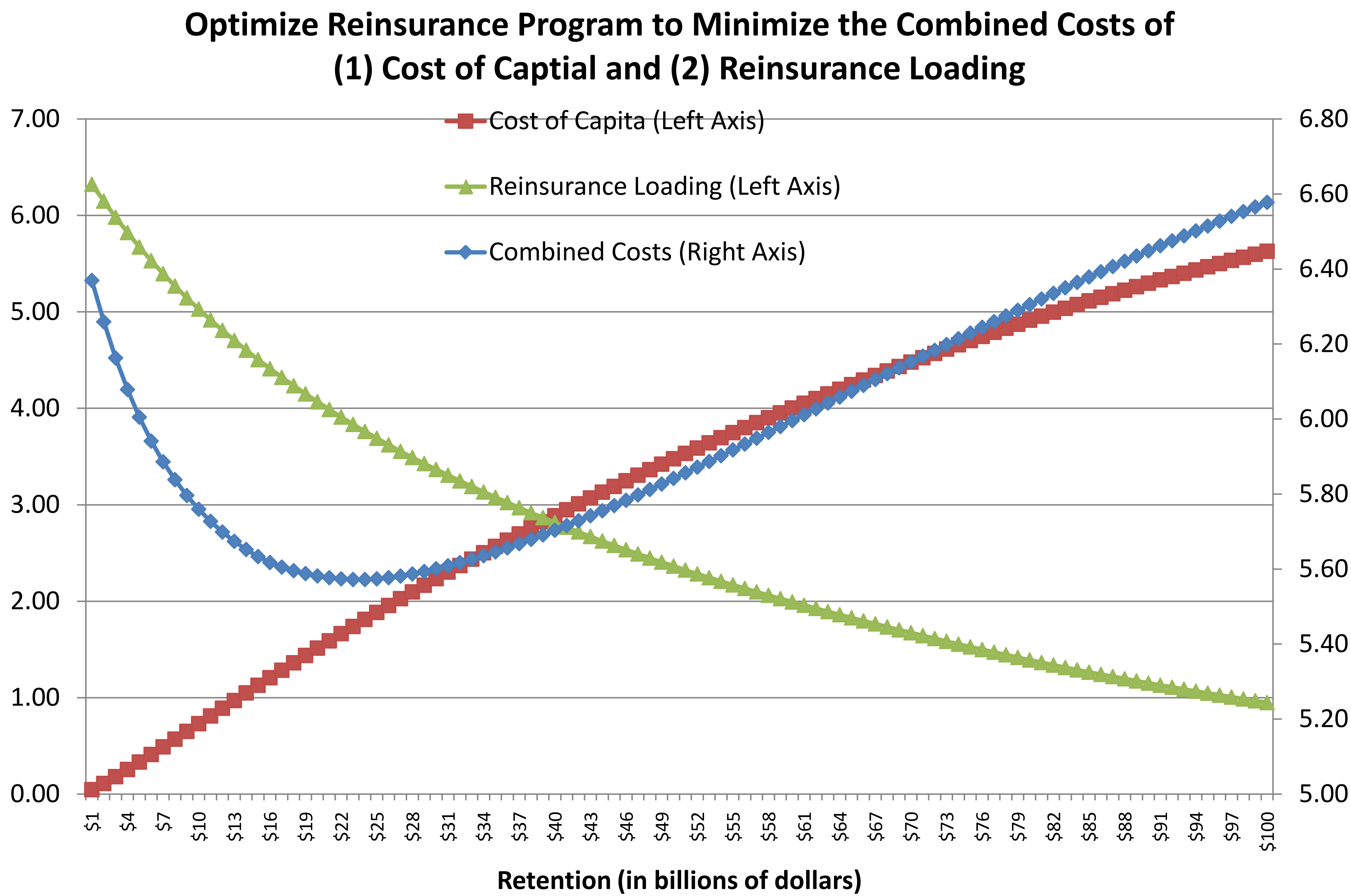
  Summary statistics (in billions)

mean	3.64
Stdev	9.35
Max	177.03

- Question: what is the optimal retention?

Simulated Florida Hurricane Loss Curve





Optimize Reinsurance Program to Minimize the Combined Costs of
(1) Cost of Captial and (2) Reinsurance Loading
Cost of Capita (Left Axis)
Reinsurance Loading (Left Axis)
Combined Costs (Right Axis)
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
6.80
6.60
6.40
6.20
6.00
5.80
5.60
5.40
5.20
5.00
$1 $4 $7 $10 $13 $16 $19 $22 $25 $28 $31 $34 $37 $40 $43 $46 $49 $52 $55 $58 $61 $64 $67 $70 $73 $76 $79 $82 $85 $88 $91 $94 $97 $100
Retention (in billions of dollars)

Calculated Costs for the case that retention = $20 billion

Costs	Actuarial Basis	Risk-neutral basis	Expected Loss plus Cost of Capital
Retained Loss	2.78	5.19	**4.30**
Ceded Loss	0.76	**4.83**	6.41

✓For the retained loss, the expected loss plus cost of capital is $4.30 billion, which is lower than the risk-neutral price (or cost of reinsurance) of $5.19 billion.

✓For the ceded loss, the expected loss plus cost of capital is $6.41 billion, which is higher than the risk-neutral price (or cost of reinsurance) of $4.83 billion.

Optimal Retention changes with pricing & capital requirements

- Everything else equal, if we lower the capital requirement from 99.5th to 99th percentile, the optimal retention will increase from $24 billion to $33 billion
- Everything else equal, if we lower the risk-neutral reinsurance pricing distortion parameter from 0.45 to 0.3, the optimal retention will decrease from $24 billion to $10 billion.

Example Two:
Regular vs. Disappearing Deductibles

- Ln(X) has a normal distribution with mu=4 and sigma=0.5

- ➢ Case 1. Regular Deductible = 50
- ➢ Case 2. Disappearing Deductible = 50

- Pricings are based on applying Wang transform to the net loss distributions (with lambda=0.6)

Apply Wang Transform

$$F^*(y) = \Phi\left(\Phi^{-1}(F(y)) - 0.6\right)$$

Regular Deductible

	E	E*
Retained	43.93	47.49
Ceded	17.92	35.88
Total	61.85	83.37

Disappearing Deductible

	E	E*
Retained	15.43	25.83
Ceded	46.42	74.94
Total	61.85	83.37

E* is additive

E* is non-additive

Example Three: Reinsurer Credit Risk

- Ln(X) has a normal distribution mu=4 and sigma=0.5
- Regular Deductible = 50
- Pricing is based on applying Wang transform with lambda=0.6

- Assume that the reinsurer has a 2% chance of default on paying claims (zero recovery rate).

1-Step versus 2-step Transforms

Apply 1-step distortion to the ceded loss distribution reflecting reinsurer credit risks

- Implied Premium Discount = 1.36%

Apply 2-step Transform

1) Transform ceded loss distribution w/o considering credit risk

2) Transform the Bernoulli reinsurer credit risk

- Implied Premium Discount = 7.3%

Conclusions

- Actuaries should use a composite of multiple measures (probability transforms) in pricing risks and allocating the cost of capital:
  1) Benchmark pricing
  2) Portfolio effects
  3) Risk limits constraints
  4) Etc.