GLM II

Basic Modeling Strategy

2013 CAS Ratemaking and Product Management Seminar
by Len Llaguno

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TOWERS WATSON

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Building predictive models is a multi-step process

- Ernesto walked us through the first 3 components
- We will now go through an example of the remaining steps:
  - Building component predictive models
    - We will illustrate how to build a frequency model
  - Validating component models
    - We will illustrate how to validate your component model
  - We will also briefly discuss combining models and incorporating implementation constraints
    - Goal should be to build best predictive models now and incorporate constraints later

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Building component predictive models can be separated into two steps

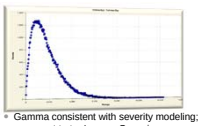
- Initial Modeling
  - Selecting error structure and link function
  - Build simple initial model
  - Testing basic modeling assumptions and methodology
- Iterative modeling
  - Refining your initial models through a series of iterative steps complicating the model, then simplifying the model, then repeating

Initial modeling

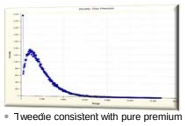
- Initial modeling is done to test basic modeling methodology
  - Is my link function appropriate?
  - Is my error structure appropriate?
  - Is my overall modeling methodology appropriate (e.g. do I need to cap losses? Exclude expense only claims? Model by peril?)

Examples of error structures

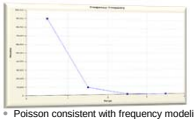
- Error functions reflect the variability of the underlying process and can be any distribution within the exponential family, for example:



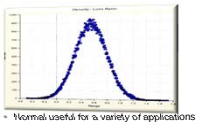
- Gamma consistent with severity modeling; may want to try Inverse Gaussian



- Tweedie consistent with pure premium modeling



- Poisson consistent with frequency modeling



- Normal useful for a variety of applications

Generally accepted error structure and link functions

- Use generally accepted standards as starting point for link functions and error structures

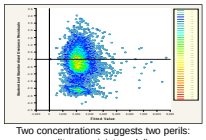
Observed Response	Most Appropriate Link Function	Most Appropriate Error Structure	Variance Function
--	--	Normal	μ^0
Claim Frequency	Log	Poisson	μ^1
Claim Severity	Log	Gamma	μ^2
Claim Severity	Log	Inverse Gaussian	μ^3
Pure Premium	Log	Gamma or Tweedie	μ^T
Retention Rate	Logit	Binomial	$\mu(1-\mu)$
Conversion Rate	Logit	Binomial	$\mu(1-\mu)$

Build an initial model

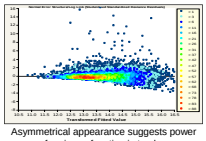
- Reasonable starting points for model structure
  - Prior model
  - Stepwise regression
  - General insurance knowledge
  - CART (Classification and Regression Trees) or similar algorithms

Test model assumptions

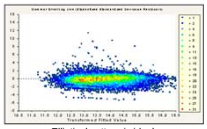
- Plot of all residuals tests selected error structure/link function



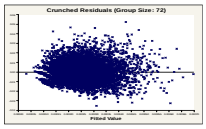
Two concentrations suggests two perils: split or use joint modeling



Asymmetrical appearance suggests power of variance function is too low



Elliptical pattern is ideal

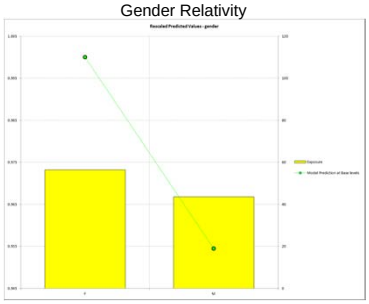


Use crunched residuals for frequency

Example: initial frequency model

- Link function: Log
- Error structure: Poisson
- Initial variable selected based on industry knowledge:
  - Gender
  - Driver age
  - Vehicle value
  - Area (territory)
- Variable NOT in initial model:
  - Vehicle body
  - Vehicle age

Gender Relativity



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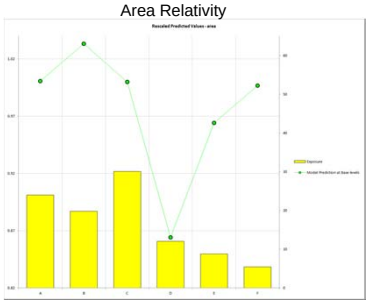
Area Relativity



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Example: initial frequency model

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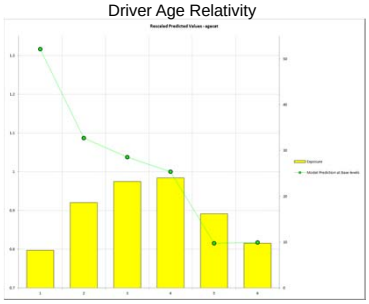
Driver Age Relativity



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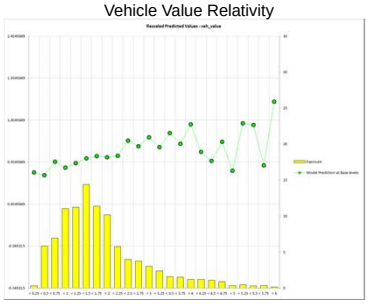
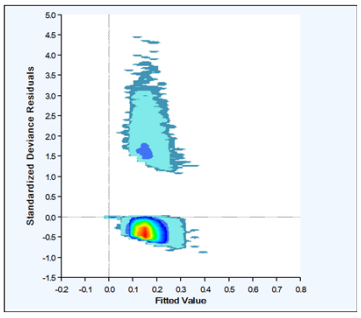
Vehicle Value Relativity

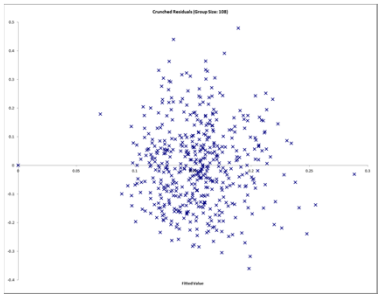


Example: initial frequency model - residuals



- Frequency residuals are hard to interpret without 'Crunching'
- Two clusters:
  - Data points with claims
  - Data point without claims

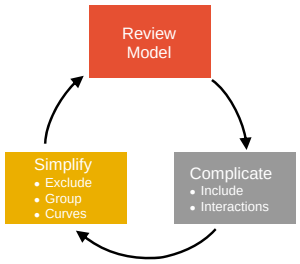
Example: initial frequency model - residuals



- Order observations from smallest to largest predicted value
- Group residuals into 500 buckets
- The graph plots the average residual in the bucket
- Crunched residuals look good!

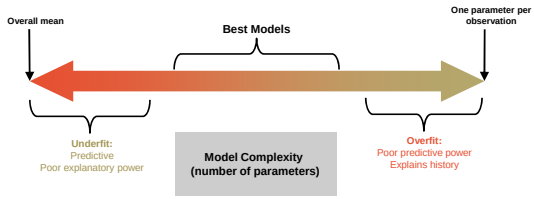
Iterative Modeling

- Initial models are refined using an iterative modeling approach
- Iterative modeling involves many decisions to complicate and simplify the models
- Your modeling toolbox can help you make these decisions
  - We will discuss your tools shortly



Ideal Model Structure

- To produce a sensible model that explains recent historical experience and is likely to be predictive of future experience

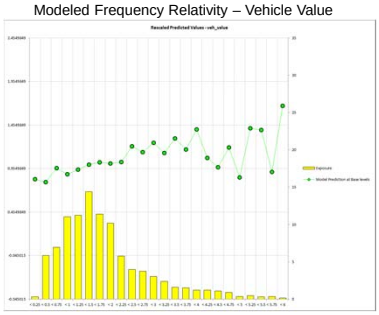


Your modeling tool box

- Model decisions include:
  - Simplification: excluding variables, grouping levels, fitting curves
  - Complication: including variables, adding interactions
- Your modeling toolbox will help you make these decisions
  - Your tools include:
    - Parameters/standard errors
    - Consistency of patterns over time or random data sets
    - Type III statistical tests (e.g., chi-square tests, F-tests)
    - Balance tests (i.e. actual vs. expected test)
    - Judgment (e.g., do the trends make sense?)

Modeling toolbox: judgment

Modeled Frequency Relativity – Vehicle Value



- The modeler should also ask, 'does this pattern make sense?'
- Patterns may often be counterintuitive, but become reasonable after investigation
- Uses:
  - Inclusion/exclusion
  - Grouping
  - Fitting curves
  - Assessing interactions

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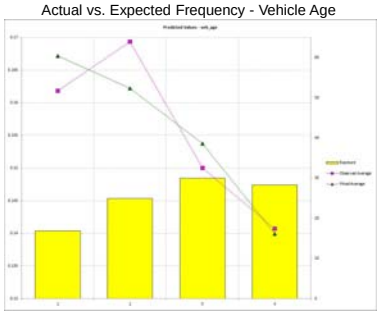
Modeling toolbox: balance test

Actual vs. Expected Frequency - Vehicle Age



- Balance test is essentially an actual vs. expected
- Can identify variables that are not in the model where the model is not in 'balance'
  - Indicates variable may be explaining something not in the model
- Uses:
  - Inclusion

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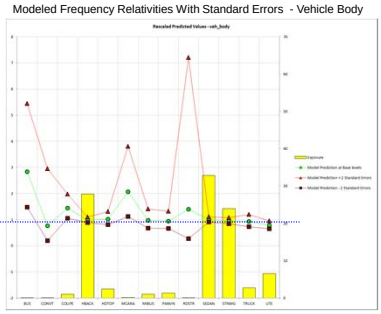
Modeling toolbox: parameters/standard errors

Modeled Frequency Relativities With Standard Errors - Vehicle Body



- Parameters and standard errors provide confidence in the pattern exhibited by the data
- Uses:
  - Horizontal line test for exclusion
  - Plateaus for grouping
  - A measure of credibility

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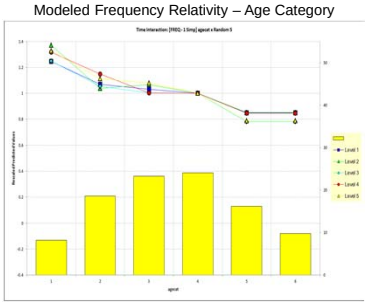
Modeling toolbox: consistency of patterns

- Checking for consistency of patterns over time or across random parts of a data set is a good practical test
- Uses:
  - Validating modeling decisions
    - Including/excluding factors
    - Grouping levels
    - Fitting curves
    - Adding Interactions

Modeled Frequency Relativity – Age Category



Modeling toolbox: type III tests

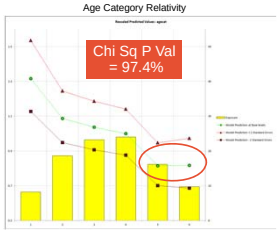
- Chi test and/or F-Test is a good statistical test to compare nested models
  - H_o: Two models are essentially the same
  - H_1: Two models are not the same
  - Principle of parsimony: If two models are the same, choose the simpler model
- Uses:
  - Inclusion/exclusion

Chi-Square Percentage	Meaning	Action*
<5%	Reject H_o	Use More Complex Model
5%-15%	Grey Area	???
15%-30%	Grey Area	???
>30%	Accept H_o	Use Simpler Model

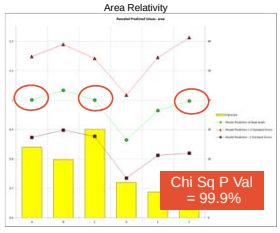
Example: frequency model iteration 1 – simplification

- Modeling decision: Grouping Age Category and Area
- Tools Used: judgment, parameter estimates/std deviations, type III test

Age Category Relativity



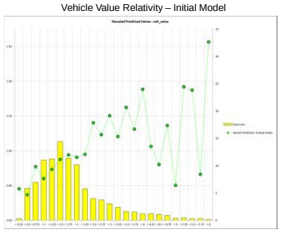
Area Relativity



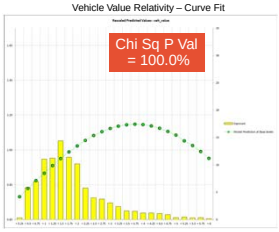
Example: frequency model iteration 1 – simplification

- Modeling decision: fitting a curve to vehicle value
- Tools used: judgment, type III test, consistency test

Vehicle Value Relativity – Initial Model



Vehicle Value Relativity – Curve Fit



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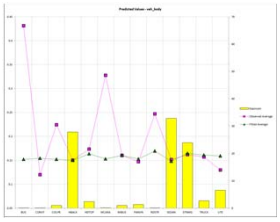
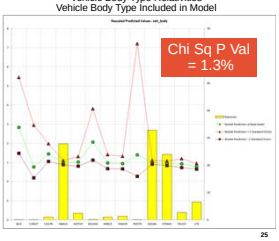
Example: frequency model iteration 2 – complication

- Modeling decision: adding vehicle body type
- Tools used: balance test, parameter estimates/std deviations, type III test

Balance Test:
Actual vs. Expected Across Vehicle Body Type
Vehicle Body Type Not In Model



Vehicle Body Type Relativities
Vehicle Body Type Included in Model



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Example: iterative modeling continued….

- Iteration 3 - simplification
  - Group vehicle body type
- Iteration 4 – complication
  - Add vehicle age
- Iteration 5 – simplification
  - group vehicle age levels

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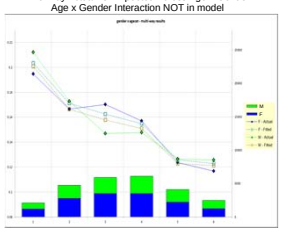
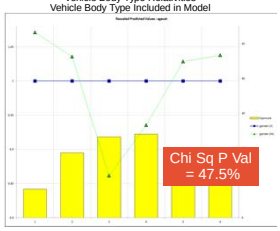
Example: frequency model iteration 6 – complication

- Action: adding age x gender interaction
- Tools used: balance test, type III test, consistency test, judgment

Balance Test:
Two Way Actual vs. Expected Across Age x Gender
Age x Gender Interaction NOT in model



Vehicle Body Type Relativities
Vehicle Body Type Included in Model

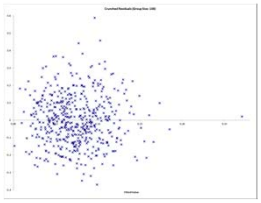


Predictive models must be validated to have confidence in the predictive power of the models

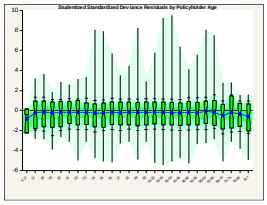
- Model validation techniques include:
  - Examining residuals
  - Examining gains curves
  - Examining hold out samples
    - Changes in parameter estimates
    - Actual vs. expected on hold out sample
- Component models and combined risk premium model should be validated

Model validation: residual analysis

- Recheck residuals to ensure appropriate shape



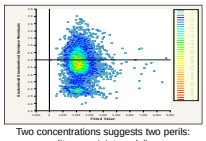
- Crunched residuals are symmetric



- For Severity - Does the Box-Whisker show symmetry across levels?

Model validation: residual analysis (cont'd)

- Common issues with residual plots



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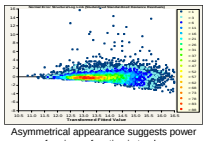

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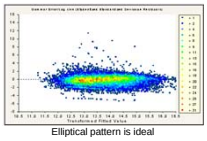

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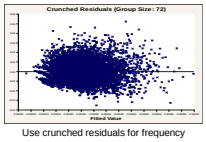
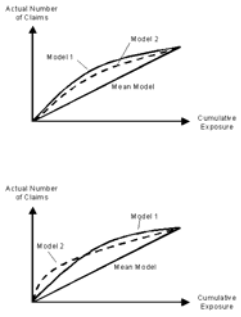


Use crunched residuals for frequency

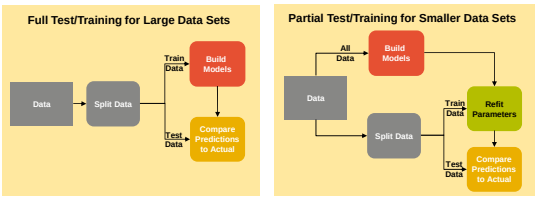
Model validation: gains curves



- Gains curve are good for comparing predictiveness of models
  - Order observations from largest to smallest predicted value on X axis
  - Cumulative actual claim counts (or losses) on Y axis
  - As you move from left to right, the better model should accumulate actual losses faster

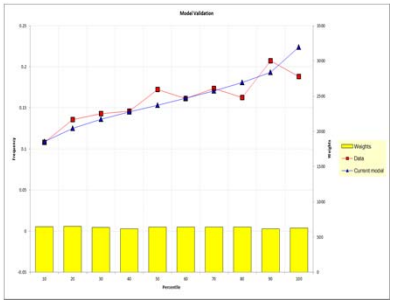
Model validation: hold out samples

- Holdout samples are effective at validating models
  - Determine estimates based on part of data set
  - Uses estimates to predict other part of data set



Predictions should be close to actuals for heavily populated cells

Model validation: lift charts on hold out data



- Actual vs. expected on holdout data is an intuitive validation technique
- Good for communicating model performance to non-technical audiences
- Can also create actual vs. expected across predictor dimensions

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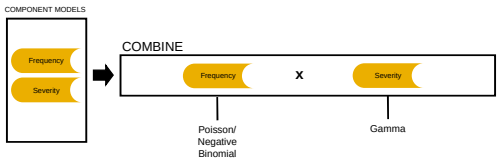
Component frequency and severity models can be combined to create pure premium models

Set project goals and review background → Gather and prepare data → Explore Data → Build Component Predictive Models → Validate Component Models → **Combine Component Models** → Incorporate Constraints

- Component models can be constructed in many different ways
  - The standard model:



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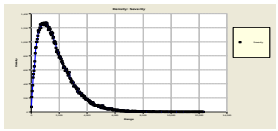
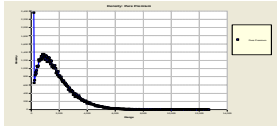
Building a model on modeled pure premium

- When using modeled pure premiums, select the gamma/log link (not the Tweedie)



- Modeled pure premiums will not have a point mass at zero



- Raw pure premiums are bimodal (i.e., have a point mass at zero) and require a distribution such as the Tweedie

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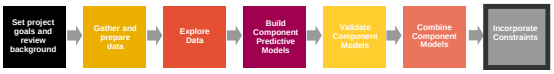
Various constraints often need to be applied to the modeled pure premiums



Goal: Convert modeled pure premiums into indications after consideration of internal and external constraints

- Not always possible or desirable to charge the fully indicated rates in the short run
  - Marketing decisions
  - Regulatory constraints
  - Systems constraints
- Need to adjust the indications for known constraints

Constraints to give desired subsidies

- Offsetting one predictor changes parameters of other correlated predictors to make up for the restrictions
  - The stronger the exposure correlation, the more that can be made up through the other variable
  - Consequently, the modeler should not refit models when a desired subsidy is incorporated into the rating plan

	Insurer-Desired Subsidy	Regulatory Subsidy
Example	Sr. mgmt wants subsidy to attract drivers 65+	Regulatory constraint requires subsidy of drivers 65+
Result of refitting with constraint	Correlated factors will adjust to partially make up for the difference. For example, territories with retirement communities will increase.	
Potential action	Do not refit models with constraint	Consider implication of refitting and make a business decision