



# GLM III - The Matrix Reloaded

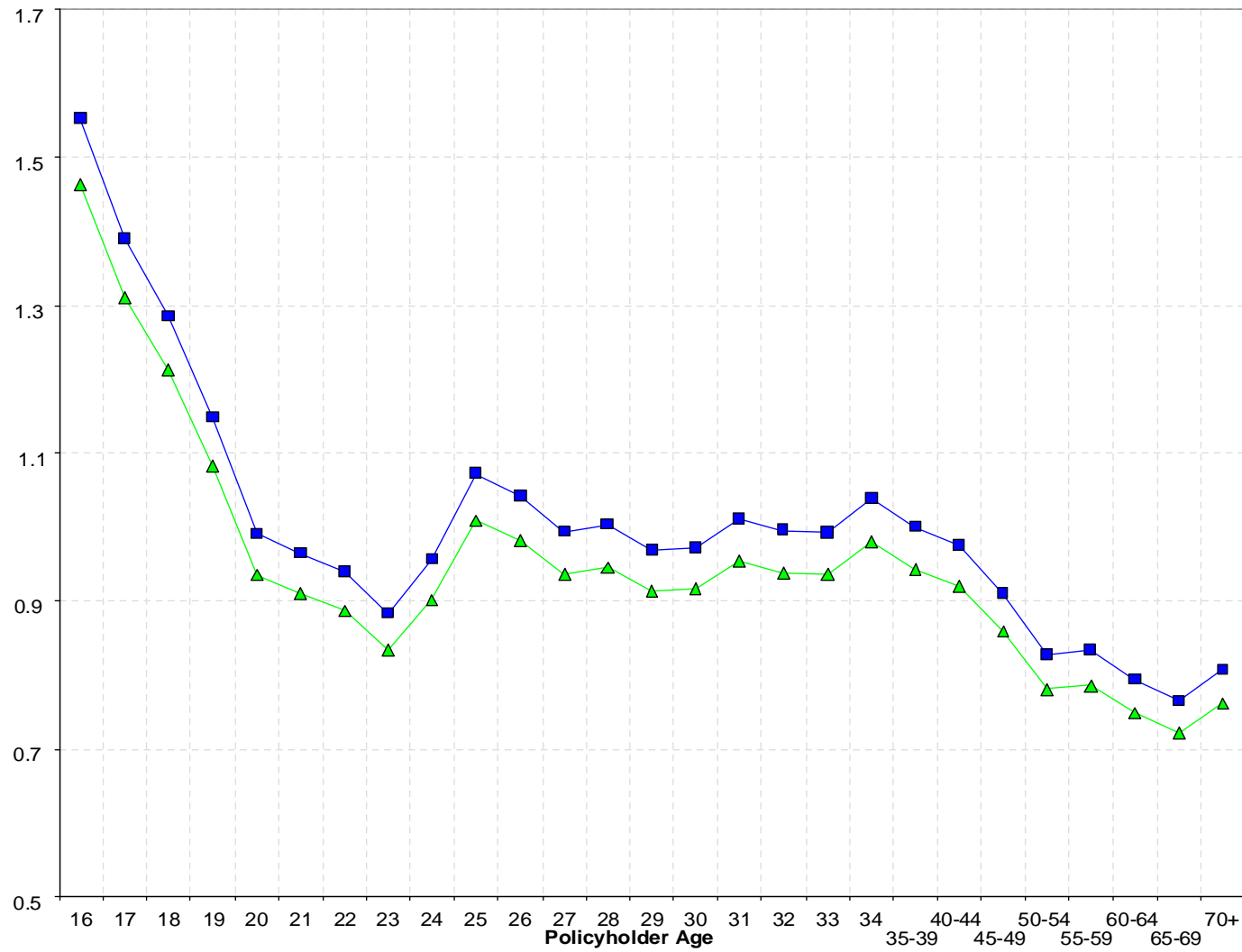
Duncan Anderson, Serhat Guven

12 March 2013

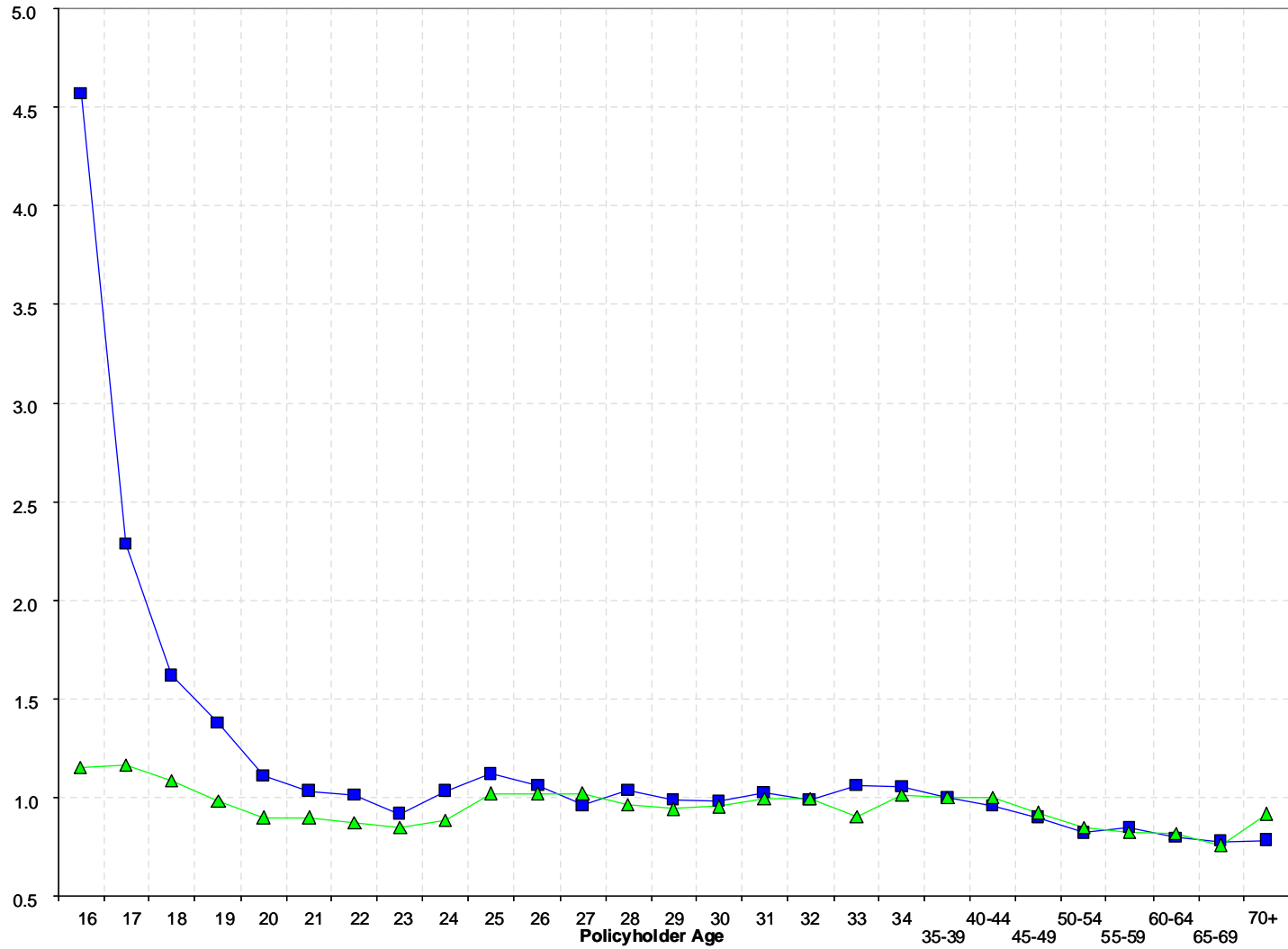
## Agenda

- "Quadrant Saddles"
- The Tweedie Distribution
- "Emergent Interactions"
- Dispersion Modeling
- Modelling sparse claim types
- Driver Averaging
- Model Validation
- Man (with GLM) vs machine

# Interactions



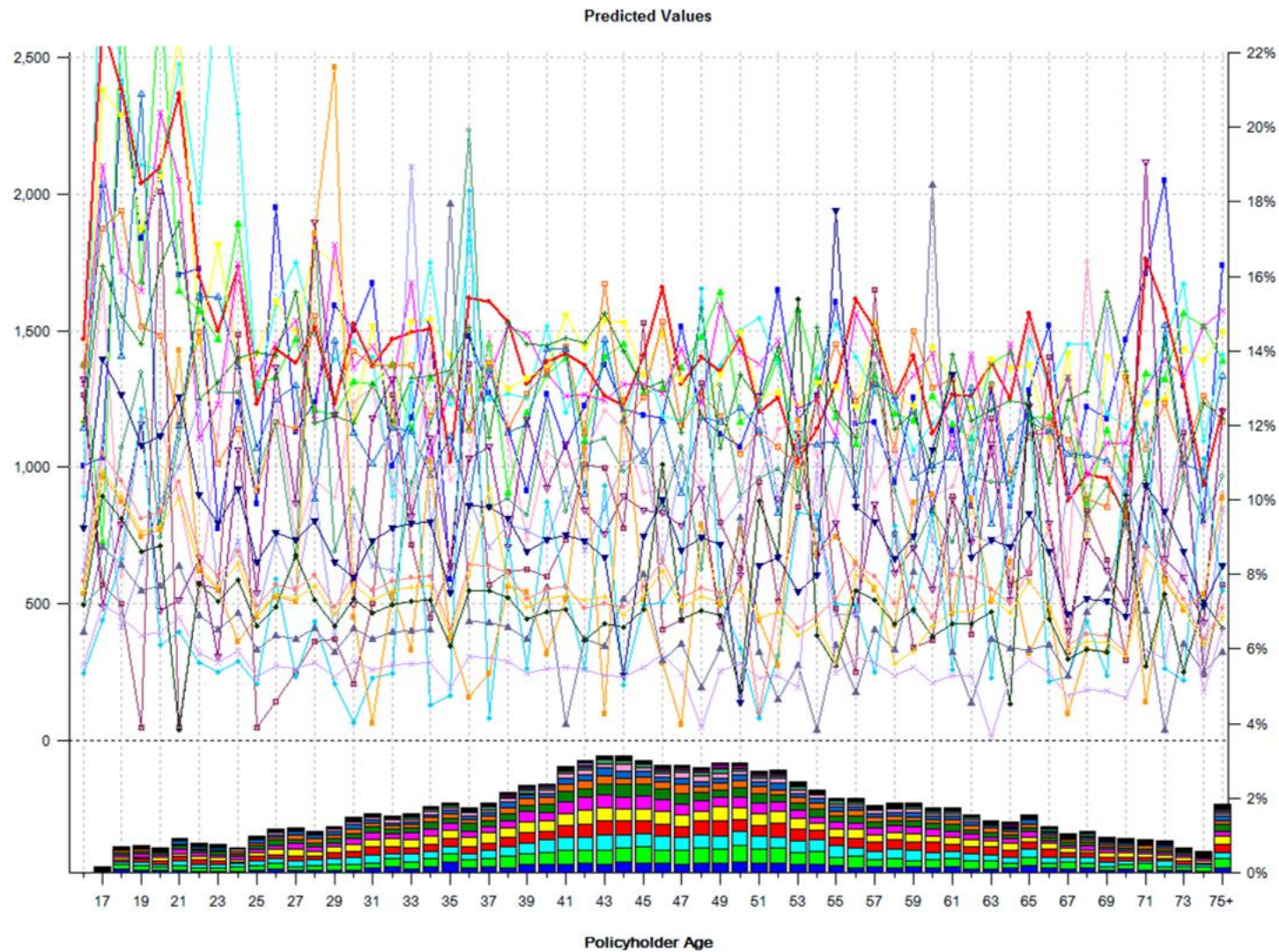
# Interactions



## Why are interactions present?

- Because that's how the factors behave
- Because the multiplicative model can go wrong at the edges
  - $1.5 * 1.4 * 1.7 * 1.5 * 1.8 * 1.5 * 1.8 = 26!$

# Interactions





# Interactions

Selected Interaction: Vehicle Age x Policyholder Age

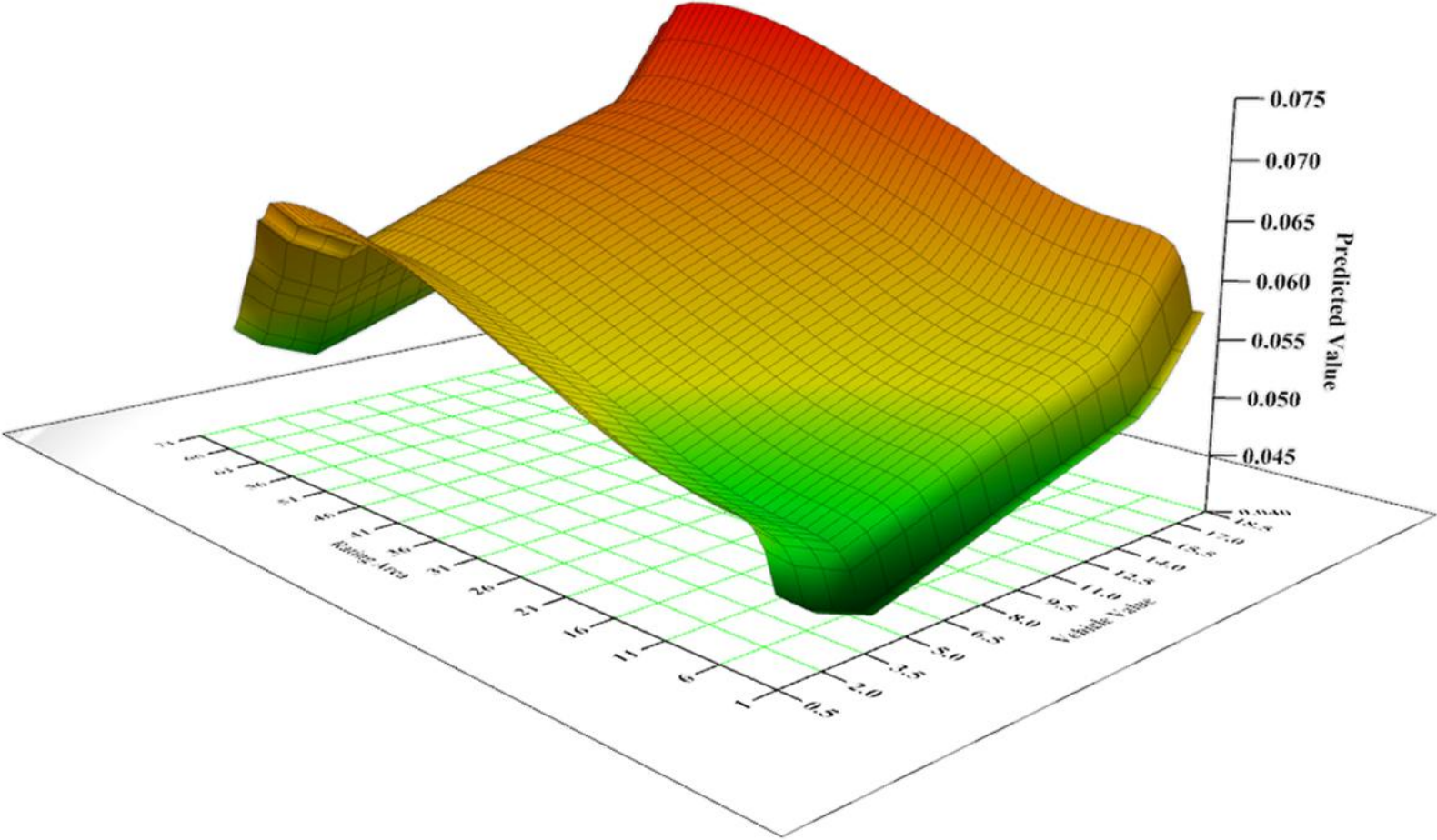
|             |        | Policyholder Age |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |      |
|-------------|--------|------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|------|
|             |        | N/A              | 17      | 18      | 19      | 20      | 21      | 22      | 23      | 24      | 25      | 26      | 27      | 28      | 29      | 30      | 31      | 32      | 33   |
| Vehicle Age | 0      | 0.0000           | 0.0861  | 1.3433  | -0.2833 | -0.1112 | -0.6276 | -0.4940 | -0.9551 | 0.9797  | -0.6060 | -0.8082 | -0.6622 | -0.8114 | -1.2990 | -0.4380 | 1.0881  | 0.9502  | 1.19 |
|             | 1      | 0.0000           | -0.3007 | -0.3838 | 0.6784  | -1.0827 | 1.5070  | 0.2896  | -0.3146 | 0.4689  | 1.1002  | 0.0152  | -1.2990 | 0.2485  | -0.1511 | -0.4321 | -1.5809 | 0.2436  | 0.87 |
|             | 2      | 0.0000           | 0.5942  | -0.5767 | -1.3743 | 0.2555  | 1.5679  | -0.3335 | -0.3765 | 1.3385  | 1.3492  | 1.2615  | -0.4747 | 0.9228  | -0.1730 | 1.0359  | -0.8213 | -0.4516 | 1.09 |
|             | 3      | 0.0000           | -0.0944 | -1.6225 | 0.5895  | -0.6255 | 0.4782  | -1.4383 | 0.8664  | -1.1652 | 0.3511  | 0.4272  | -0.3256 | -0.5740 | -0.2573 | -0.3694 | 0.3803  | -0.6589 | -0.7 |
|             | 4      | 0.0000           | 0.7341  | -0.3873 | -0.7813 | 0.1535  | -1.0952 | 1.2768  | 0.0124  | -0.2584 | -0.2420 | 0.9662  | 0.8497  | -0.9958 | -1.4329 | 0.4942  | -0.1446 | 1.6775  | -0.3 |
|             | 5      | 0.0000           | 0.7834  | -0.1512 | 0.6603  | 0.1302  | -1.0948 | -1.7072 | -2.7967 | -0.9688 | -0.1618 | -2.4681 | -0.0443 | -1.2045 | 0.6146  | 1.1614  | 1.8351  | -1.5661 | -0.2 |
|             | 6      | 0.0000           | 2.2230  | -0.7901 | -0.2768 | -1.3420 | -0.6245 | 1.1232  | 1.5019  | 0.5902  | -0.0202 | 2.1912  | 0.1612  | -1.7502 | 0.1939  | -1.9662 | 1.5626  | 0.6414  | -0.1 |
|             | 7      | 0.0000           | -0.6514 | 0.4566  | -1.4507 | 0.2114  | -0.7806 | -0.8530 | -0.8567 | 0.0683  | 0.2886  | -0.1552 | 1.3343  | 1.5336  | -1.3740 | 1.5809  | -1.1128 | -0.8446 | 0.22 |
|             | 8      | 0.0000           | -0.2580 | 0.2443  | -1.0002 | 0.2211  | 0.6874  | 0.8802  | 1.4942  | -1.7197 | 2.0866  | -0.1492 | 0.5193  | 1.0116  | 2.4261  | 0.1893  | 1.9269  | -1.1336 | -0.7 |
|             | 9      | 0.0000           | -1.4664 | 1.0500  | -1.7925 | -0.6770 | 0.6516  | -1.4137 | 0.1120  | -0.4953 | -1.2055 | -0.9852 | -0.0994 | 0.4017  | 0.3569  | -0.8758 | -0.1560 | 0.6361  | 1.09 |
|             | 10     | 0.0000           | -0.3617 | -0.4029 | 1.1780  | 2.4451  | -0.3206 | 0.3776  | 1.2449  | 2.3793  | -1.0895 | -0.5111 | -1.7375 | 1.5678  | 0.9283  | -0.4278 | -1.9636 | 1.7165  | -1.8 |
|             | 11     | 0.0000           | -1.0350 | 1.2051  | -0.2111 | 0.3438  | -2.0220 | 1.4628  | -0.7946 | -0.1623 | -0.4664 | -0.0784 | -0.2583 | -1.3273 | 0.7165  | -0.1358 | -0.3915 | 0.2076  | -0.6 |
|             | 12     | 0.0000           | 1.1877  | -0.1965 | 1.8634  | -0.2171 | -0.4839 | -0.7580 | -0.2611 | 0.1789  | -2.0832 | 0.1933  | -0.9373 | 0.1420  | -0.8524 | 1.2916  | -1.8746 | 0.3296  | 1.12 |
|             | 13     | 0.0000           | -0.7453 | 0.8332  | 0.7936  | 0.6080  | -0.1837 | -0.3785 | -0.1706 | -0.1664 | -0.3504 | -0.4945 | 2.2726  | 0.5549  | -0.6972 | 0.0935  | 0.4392  | -0.6115 | -1.6 |
|             | 14     | 0.0000           | -0.3047 | 0.8316  | -0.8146 | 0.3477  | 0.3955  | -0.2695 | 0.7418  | -1.0084 | -0.7375 | -0.3714 | 1.5842  | 0.0045  | 0.6473  | -0.9911 | 0.0465  | -0.2720 | 0.33 |
|             | 15     | 0.0000           | -0.1280 | -1.0783 | 1.4620  | -0.0174 | 1.8673  | 2.3668  | 0.1103  | 0.3009  | 0.8215  | -0.7305 | -0.6188 | 0.3240  | -1.0383 | -1.2315 | -1.0858 | -1.6775 | 0.17 |
|             | 16     | 0.0000           | -0.7361 | 1.4342  | 2.1929  | 1.0053  | 1.2026  | 0.5048  | 0.0824  | -0.1974 | 0.1701  | 1.3634  | 1.4209  | 1.2107  | 0.9176  | 0.2875  | 0.8263  | -1.4325 | 1.85 |
|             | 17     | 0.0000           | -0.6590 | -0.6840 | 3.9395  | -0.2883 | 1.5156  | 1.8105  | 4.0477  | 0.1852  | -0.9165 | 2.6069  | -0.0802 | 0.0289  | 0.1457  | 0.0409  | 0.2029  | 0.1843  | -0.1 |
|             | 18     | 0.0000           | -0.3412 | -0.9500 | 0.6935  | -0.5550 | 1.0272  | 1.5322  | -0.6856 | -0.6104 | -0.4651 | -0.4640 | -0.3911 | -0.5845 | 3.5499  | -0.4948 | -0.6204 | -0.6273 | -0.5 |
|             | 19     | 0.0000           | -0.4783 | -0.8589 | -0.6467 | -0.5014 | -0.6032 | 0.7630  | -0.5475 | -0.5006 | -0.5502 | -0.5398 | -0.4846 | -0.3470 | 1.6578  | -0.4972 | -0.4311 | 1.8195  | -0.6 |
| 20+         | 0.0000 | -0.4626          | -0.7337 | 0.9166  | -0.4947 | 0.9693  | -0.4324 | -0.5751 | -0.4146 | -0.4206 | -0.5885 | -0.3000 | 1.7336  | -0.2879 | -0.4496 | -0.2557 | -0.4791 | -0.4    |      |
| Unknown     | 0.0000 | 2.4147           | -0.7042 | -0.6024 | -0.5451 | -0.4055 | 1.4308  | -0.4851 | -0.3577 | 1.5886  | -0.4599 | -0.4927 | 1.7167  | 2.6352  | -0.4718 | 2.0103  | 1.5534  | -0.4    |      |

Graph Log Un-Group Group Save Hide Values Minimum Weight 0 Shortlist Previous Next

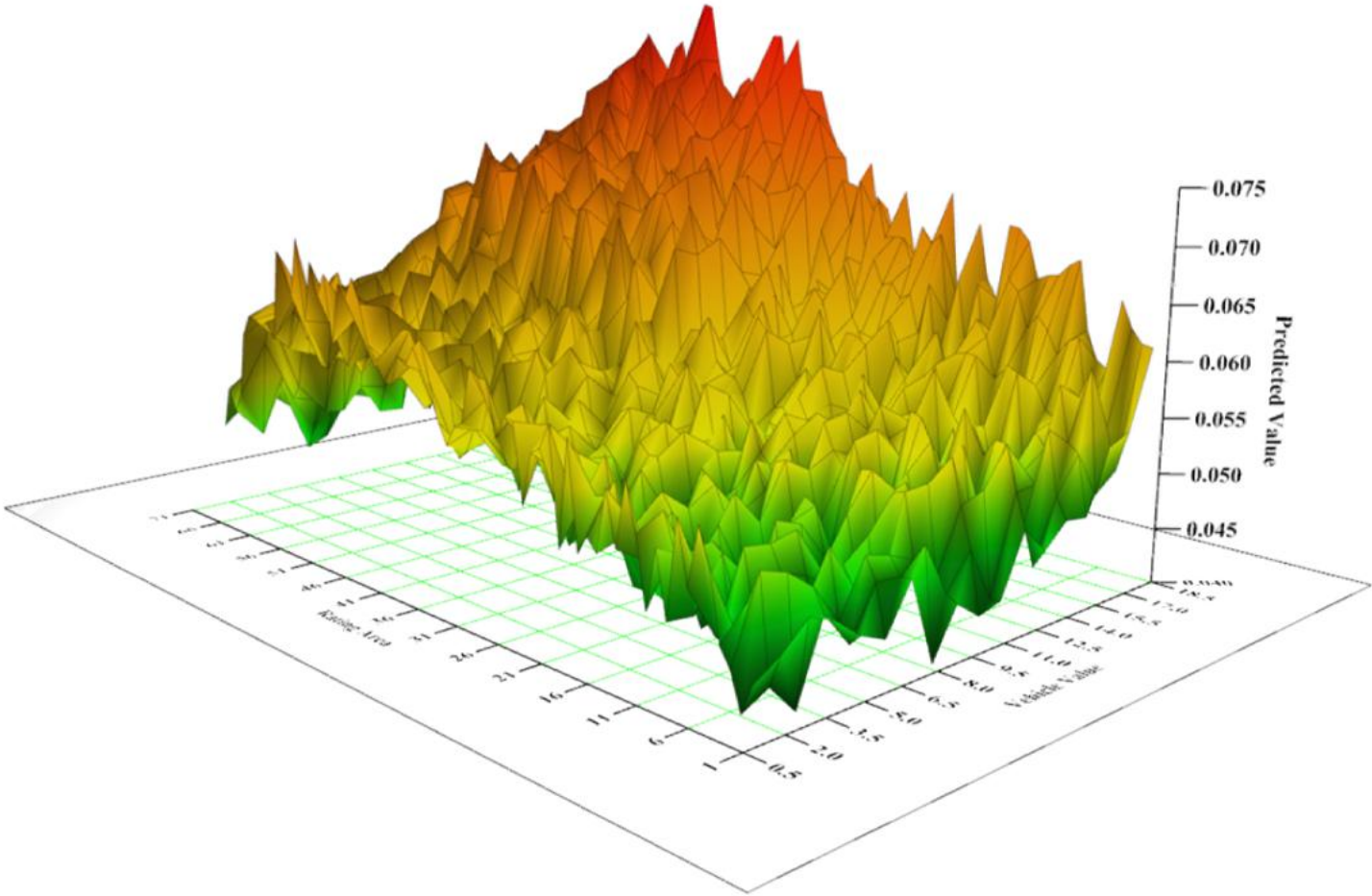




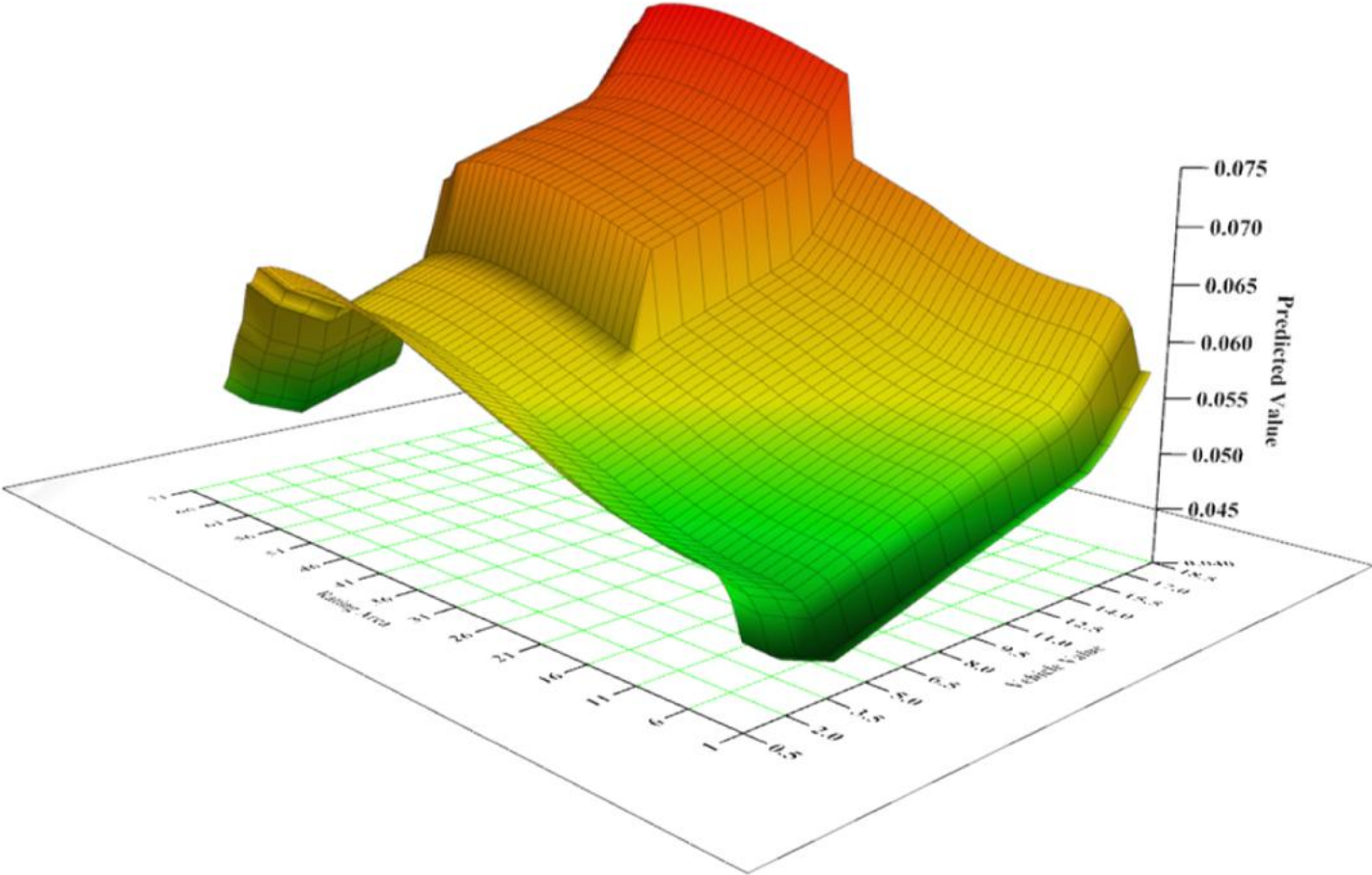
# Example



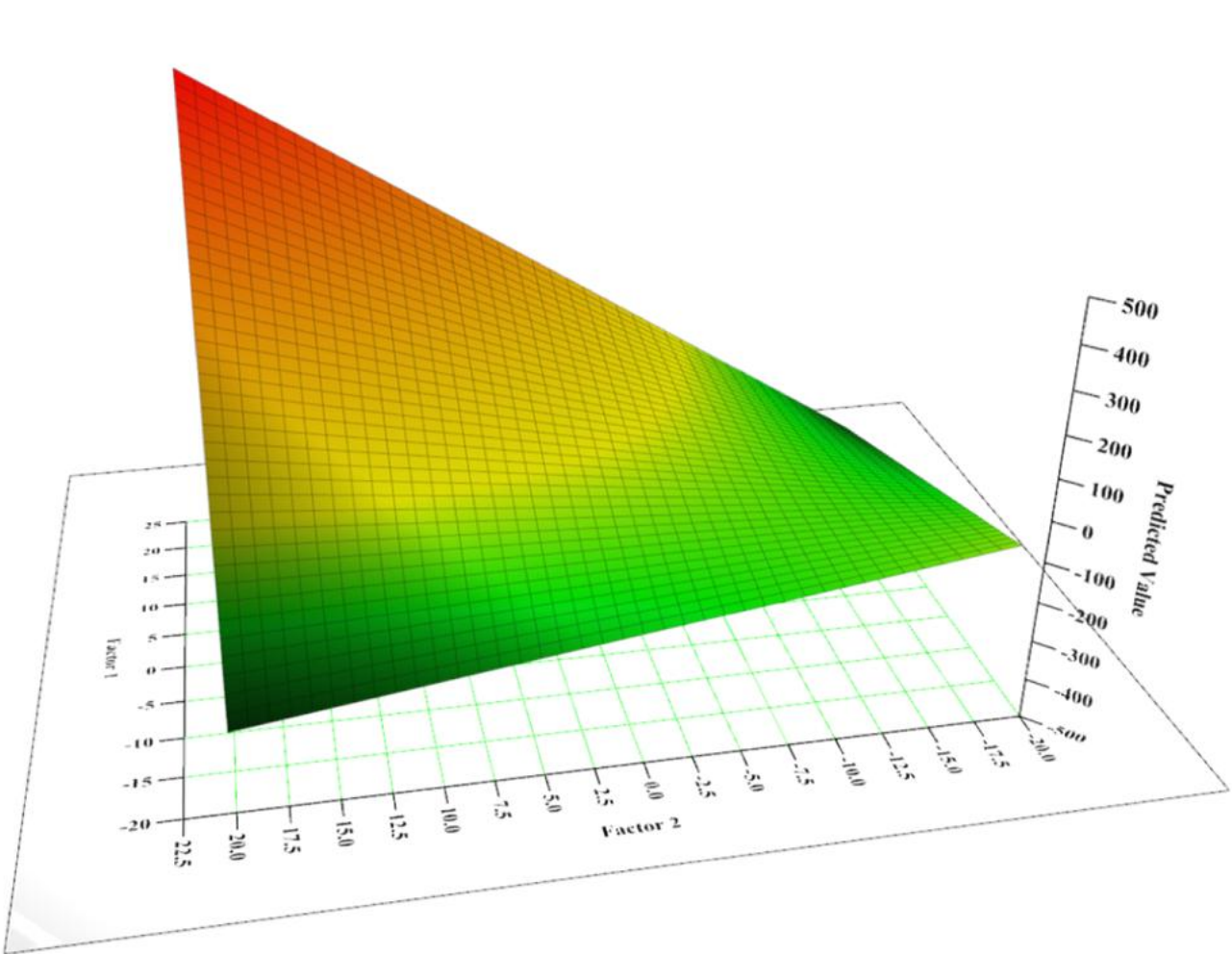
# Example



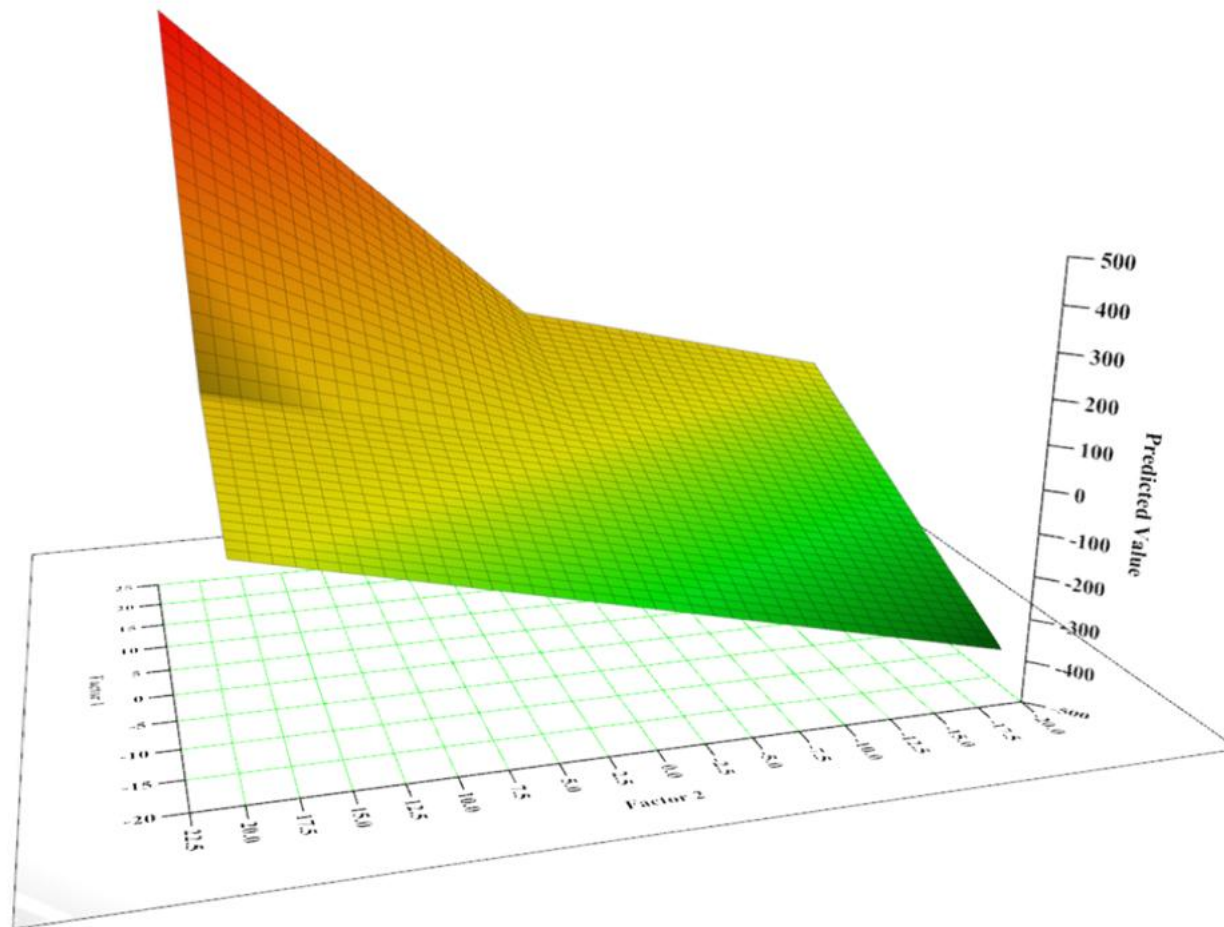
# Example



# Saddles

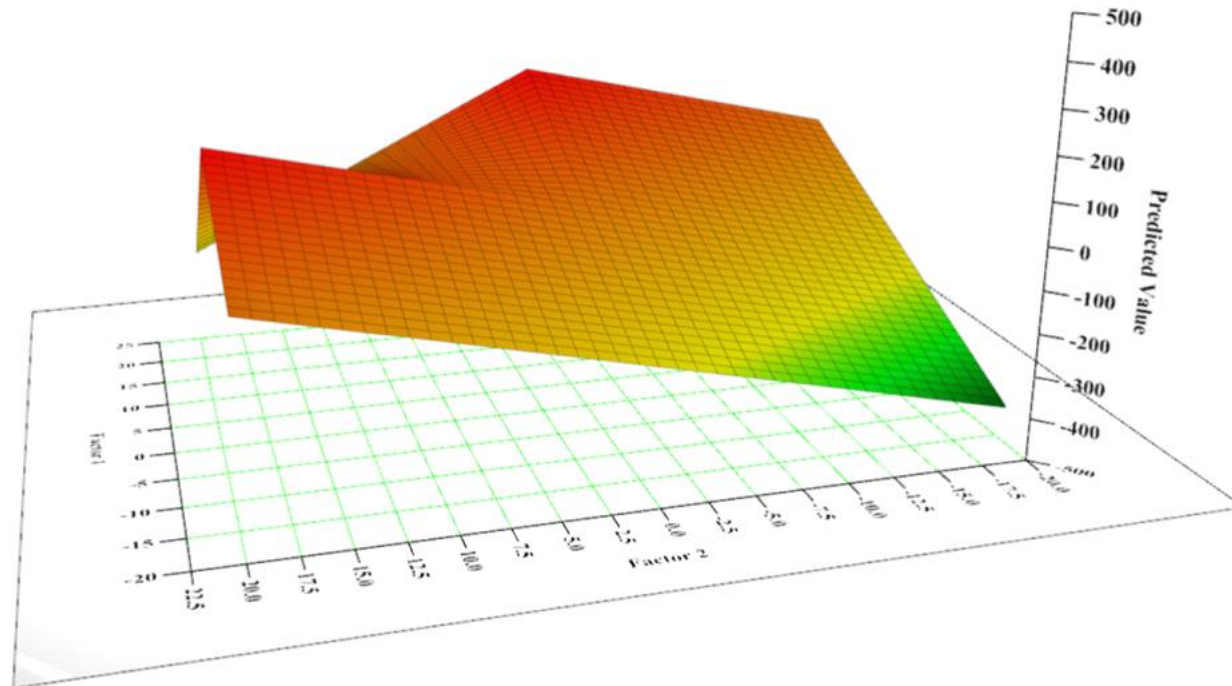


# Saddles

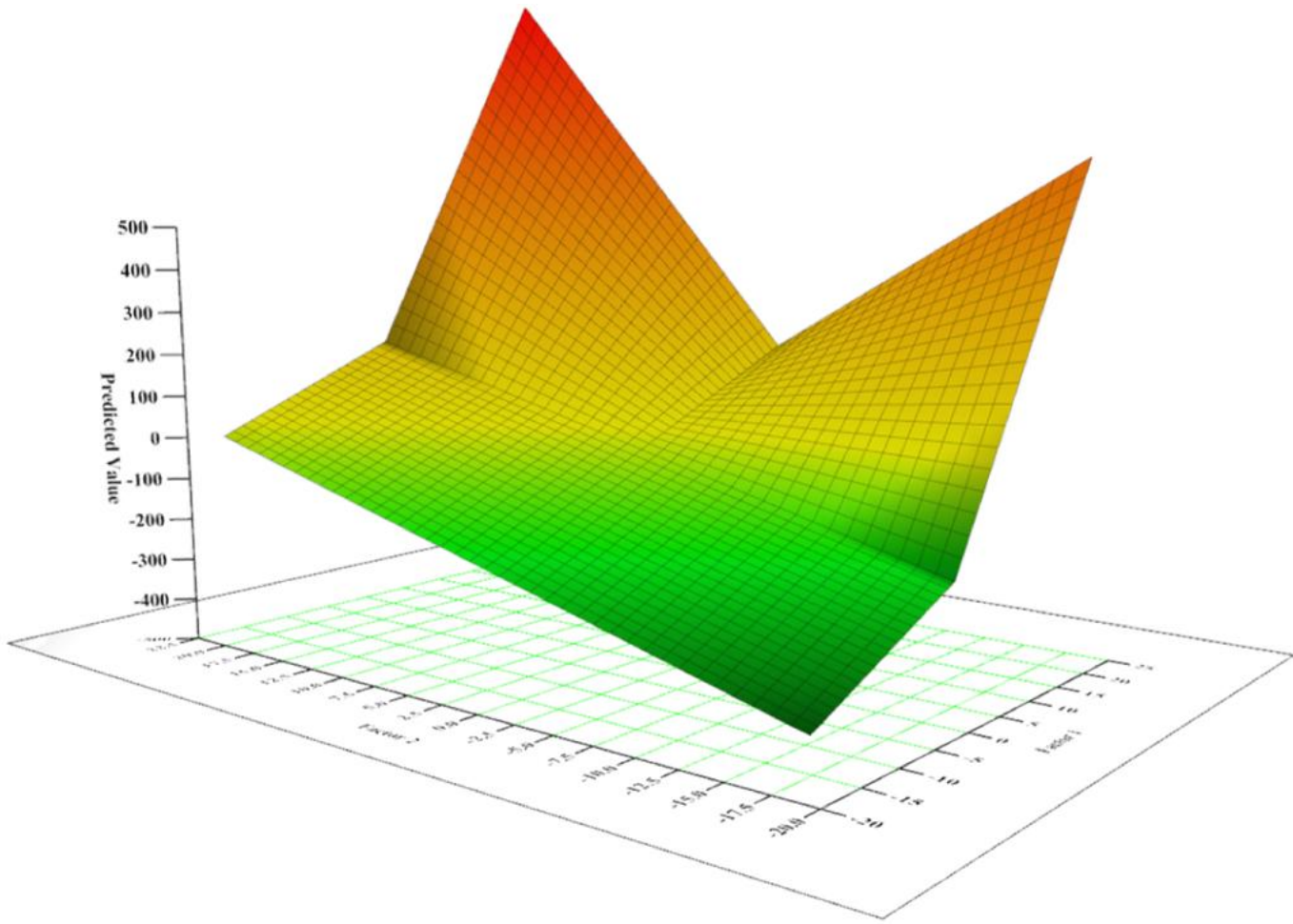




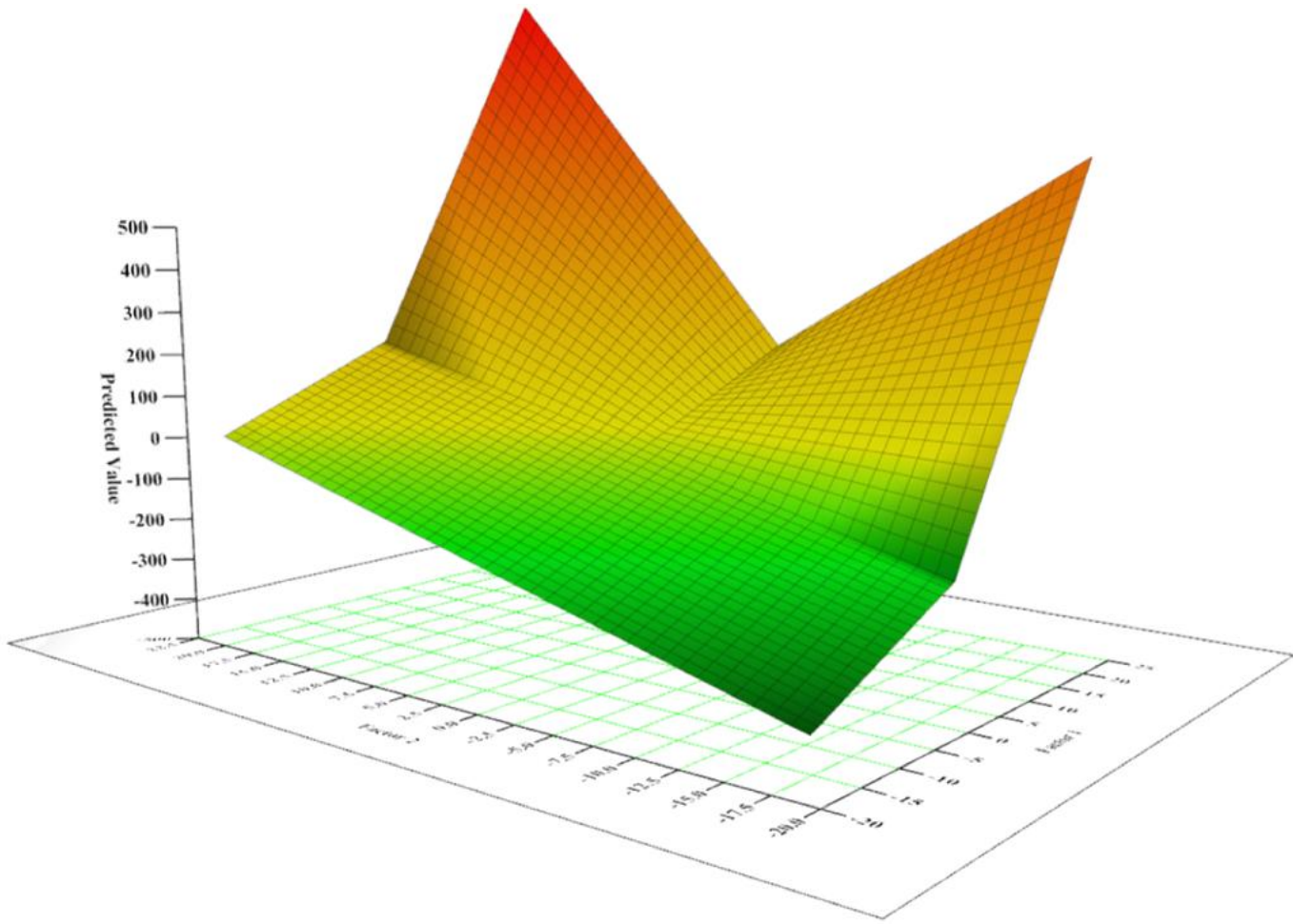
# Saddles



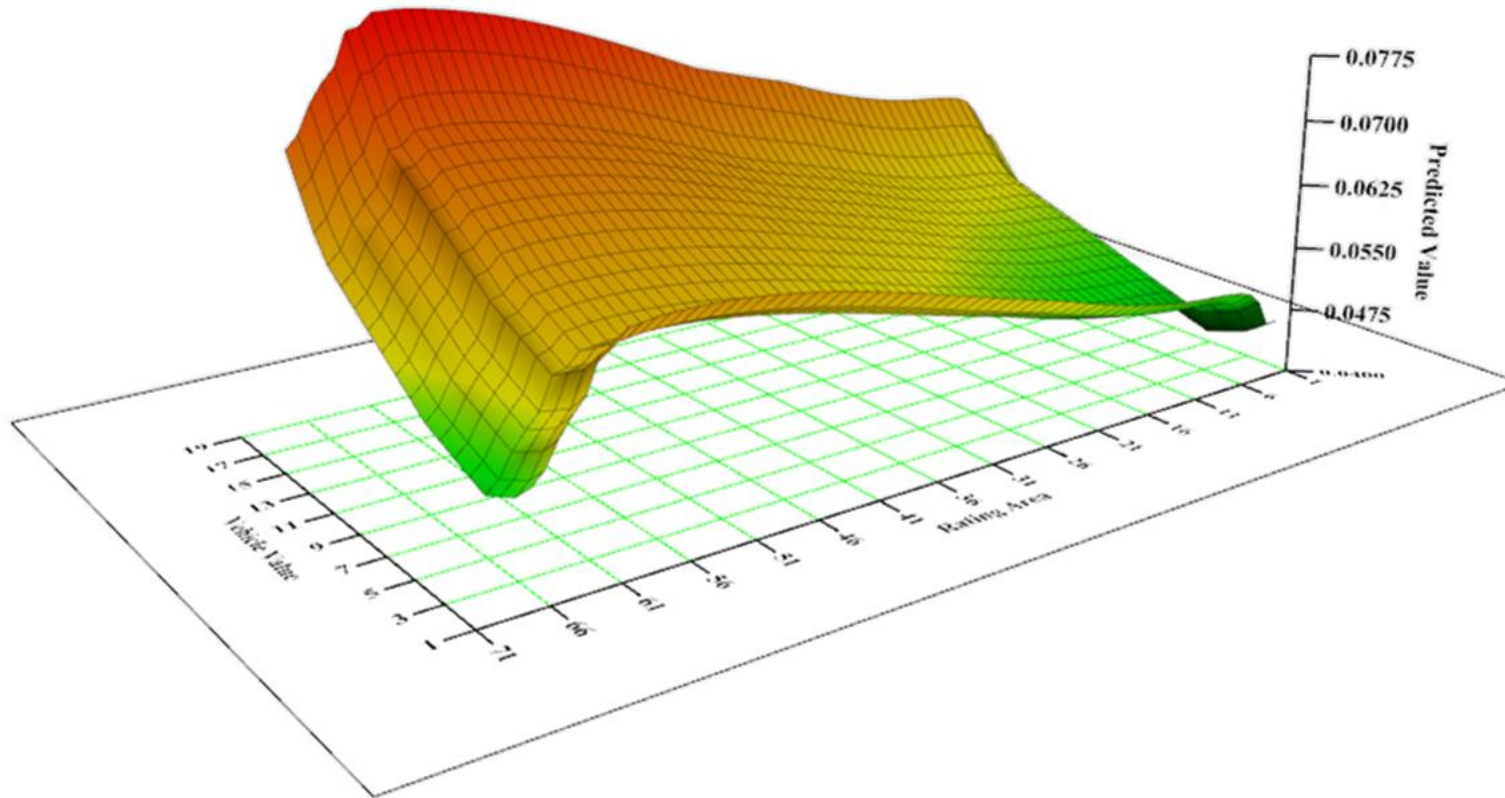
# Saddles



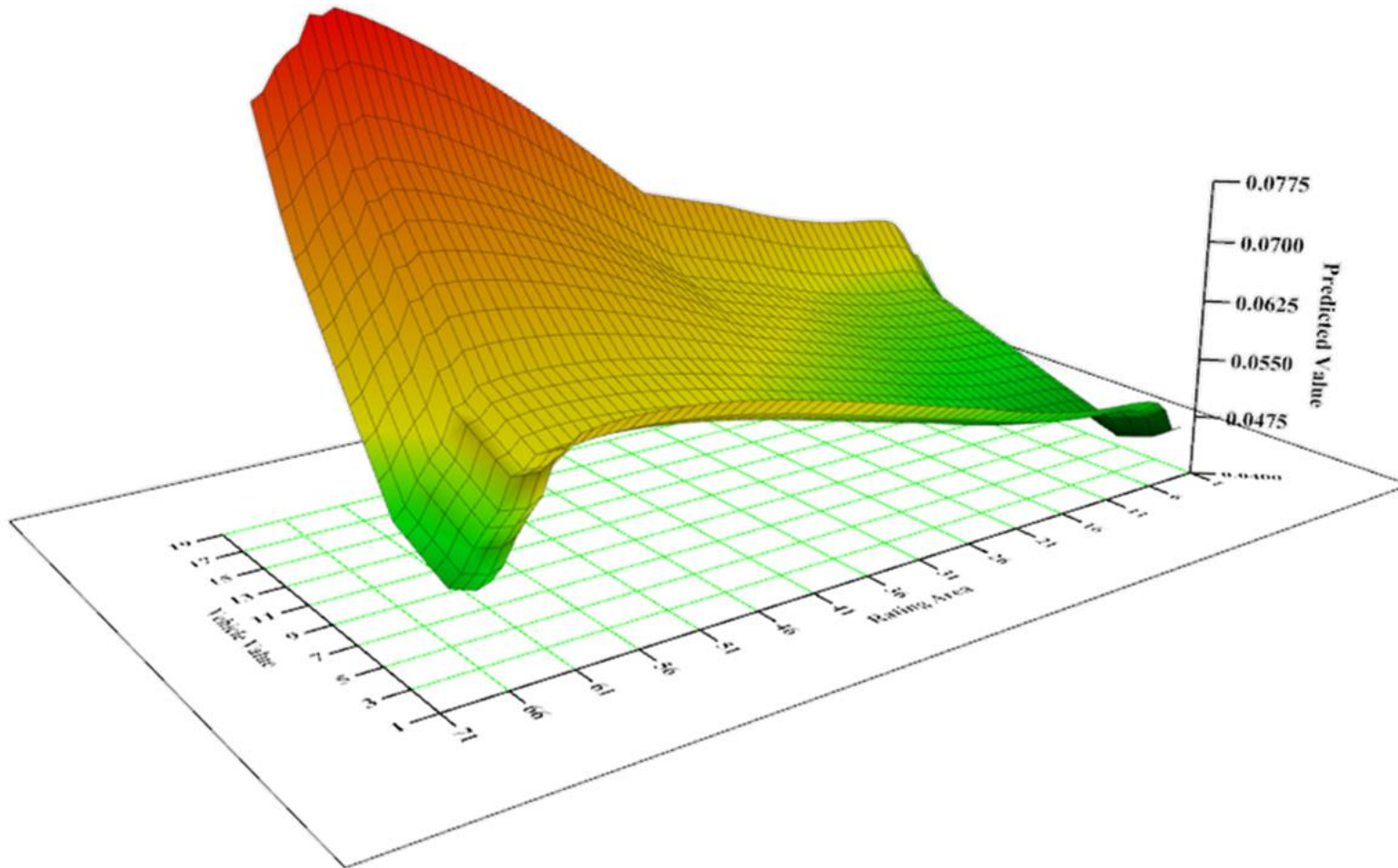
# Saddles



# Saddles

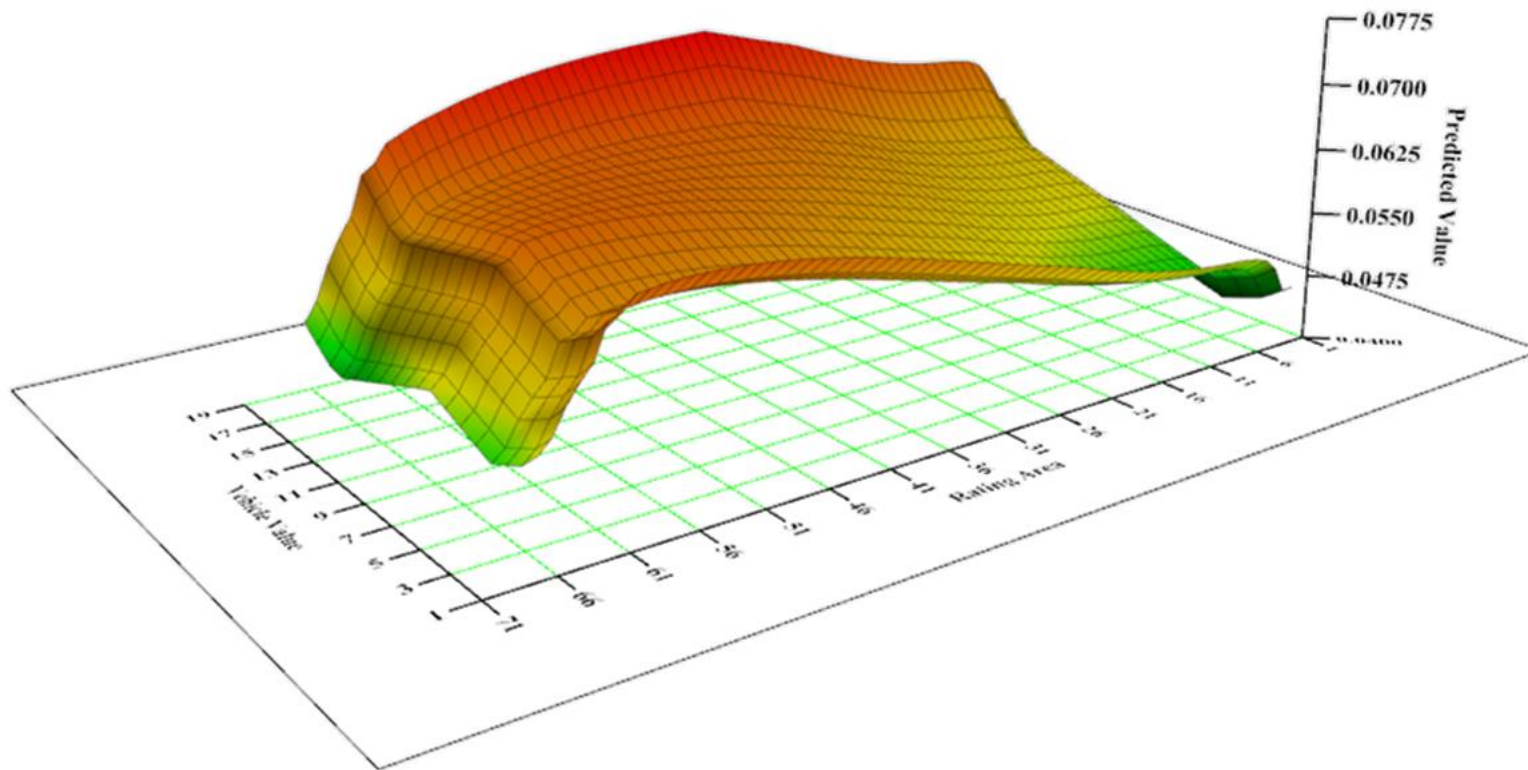


# Saddles

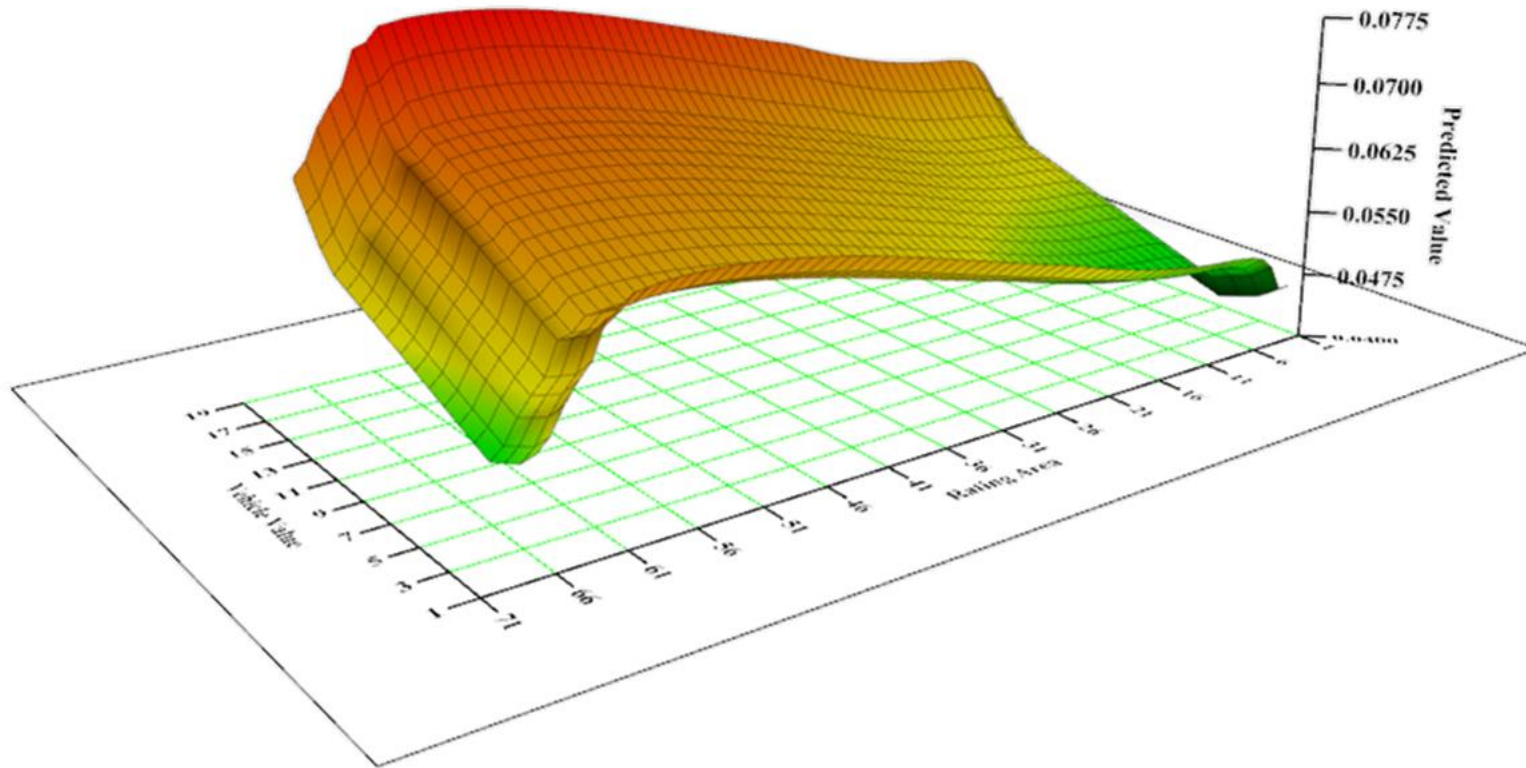




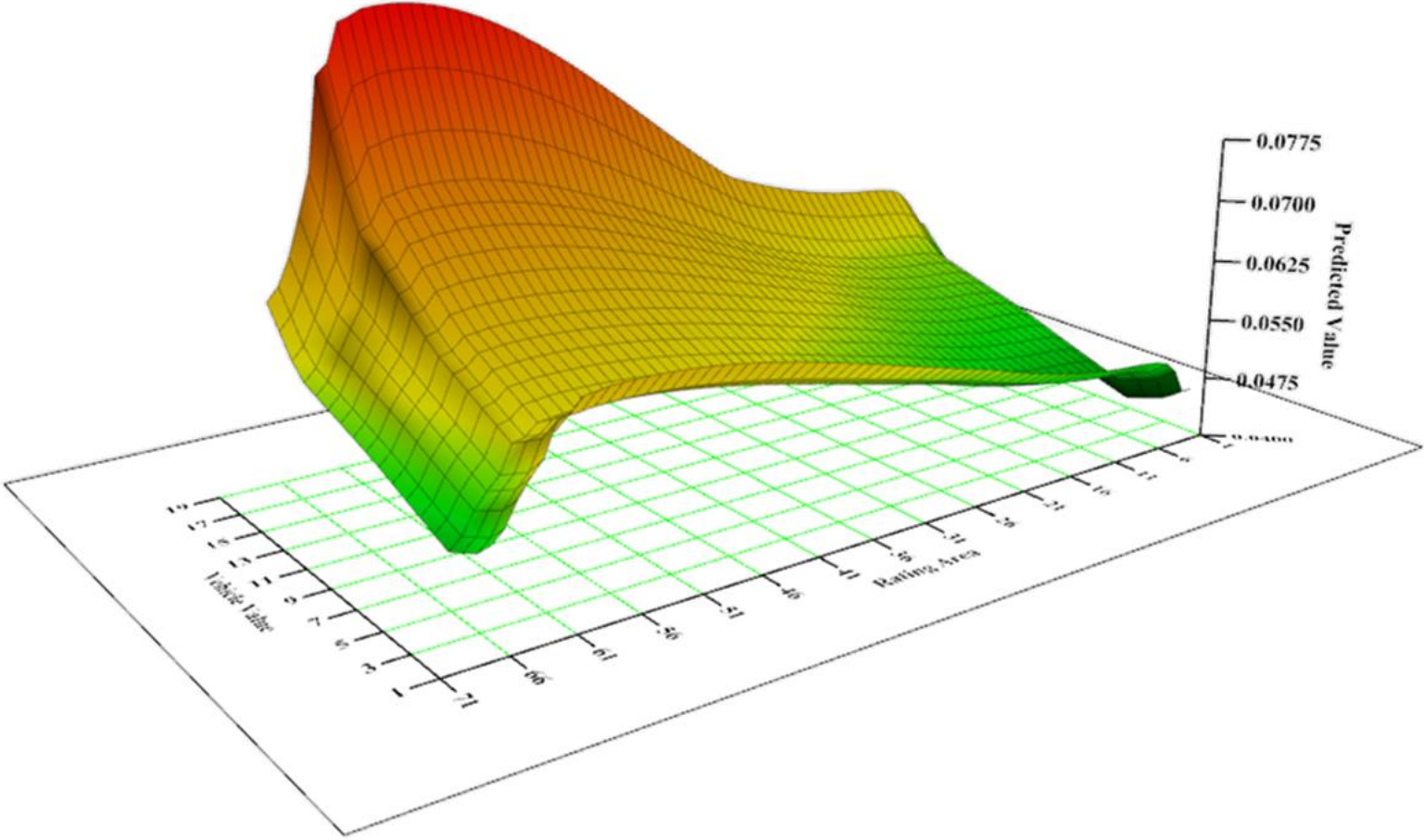
# Saddles



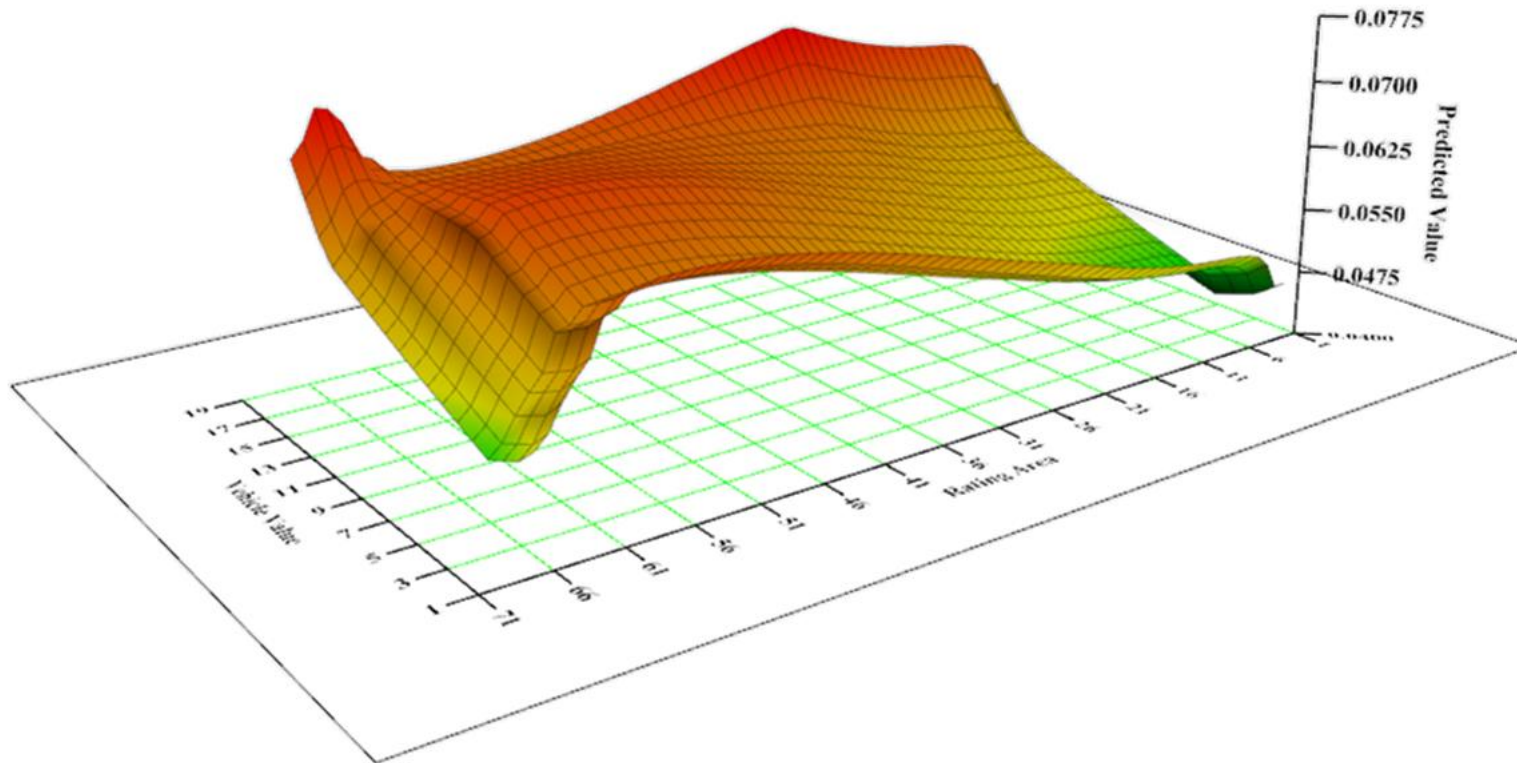
# Saddles



# Saddles

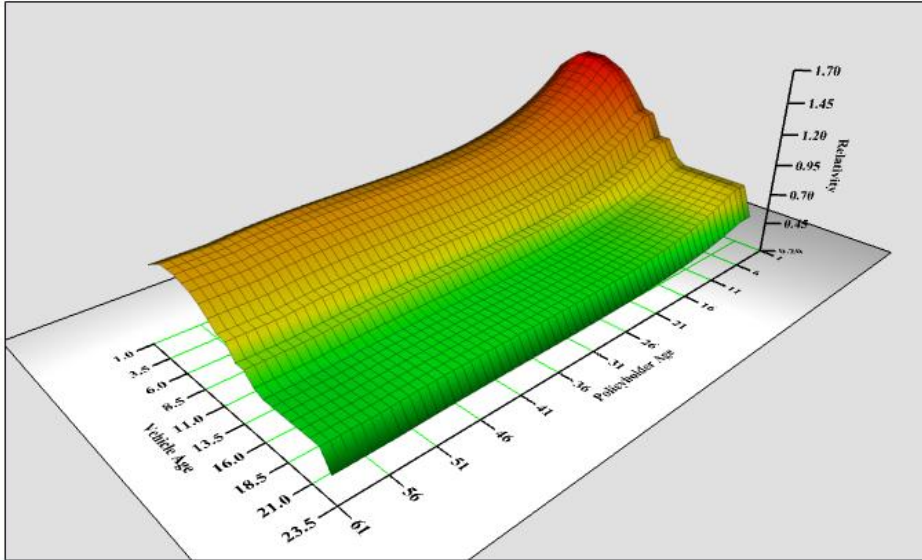


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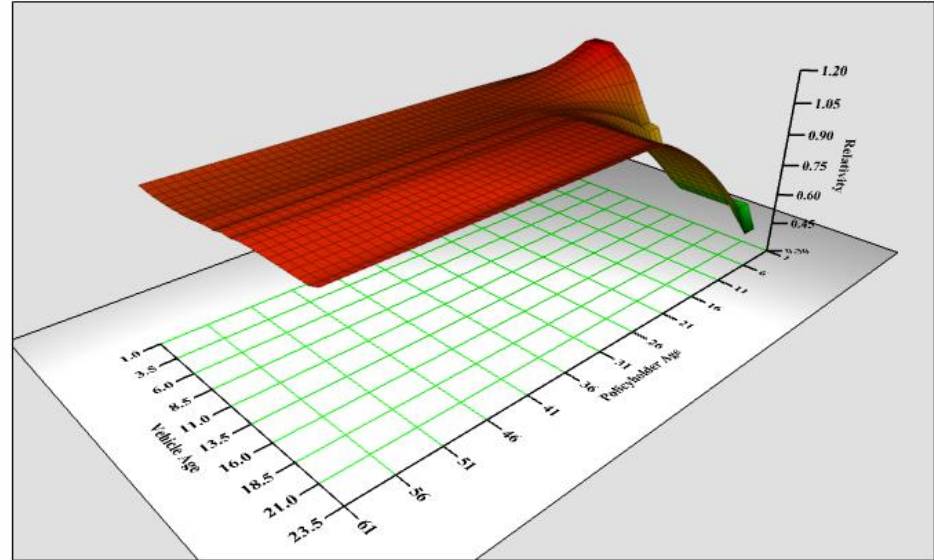




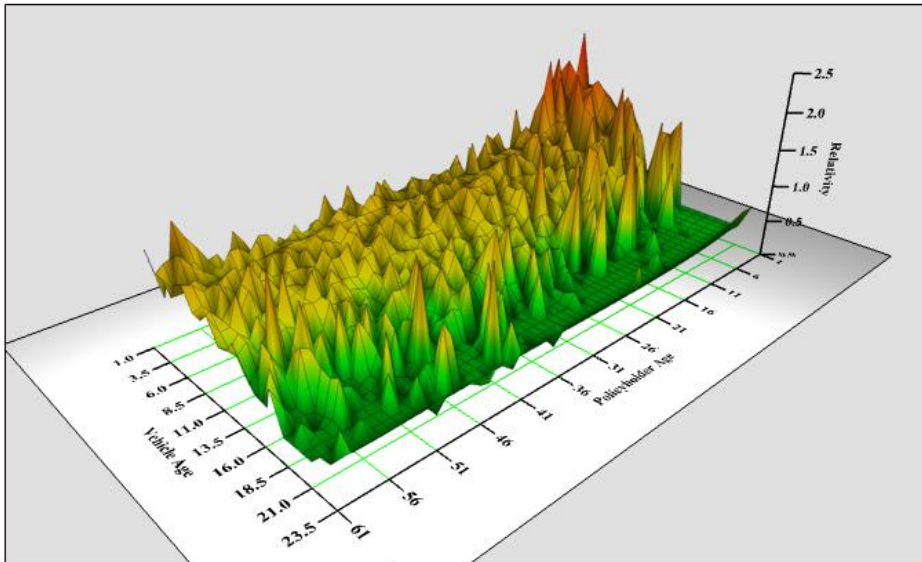
Original



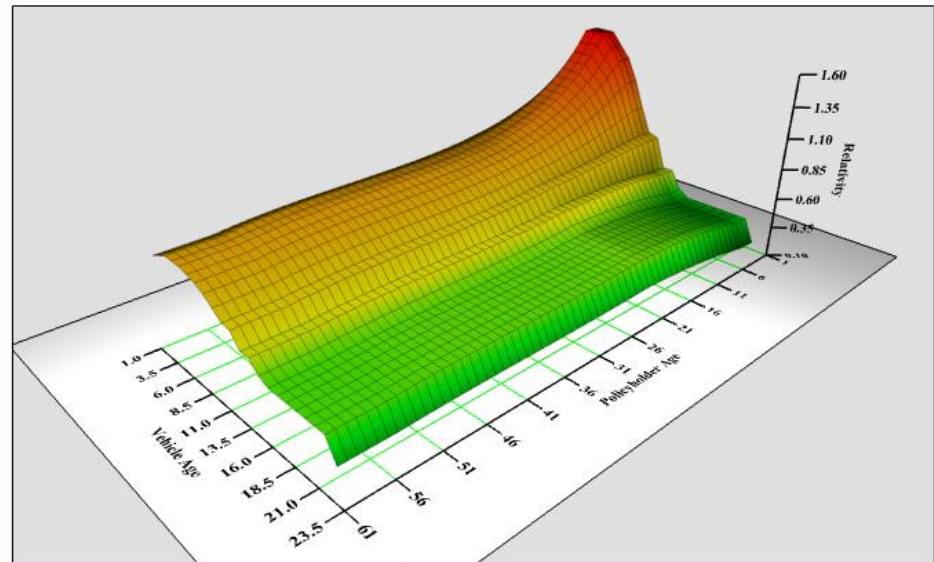
Saddle Parameter



Unsimplified



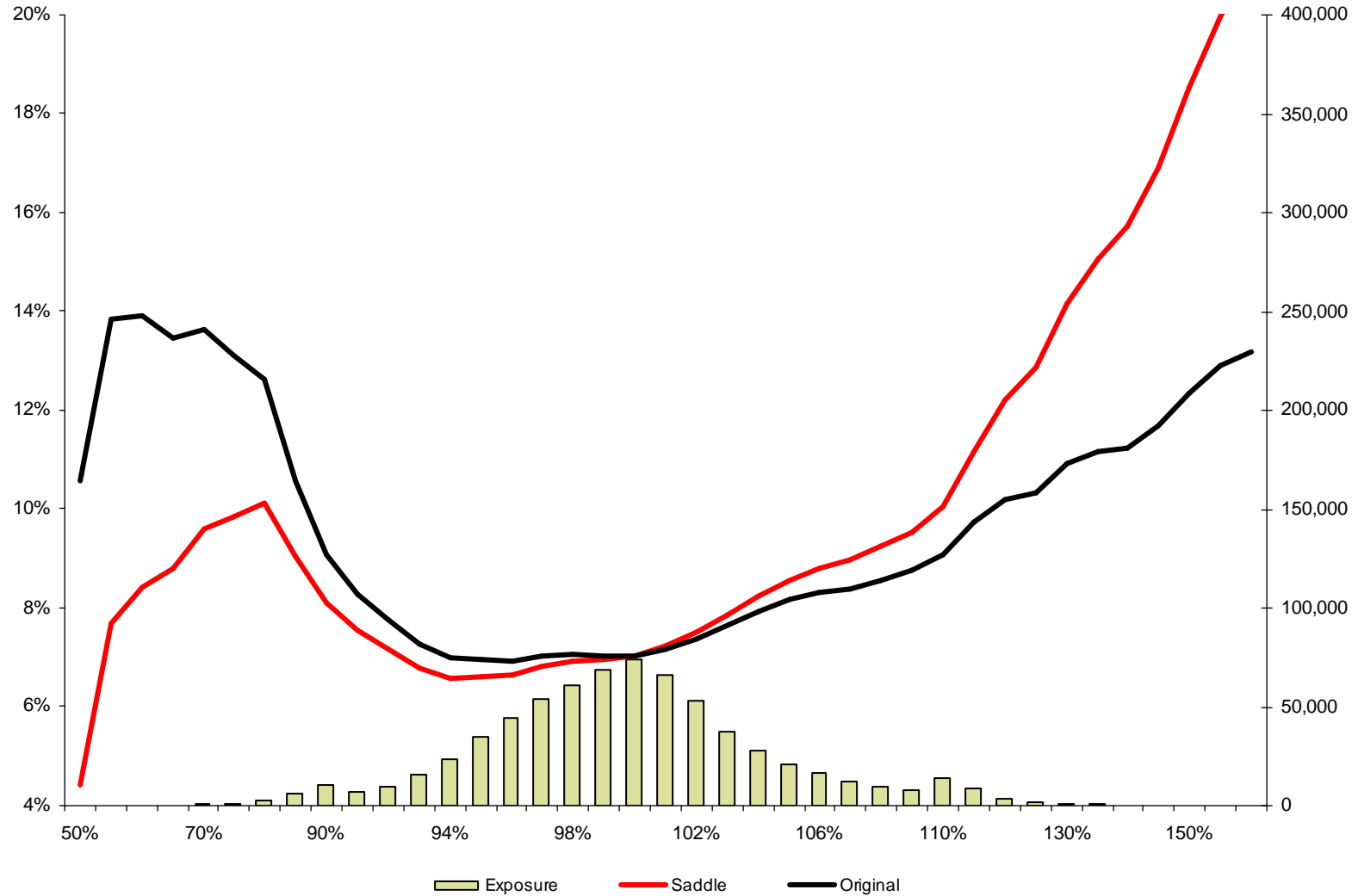
With Saddle





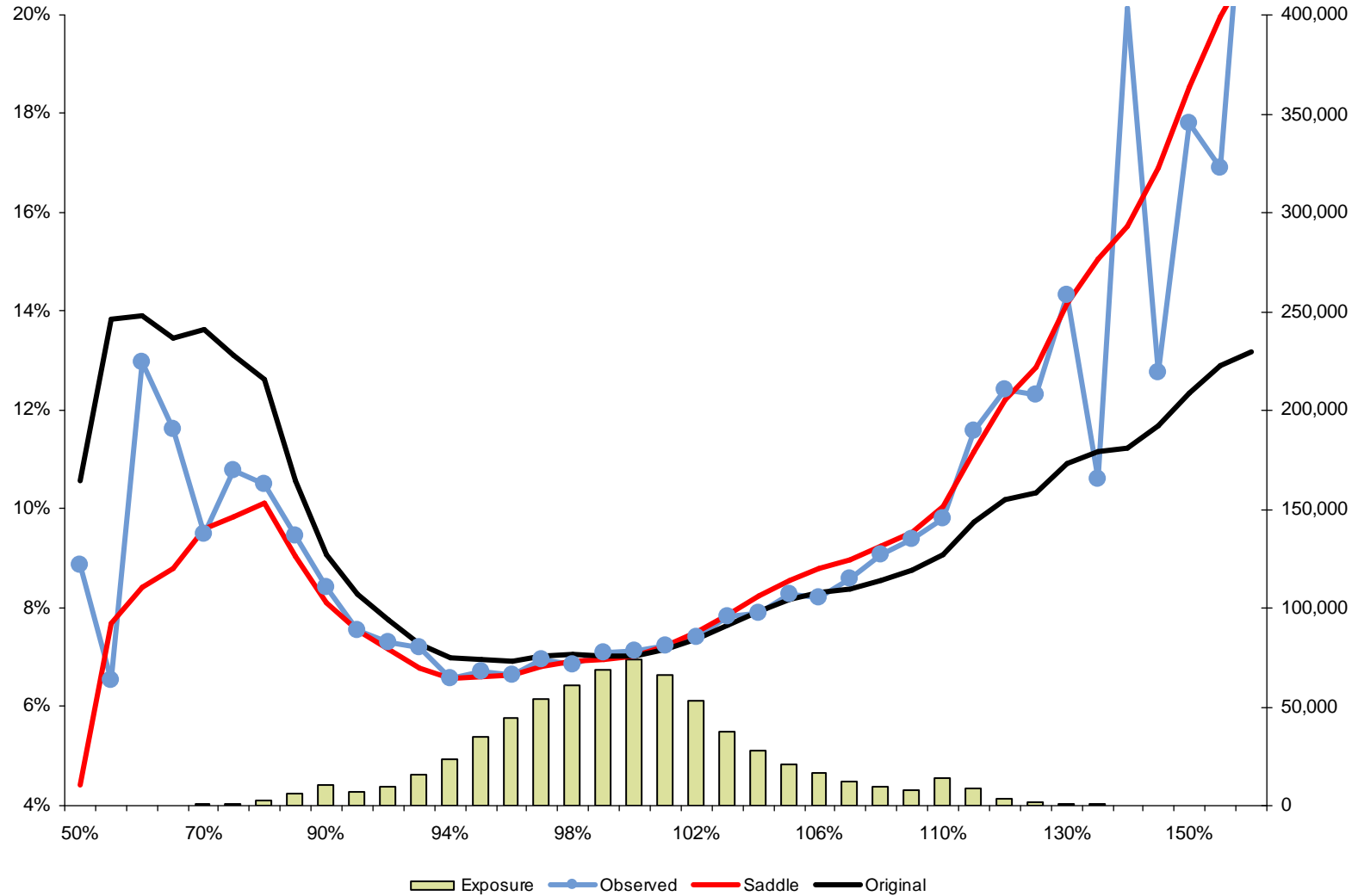
# Saddles - model comparison

## Motor frequency - out of sample



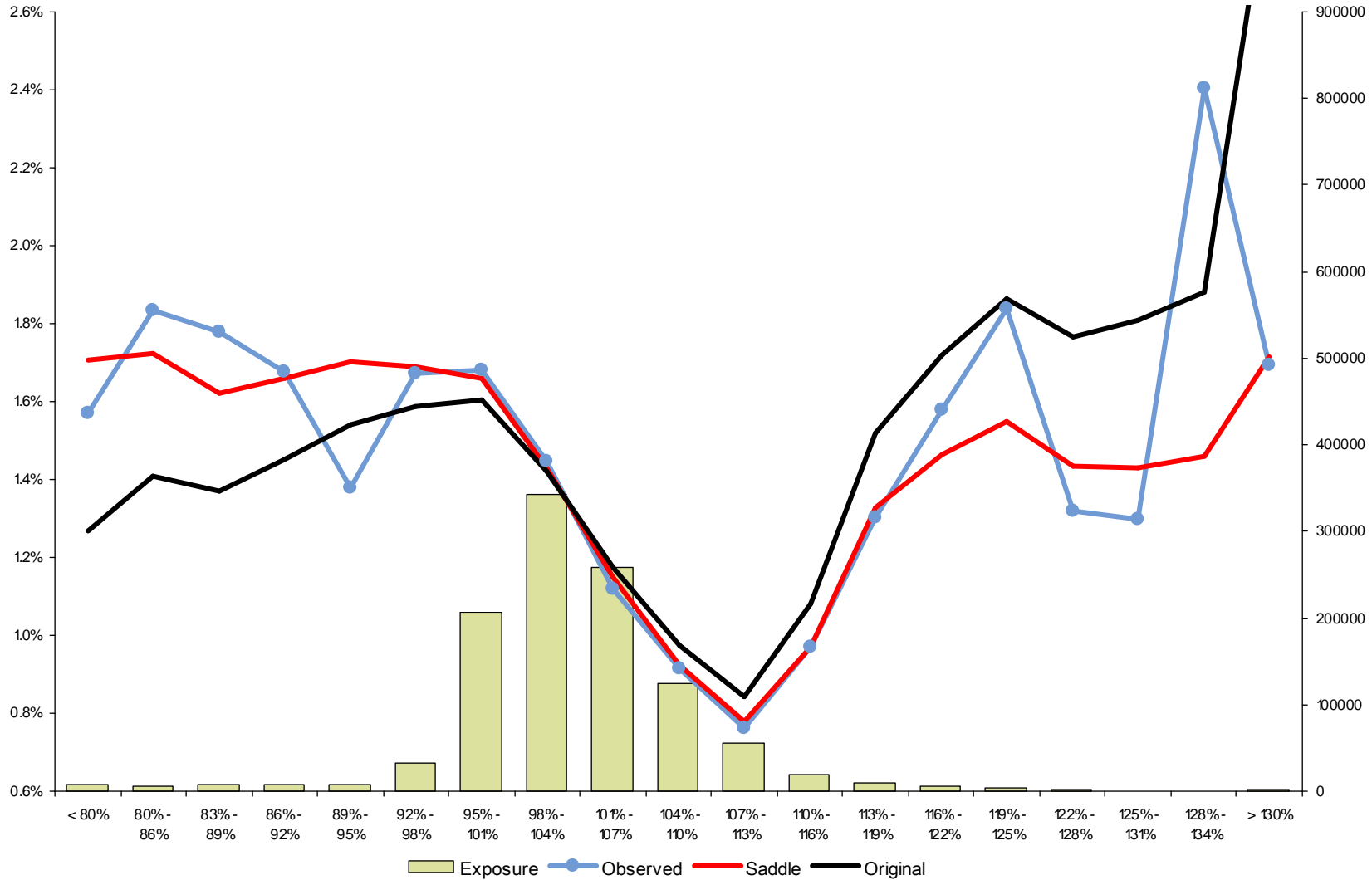
# Saddles - model comparison

## Motor frequency - out of sample



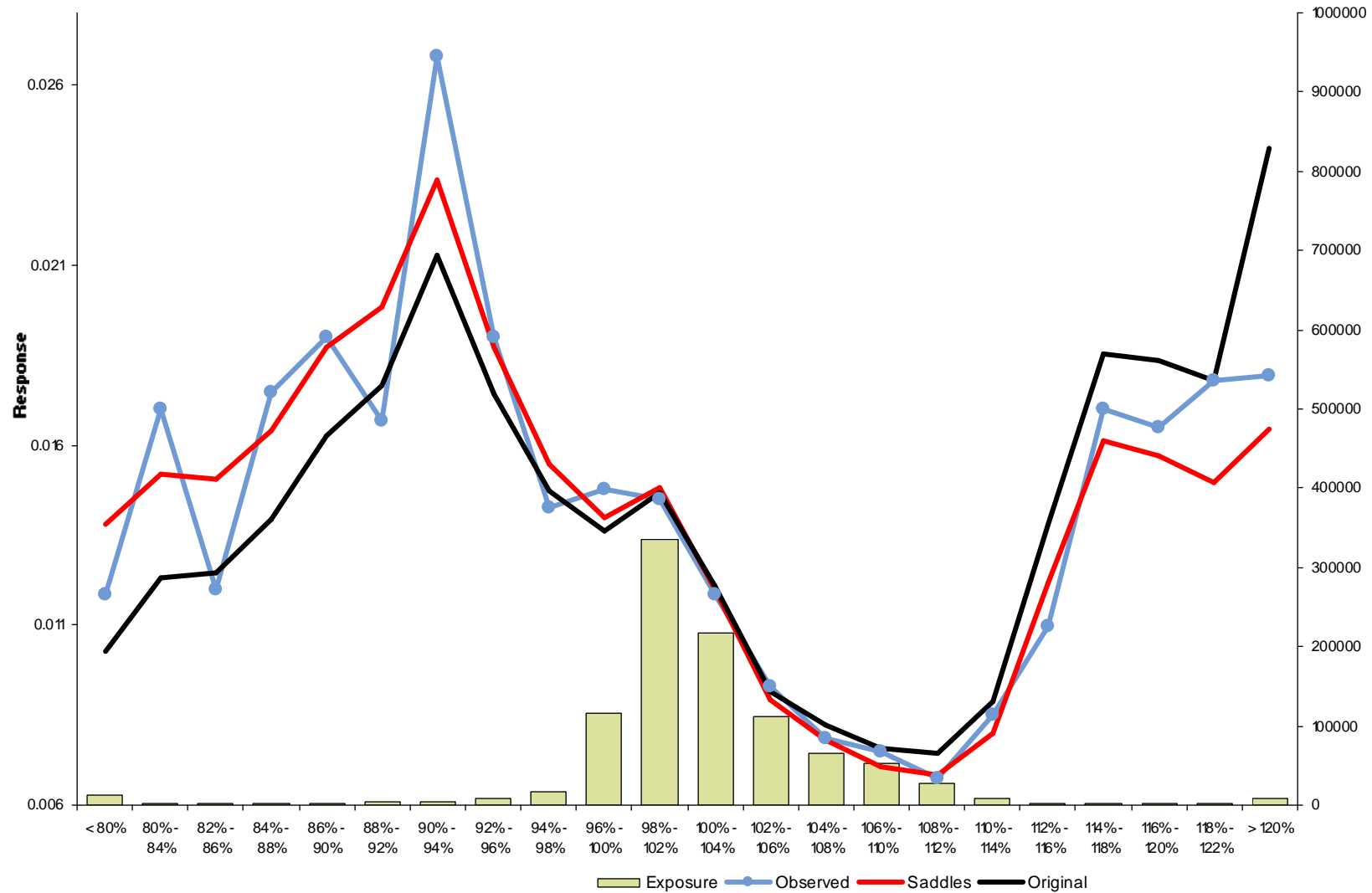
# Saddles - model comparison

## Motor frequency - out of sample



# Saddles - model comparison

## Motor frequency - out of time



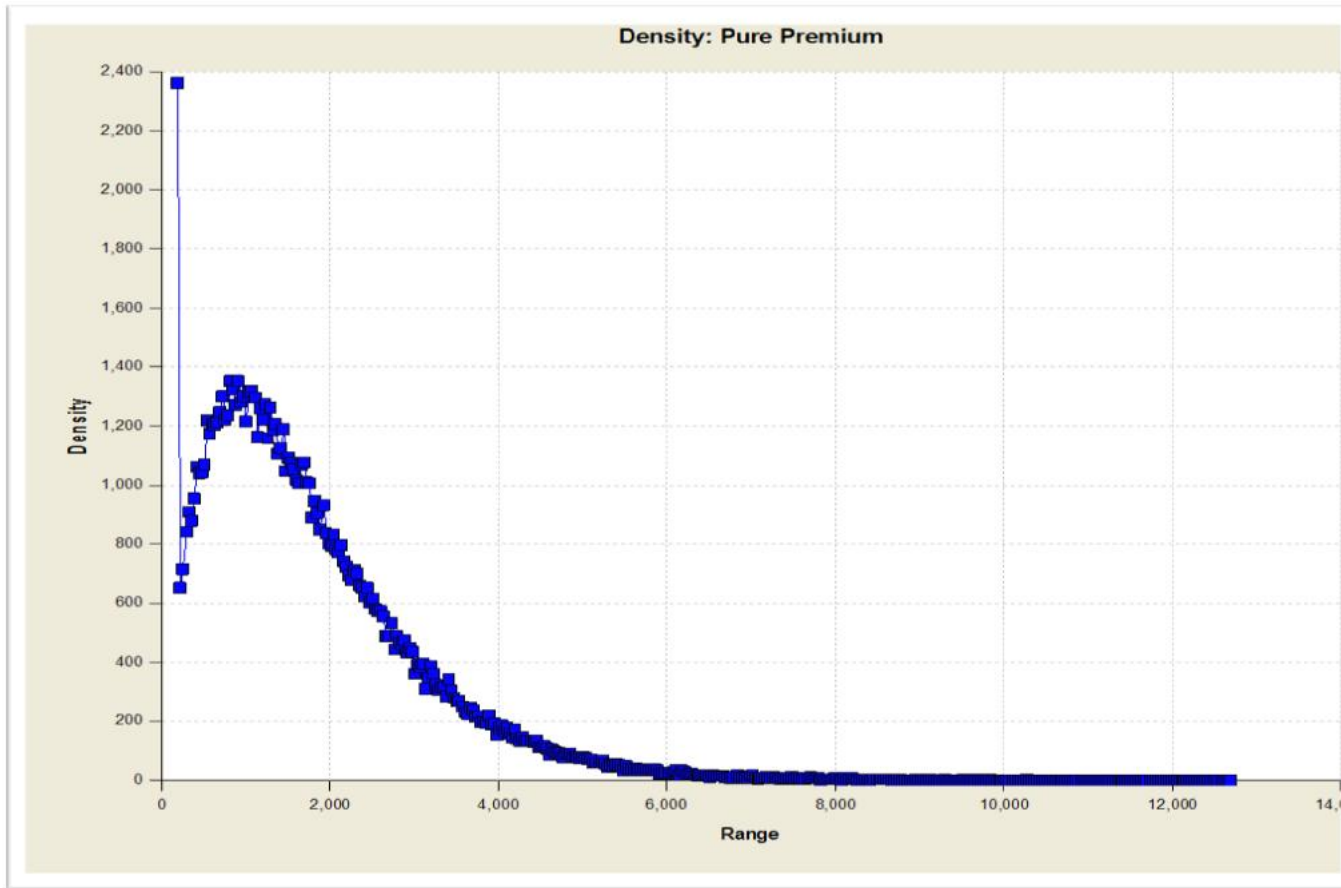
## Agenda

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- "Emergent Interactions"
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- Man (with GLM) vs machine



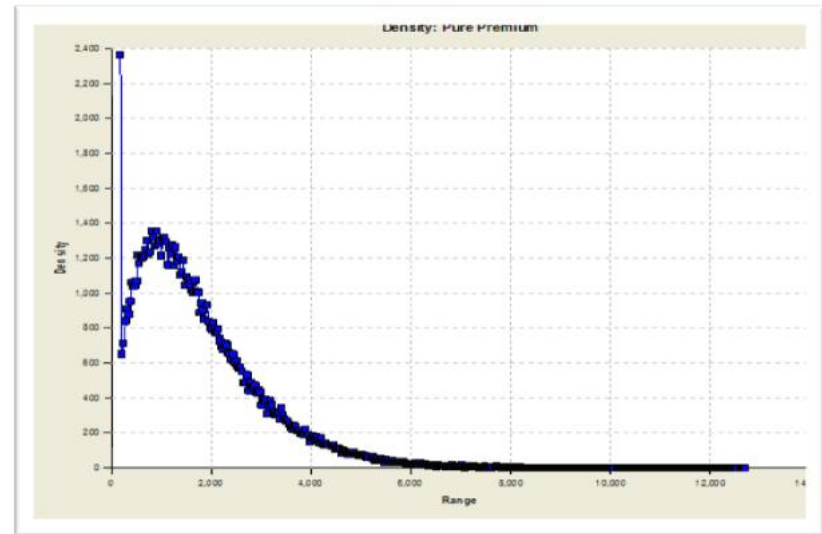
# Tweedie GLMs

- Consider the following empirical probability distribution function



# Tweedie GLMs

- Raw pure premiums
  - Incurred losses have a point mass at zero and then a continuous distribution
  - Poisson and gamma not suited to this
  - Tweedie distribution has
    - point mass at zero
    - a parameter which changes shape above zero



$$f_Y(y; \theta, \lambda, \alpha) = \sum_{n=1}^{\infty} \frac{\{(\lambda \omega)^{1-\alpha} \kappa_{\alpha}(-1/y)\}^n}{\Gamma(-n\alpha)n! y} \exp\{\lambda \omega [\theta_0 y - \kappa_{\alpha}(\theta_0)]\} \quad \text{for } y > 0$$

$$p(Y = 0) = \exp\{-\lambda \omega \kappa_{\alpha}(\theta_0)\}$$

## Formulization of GLMs

- Generally accepted standards for link functions and error distribution

| Observed Response | Most Appropriate Link Function | Most Appropriate Error Structure | Variance Function |
|-------------------|--------------------------------|----------------------------------|-------------------|
| --                | --                             | Normal                           | $\mu^0$           |
| Claim Frequency   | Log                            | Poisson                          | $\mu^1$           |
| Claim Severity    | Log                            | Gamma                            | $\mu^2$           |
| Claim Severity    | Log                            | Inverse Gaussian                 | $\mu^3$           |
| Raw Pure Premium  | Log                            | Tweedie                          | $\mu^T$           |
| Retention Rate    | Logit                          | Binomial                         | $\mu(1-\mu)$      |
| Conversion Rate   | Logit                          | Binomial                         | $\mu(1-\mu)$      |

## Formulization of GLMs

- More formally:

$$\text{Var}(Y) = \frac{\phi V(\hat{\mu})}{\omega}$$

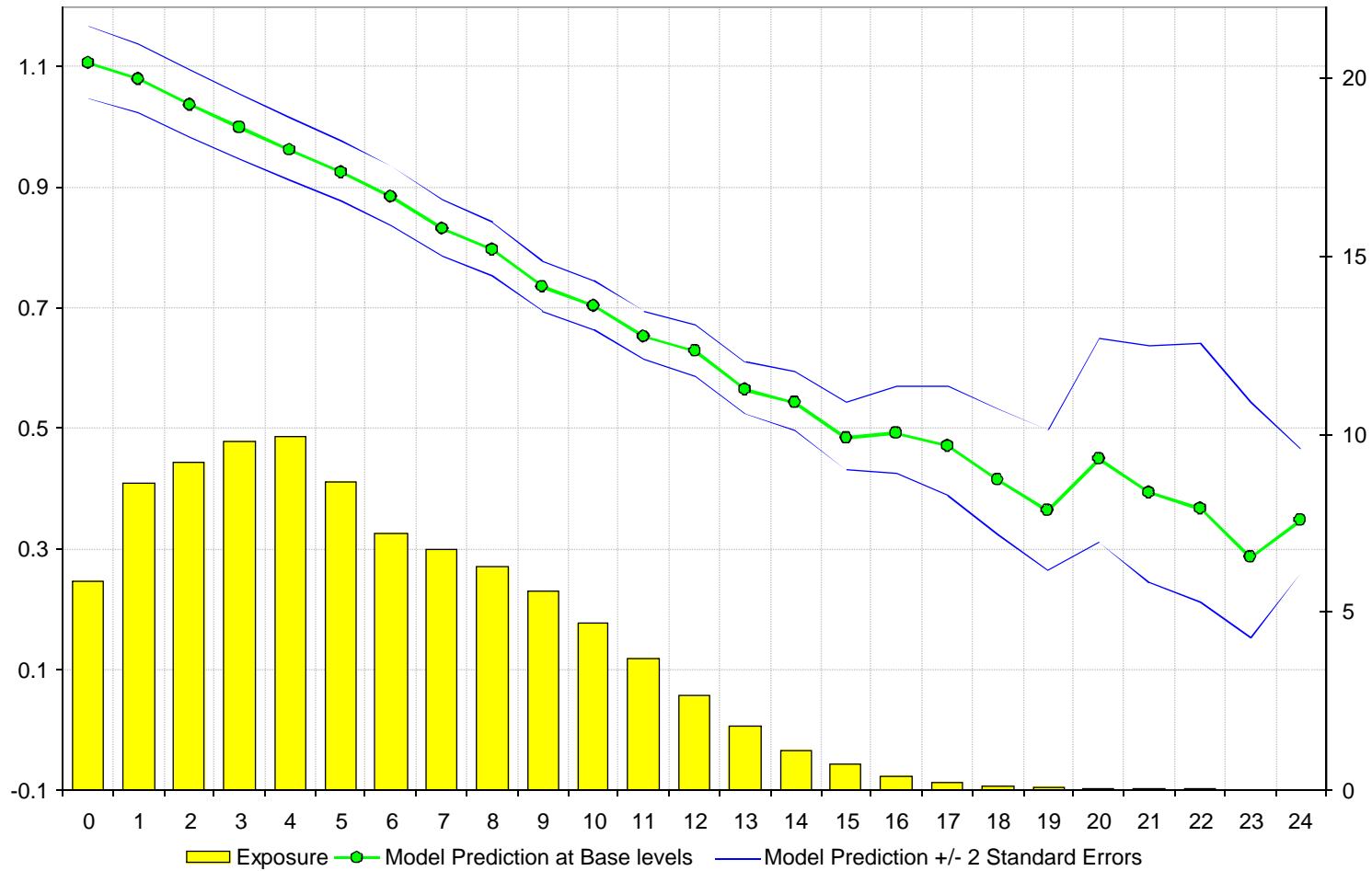
Diagram illustrating the components of the variance formula:

- $\phi$  is labeled as the Scale Parameter.
- $V(\hat{\mu})$  is labeled as the Variance Function.
- $\omega$  is labeled as the Prior Weights.

- Tweedie's Variance function:  $V(\mu) = \mu^p$ 
  - $p=1$  Poisson
  - $p=2$  Gamma
  - $1 < p < 2$  Poisson/Gamma process
- Other concerns
  - Need to estimate both  $\phi$  and  $p$  when fitting models
  - Typically  $p \approx 1.5$  for incurred claims

# Example 1

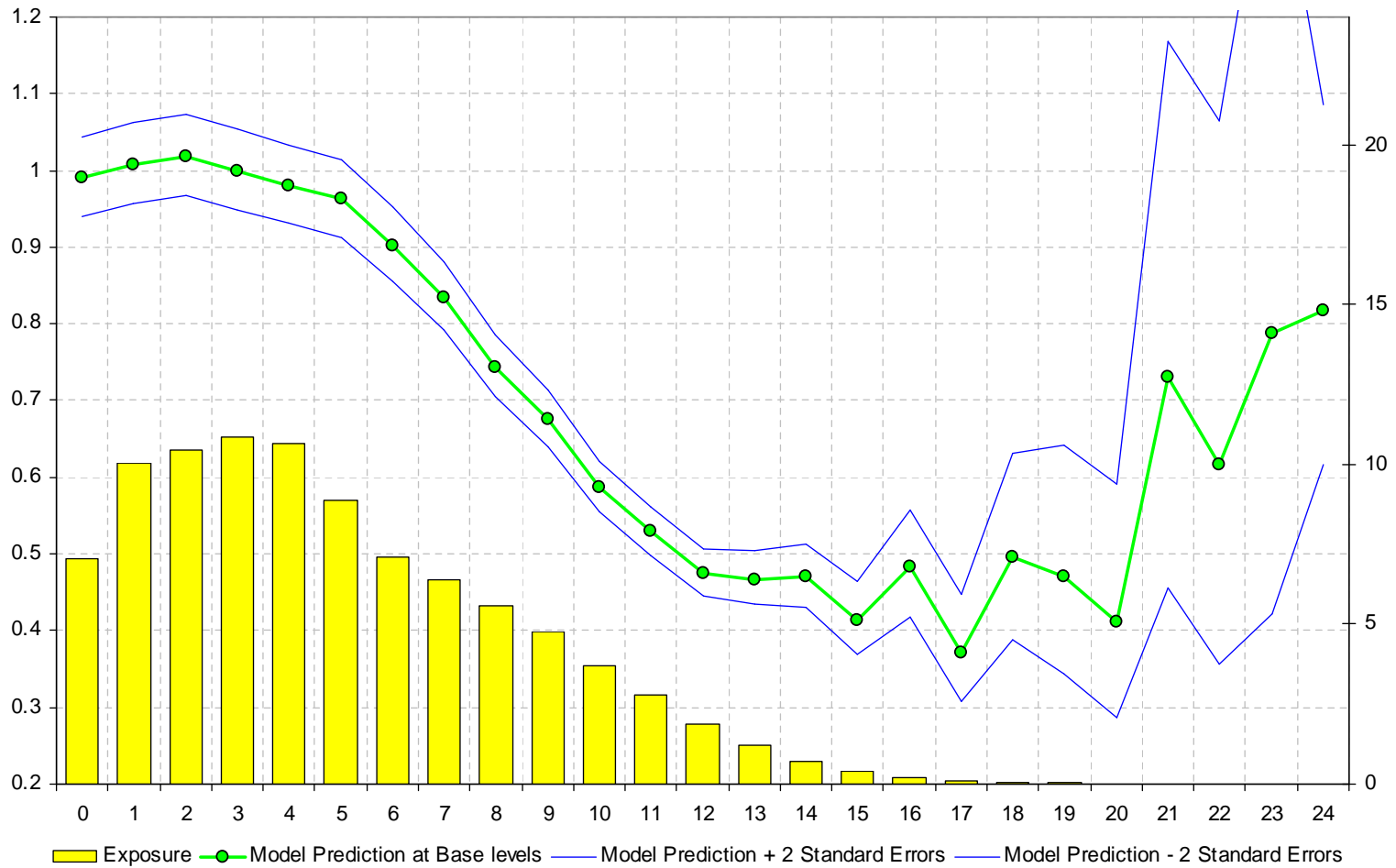
## Vehicle age - frequency





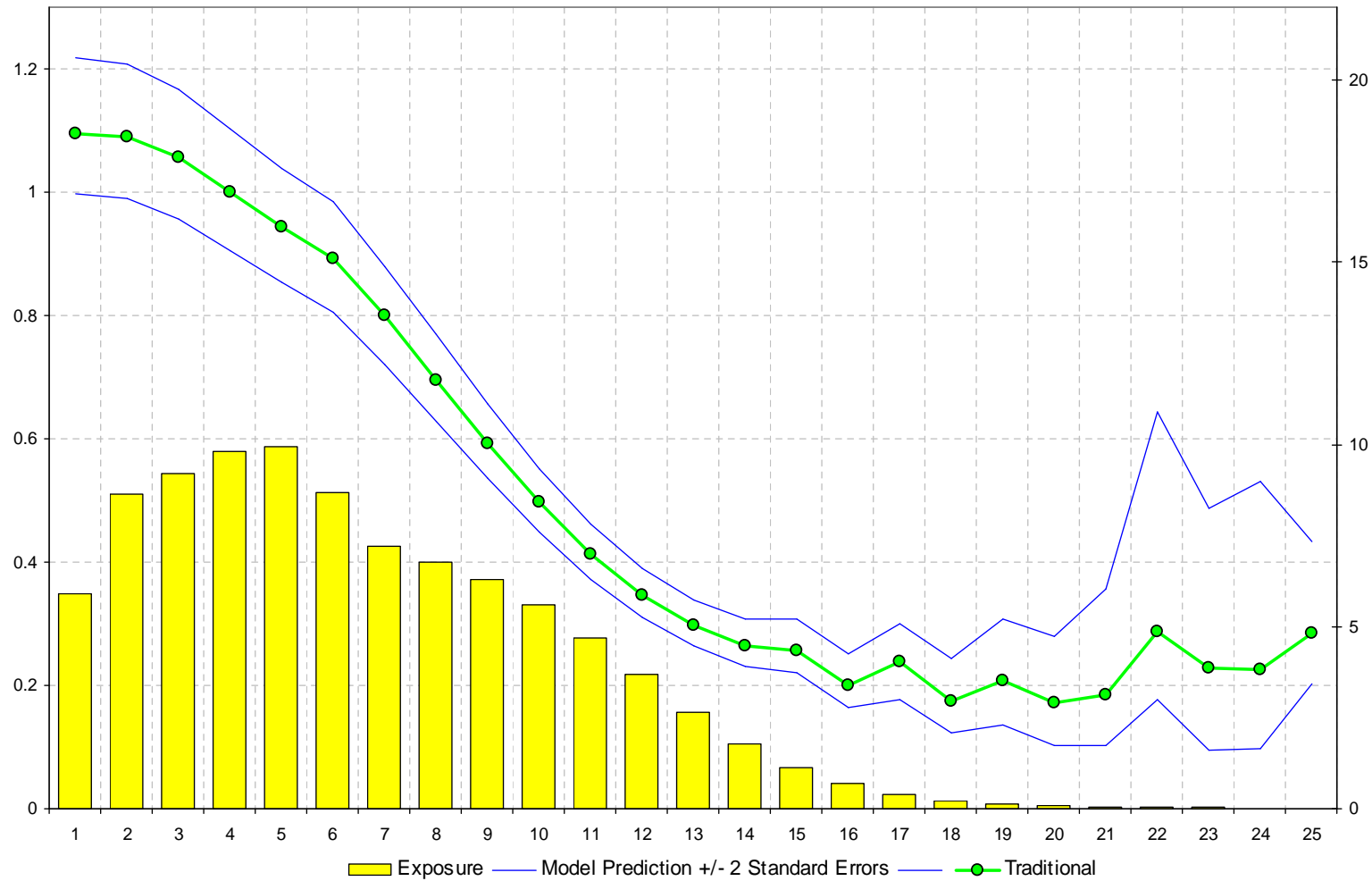
# Example 1

## Vehicle age - amounts



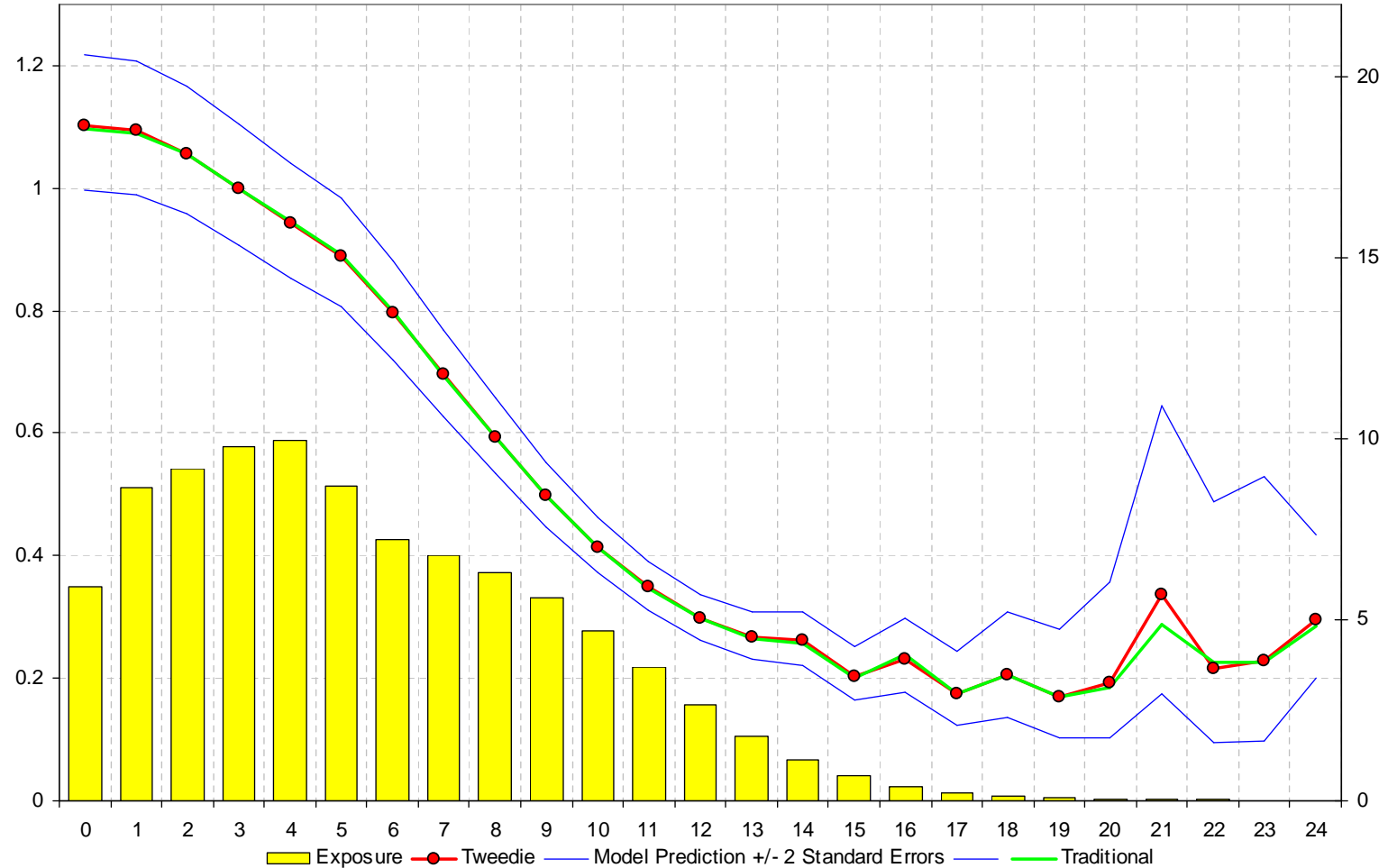
# Example 1

## Vehicle age - pure premium



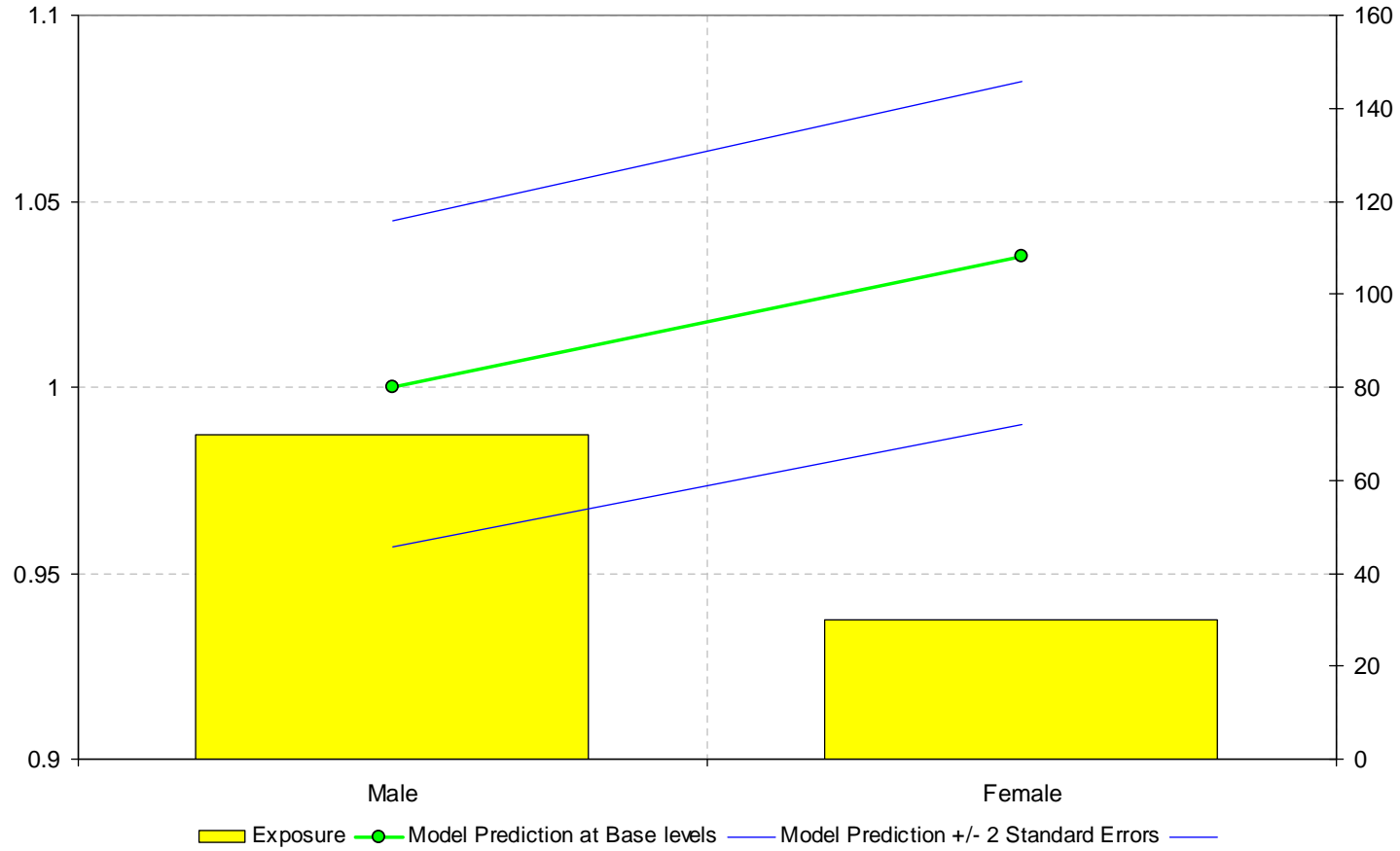
# Example 1

Vehicle age - pure premium



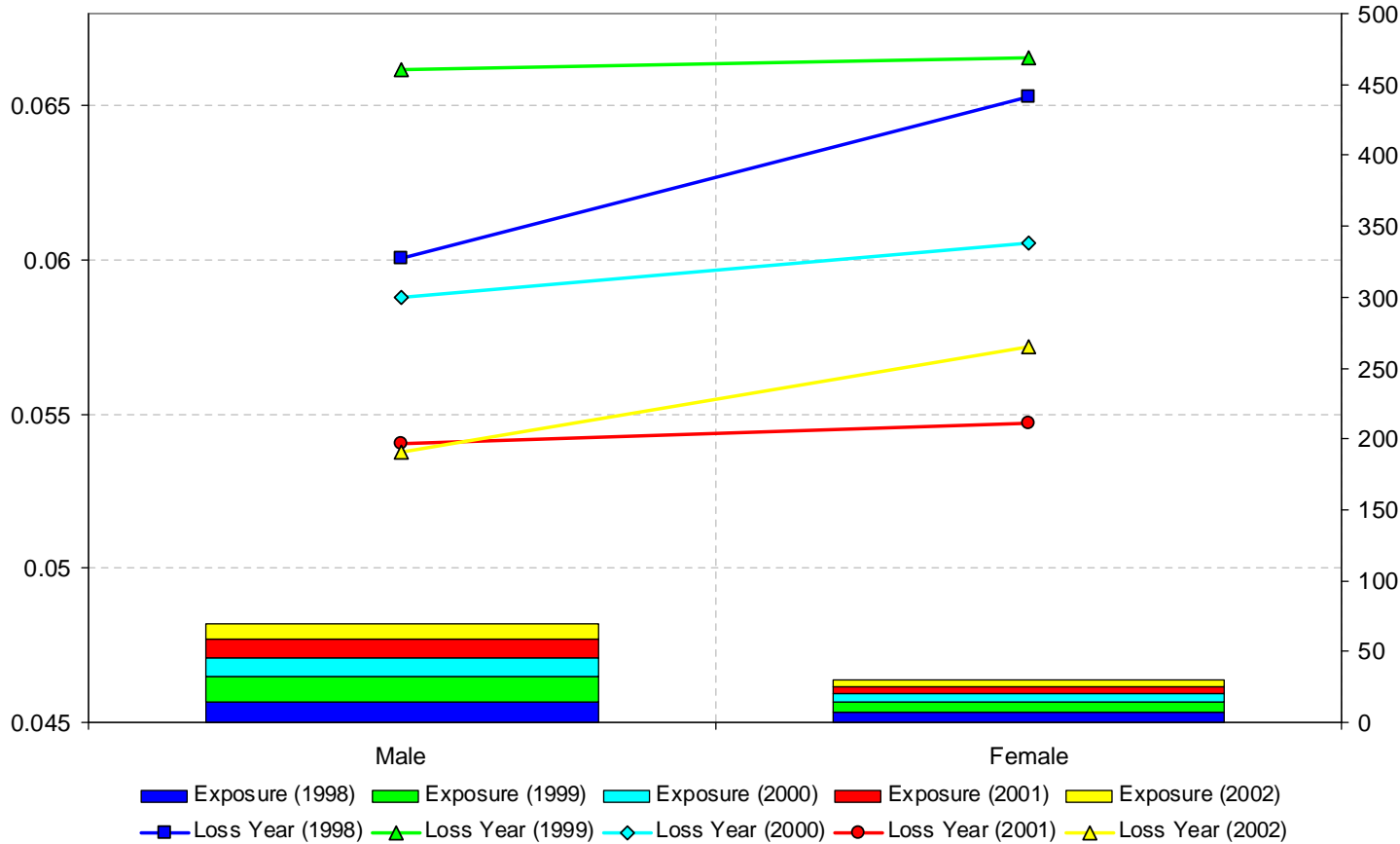
# Example 2

## Gender - frequency



# Example 2

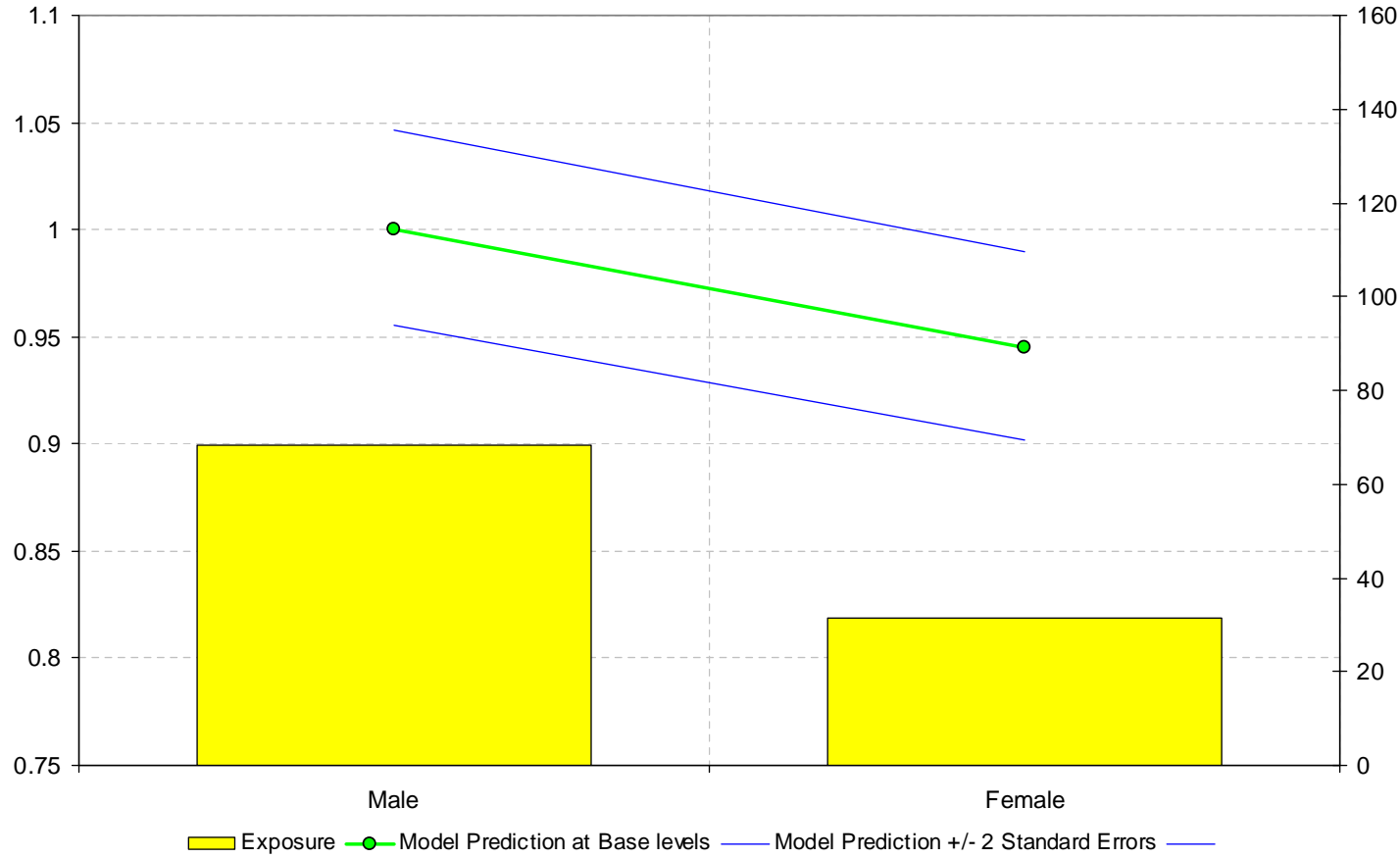
Gender - frequency





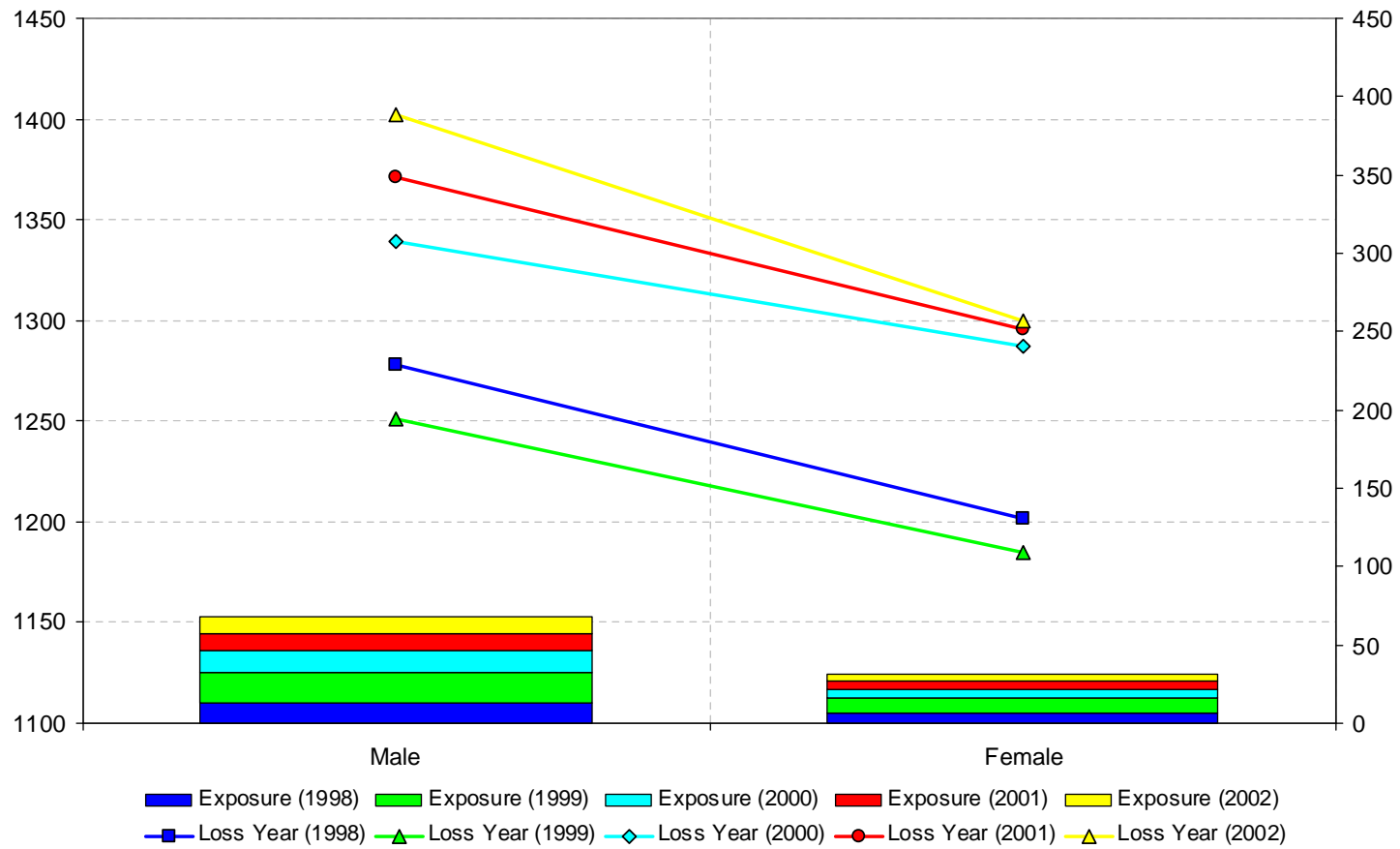
# Example 2

## Gender - amounts



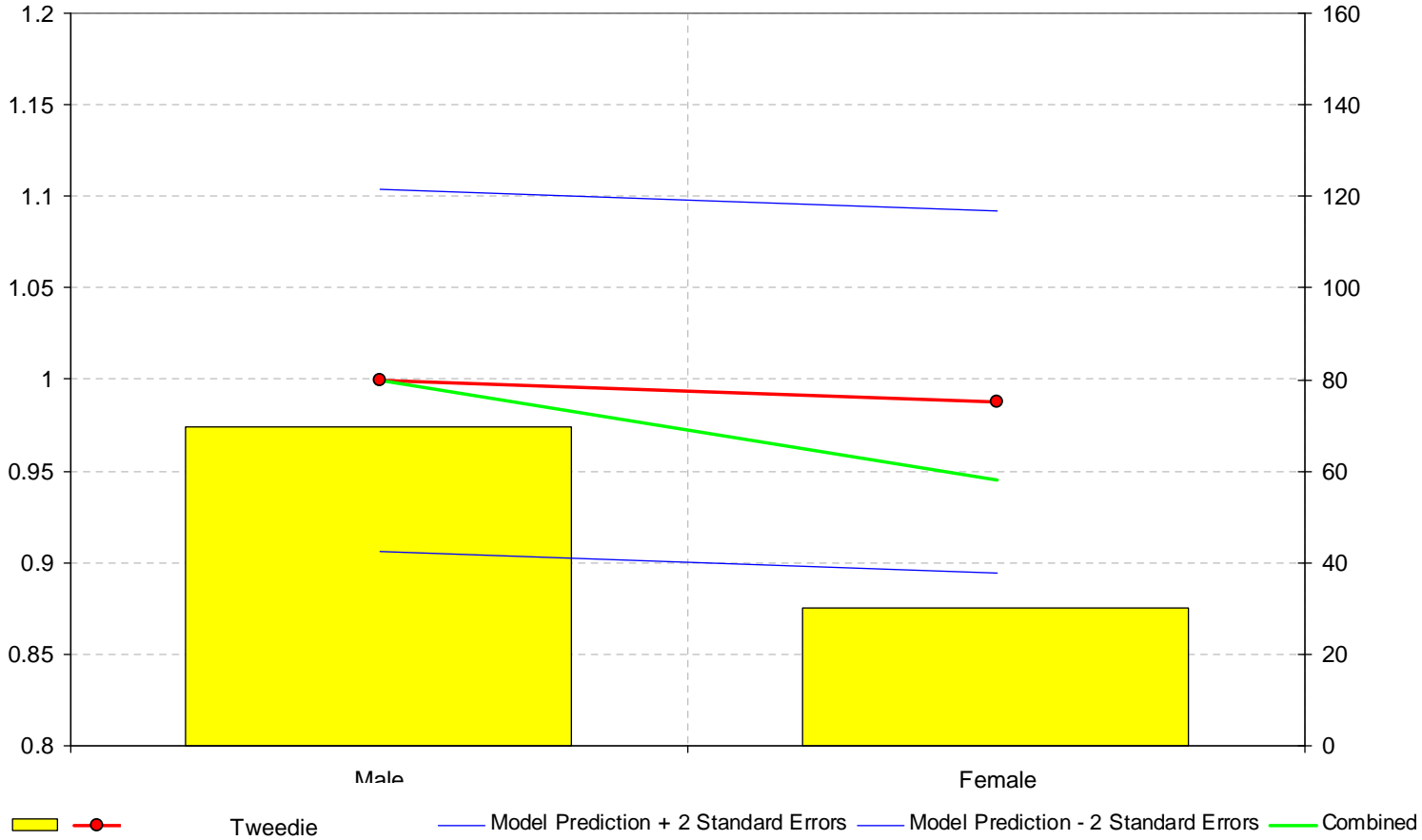
# Example 2

## Gender - amounts



# Example 2

## Gender – pure premium



## Tweedie GLMs

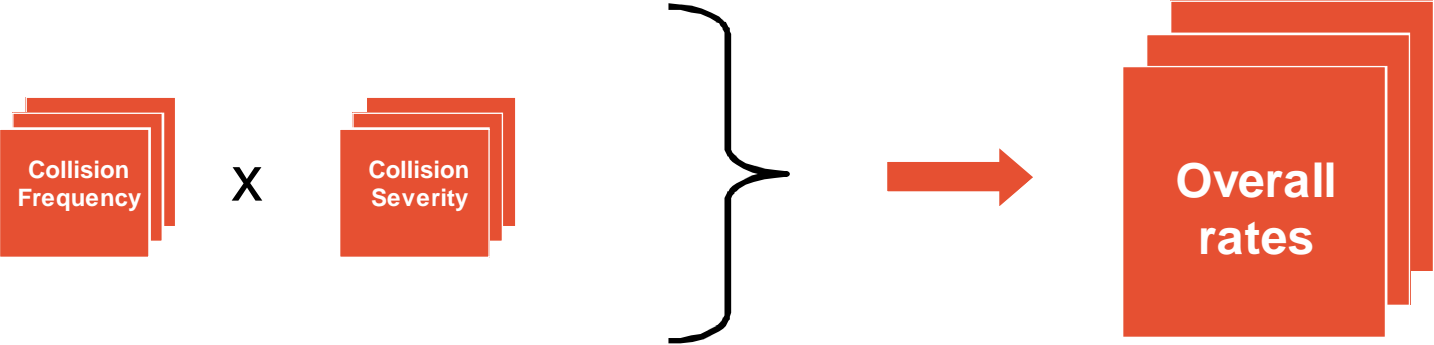
- Helpful when it's important to fit to incurred costs directly
- Similar results to frequency/severity traditional approach if frequency and amounts effects are clearly weak or clearly strong
- Distorted by large insignificant effects
- Removes understanding of what is driving results
- Smoothing harder

## Agenda

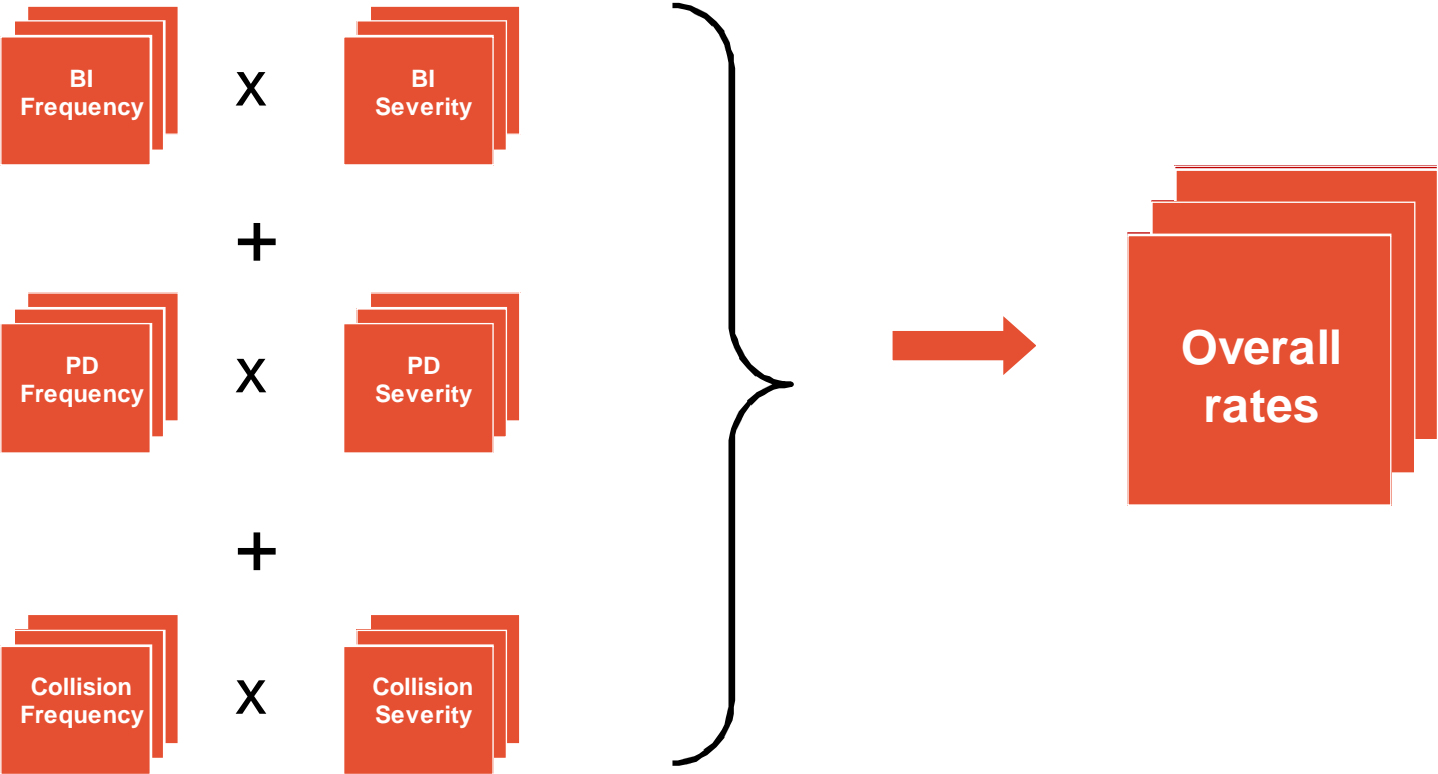
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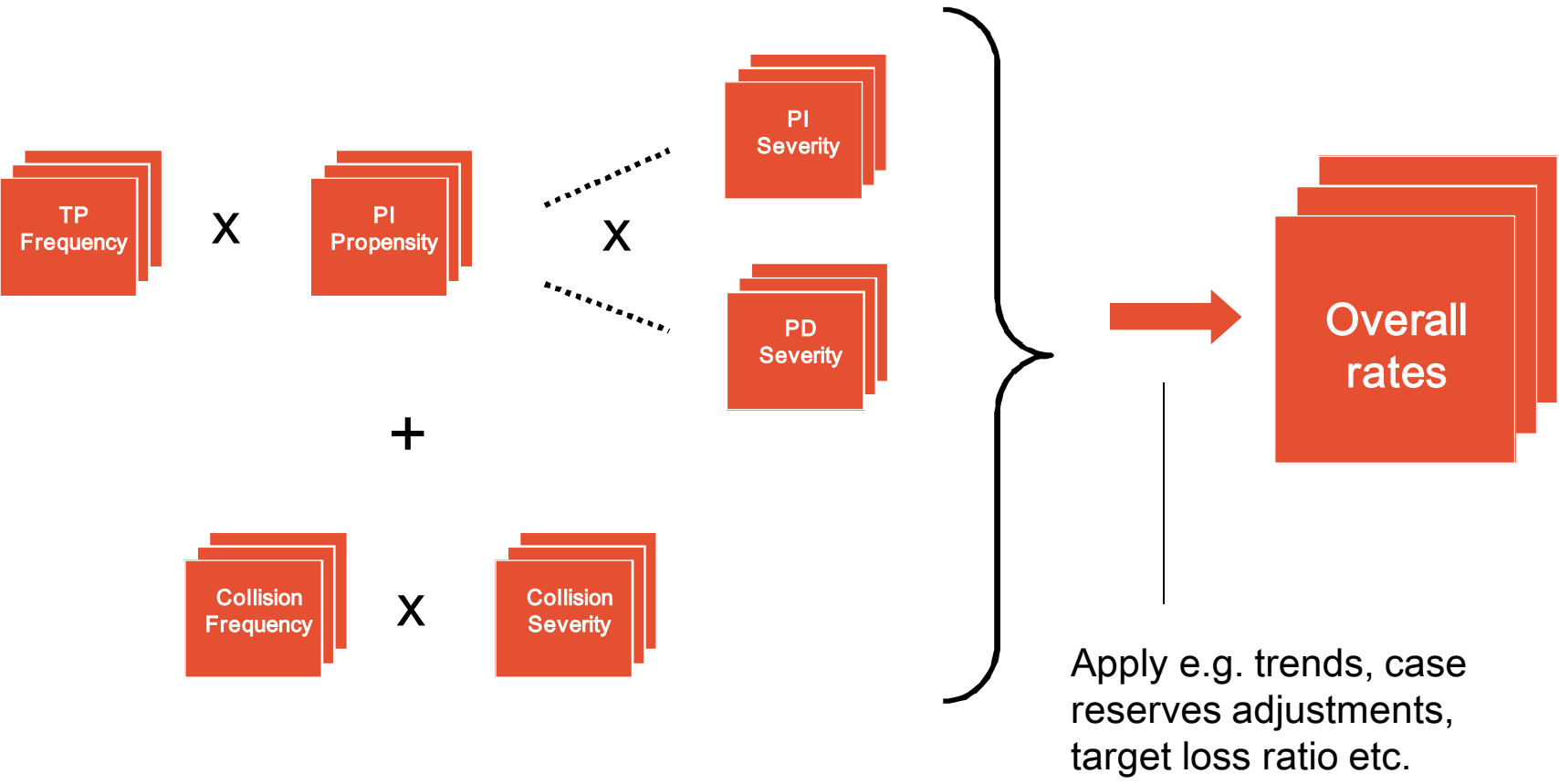
# Combining models



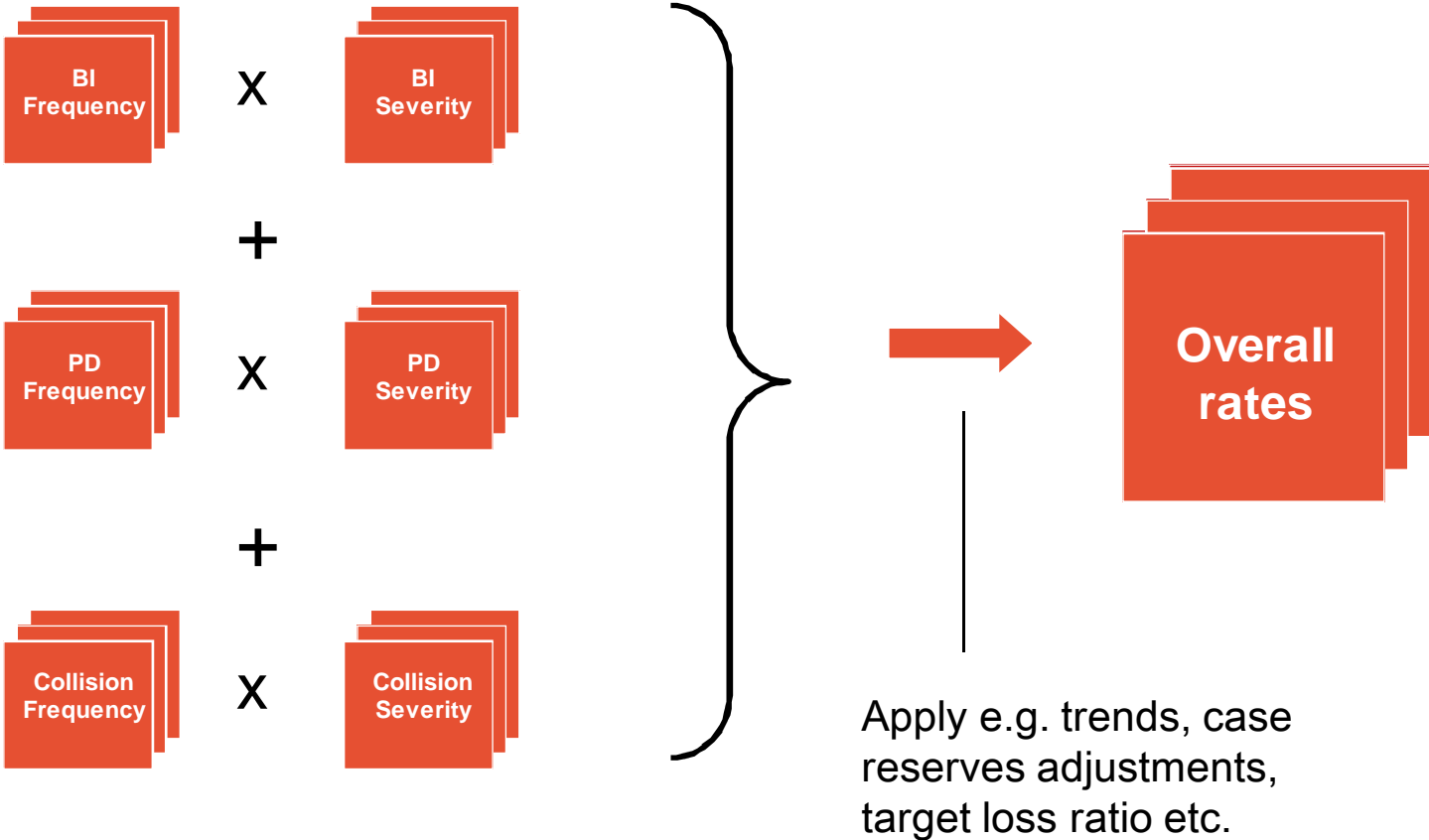
# Combining models



# Combining models



# Combining models



## Combining models

- Take models
- Take relevant mix of business
  - eg current in force policies
- For each record calculate expected frequencies and severities according to the models
- For each record, calculate expected total cost of claims "C"
- Fit a GLM to "C" using all available factors



## Combining models

|           |         | PD<br>Numbers | PD<br>Amounts | BI<br>Numbers | BI<br>Amounts |
|-----------|---------|---------------|---------------|---------------|---------------|
| Intercept |         | 32%           | \$1000        | 12%           | \$4860        |
| Sex       | Male    | 1.000         | 1.000         | 1.000         | 1.000         |
|           | Female  | 0.750         | 1.200         | 0.667         | 0.900         |
| Area      | Town    | 1.000         | 1.000         | 1.000         | 1.000         |
|           | Country | 1.250         | 0.700         | 0.750         | 0.833         |

| Policy   | Sex | Area | NUM1 | AMT1 | NUM2 | AMT2 | CC1 | CC2    | RISKPREM |
|----------|-----|------|------|------|------|------|-----|--------|----------|
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |
| 82155654 | M   | T    | 32%  | 1000 | 12%  | 4860 | 320 | 583.20 | 903.20   |
| 82168746 | F   | T    | 24%  | 1200 | 8%   | 4374 | 288 | 349.92 | 637.92   |
| 82179481 | M   | C    | 40%  | 700  | 9%   | 4050 | 280 | 364.50 | 644.50   |
| 82186845 | F   | C    | 30%  | 840  | 6%   | 3645 | 252 | 218.70 | 470.70   |
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |

## Except...

| Policy   | Sex | Area | NUM1 | AMT1 | NUM2 | AMT2 | CC1 | CC2    | RISKPREM |
|----------|-----|------|------|------|------|------|-----|--------|----------|
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |
| 82155654 | M   | T    | 32%  | 1000 | 12%  | 4860 | 320 | 583.20 | 903.20   |
| 82168746 | F   | T    | 24%  | 1200 | 8%   | 4374 | 288 | 349.92 | 637.92   |
| 82179481 | M   | C    | 40%  | 700  | 9%   | 4050 | 280 | 364.50 | 644.50   |
| 82186845 | F   | C    | 30%  | 840  | 6%   | 3645 | 252 | 218.70 | 470.70   |
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |

- The global risk premium is not multiplicative
- In the town, women have a modelled claim cost 29% lower than men
  - $637.92/903.20=0.706$
- In the country, women have a modelled claim cost 27% lower than men
  - $470.07/644.50=0.730$

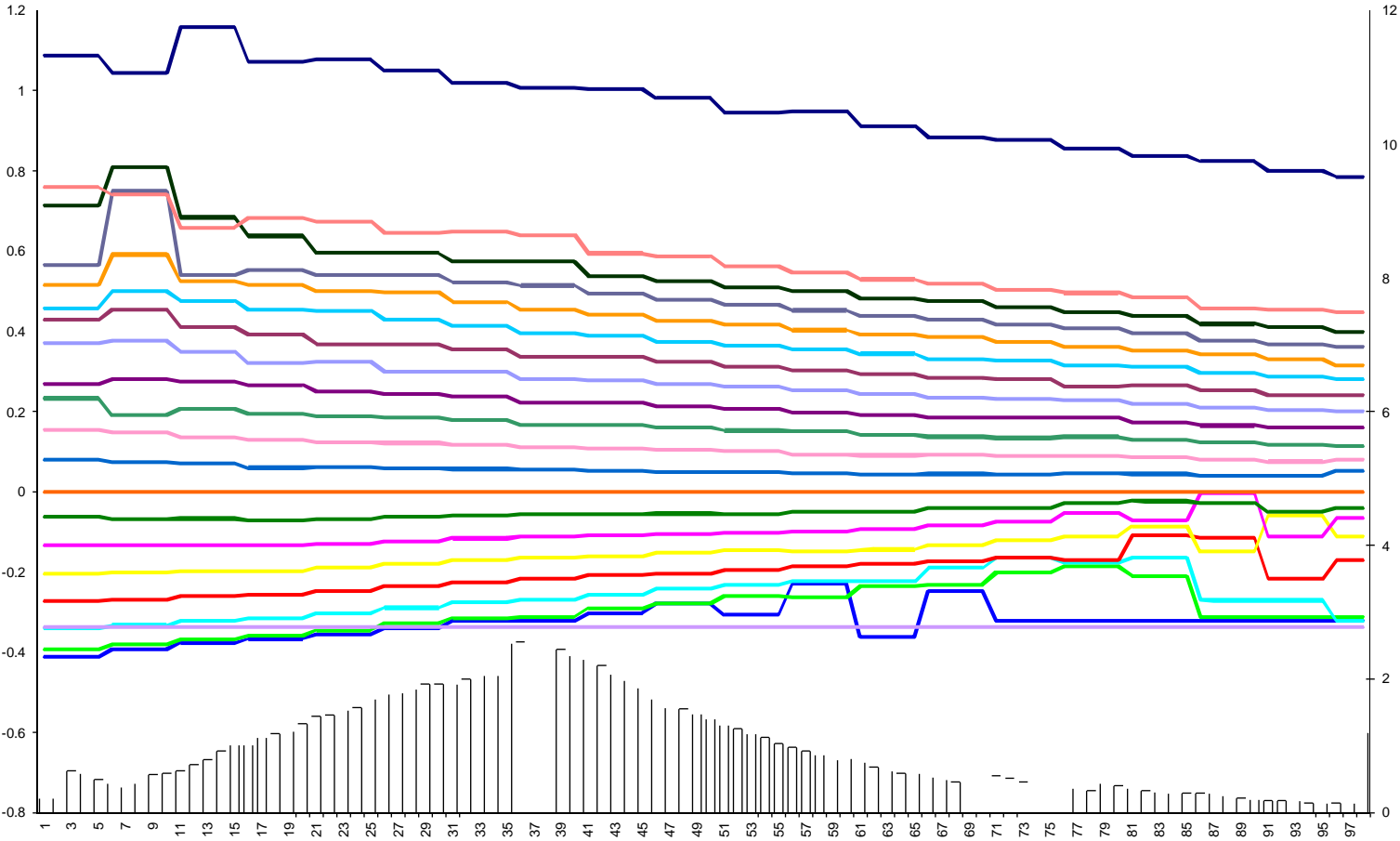
## To solve...

| Policy   | Sex | Area | NUM1 | AMT1 | NUM2 | AMT2 | CC1 | CC2    | RISKPREM |
|----------|-----|------|------|------|------|------|-----|--------|----------|
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |
| 82155654 | M   | T    | 32%  | 1000 | 12%  | 4860 | 320 | 583.20 | 903.20   |
| 82168746 | F   | T    | 24%  | 1200 | 8%   | 4374 | 288 | 349.92 | 637.92   |
| 82179481 | M   | C    | 40%  | 700  | 9%   | 4050 | 280 | 364.50 | 644.50   |
| 82186845 | F   | C    | 30%  | 840  | 6%   | 3645 | 252 | 218.70 | 470.70   |
| ...      | ... | ...  | ...  | ...  | ...  | ...  | ... | ...    | ...      |

- We can capture this result exactly with an interaction

|                    |         |          |        |
|--------------------|---------|----------|--------|
| Total risk premium |         |          |        |
| Intercept          |         | \$903.20 |        |
|                    | Sex     | Male     | Female |
| Area               | Town    | 1.000    | 0.706  |
|                    | Country | 0.714    | 0.521  |

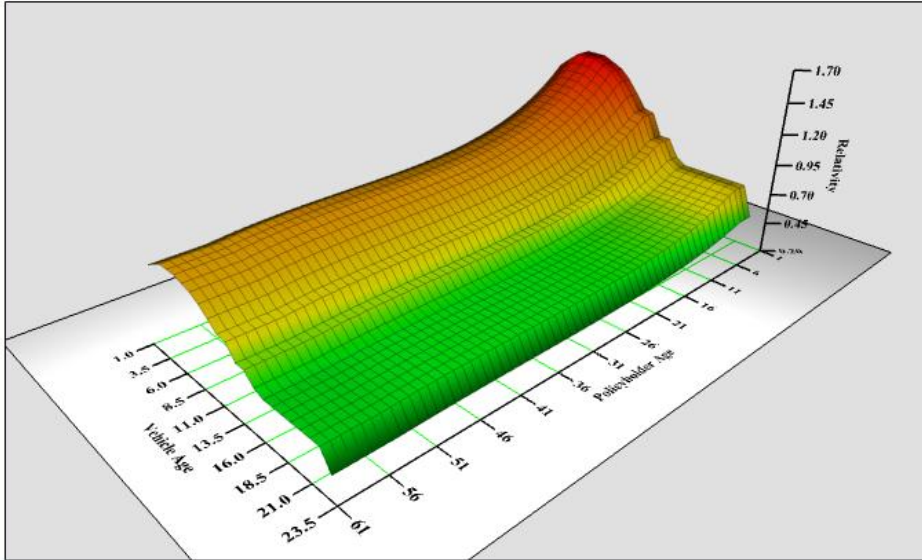
# Example "emergent" interaction



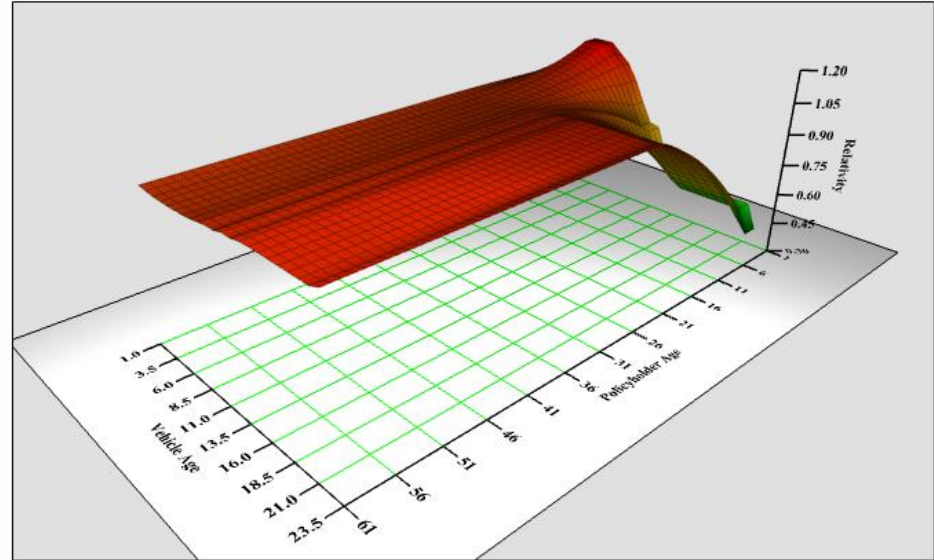
## "Emergent" interactions

- In the above examples the interaction "emerged" from the risk premium step
- Emergent interactions are **not** risk insights, there is no subtle risk effect we have just discovered
  - The different behaviour is by peril, and the rating factors are just bad proxies for the peril effects
- Emergent interactions are corrections to fix problems we have introduced
- Best solution is by peril pricing
  - Reflects true behaviour
  - Underlying models simple to understand and implement
- If not, check for emergent interactions in the risk premium

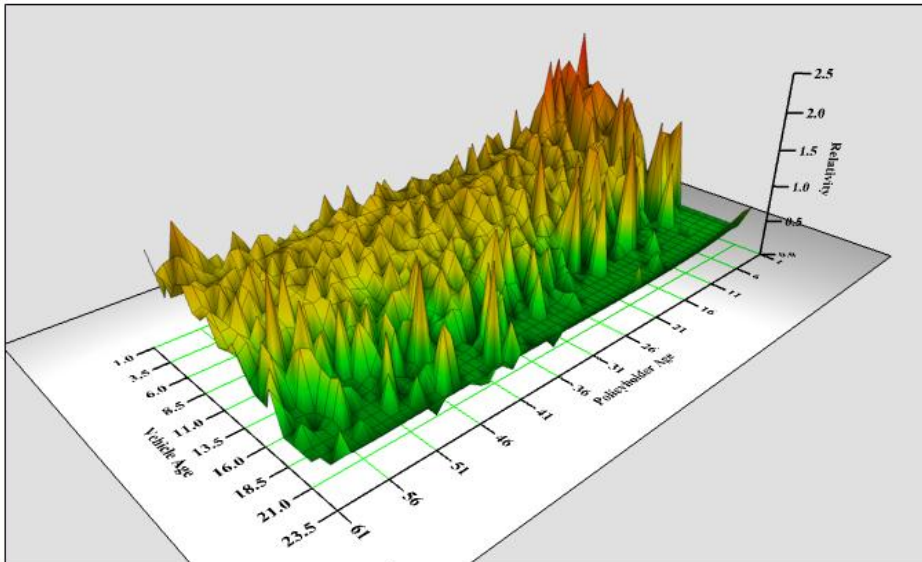
Original



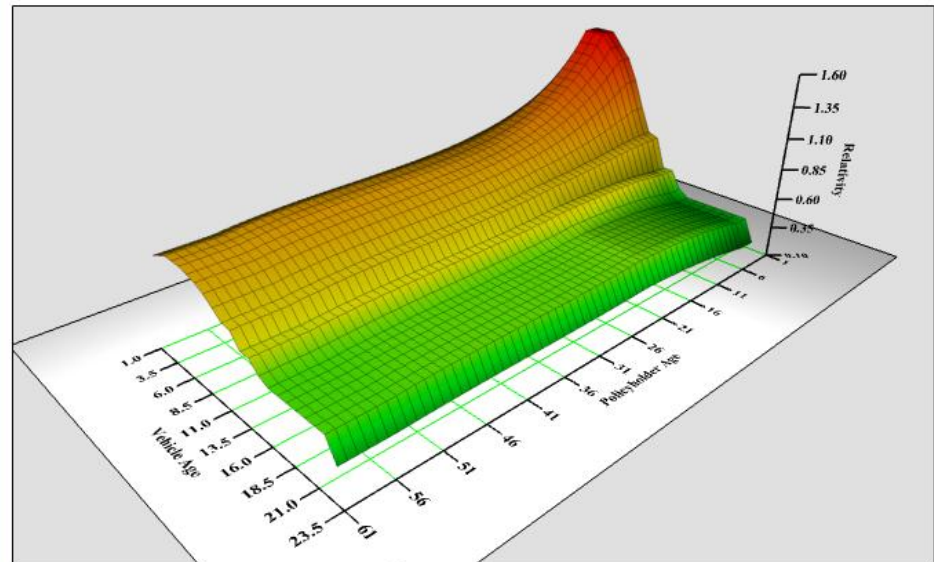
Saddle Parameter



Unsimplified



With Saddle



## Agenda

- "Quadrant Saddles"
- The Tweedie Distribution
- "Emergent Interactions"
- Dispersion Modeling
- Modelling sparse claim types
- Driver Averaging
- Model Validation
- Man (with GLM) vs machine

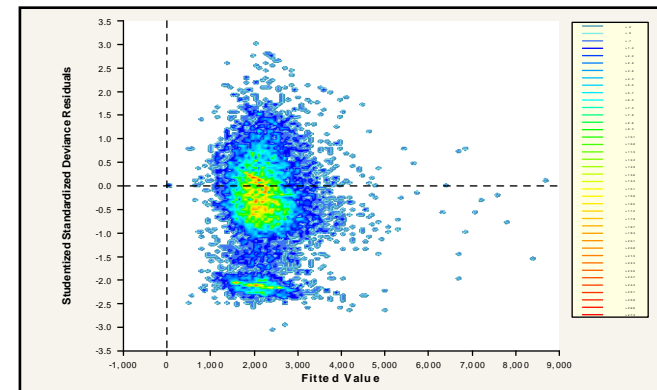


# Modeling the Insurance Risk

## ISSUE:

### Heterogeneous exposure bases

- Different policies within the same line can cover entirely different structures (i.e. commercial property)
- Goal of a predictive model
  - Ideally would like to separate the heterogenous exposure bases
  - Joint-modeling techniques and quasi-likelihood functions allow for analysis of heterogeneous environment without separation



Two concentrations suggests two perils:  
split or use joint modeling

## Heterogeneous Exposure Bases

- If possible should be modeled separately
  - If model together, exposures with high variability may mask patterns of less random risks
  - If loss trends vary by exposure class, the proportion each represents of the total will change and may mask important trends
  - Independent predictors can have different effects on different perils
- If cannot, use joint modeling techniques to improve overall fit

## Generalized Linear Models

- Formulation of deviance – logarithm of a ratio of likelihoods

$$\frac{D}{a(\varphi)} = \ln \left( \frac{\text{Act}}{\text{Exp}} \right)^2$$

Where:

$$\text{Act} = f_Y(y; \tilde{\theta}, \varphi) \ni E(Y) = y = b(\tilde{\theta})$$

$$\text{Exp} = f_Y(y; \hat{\theta}, \varphi) \ni E(Y) = \hat{\mu} = b(\hat{\theta})$$

Then:

$$\frac{D}{a(\varphi)} = \ln \left( \frac{f_Y(y; \tilde{\theta}, \varphi)}{f_Y(y; \hat{\theta}, \varphi)} \right)^2 = 2 \times \left[ \frac{y\tilde{\theta} - y\hat{\theta} - b(\tilde{\theta}) + b(\hat{\theta})}{a(\varphi)} \right]$$

## Generalized Linear Models

- Analyzing the scale parameter
  - When modeling homogeneous data

$$a(\varphi) = \frac{\varphi}{\omega} \Rightarrow \varphi = \frac{D}{\text{dof}}$$

- Heterogeneous data requires a more rigorous definition of the scale function
  - Scale parameter could vary in a systematic way with other predictors
  - Construct and fit formal models for the dependence of both the mean and the scale

## Dispersion Model Form

- Double generalized linear models
  - Response model

$$Y \sim f_Y(y; \theta; \varphi)$$

$$E(Y) = b'(\theta)$$

$$\text{Var}(Y) = \frac{\varphi b''(\theta)}{\omega}$$

- Dispersion model

$$D \sim f_D(d; \xi, \tau)$$

$$E(D) = b'(\xi)$$

$$\text{Var}(D) = \frac{\tau b''(\xi)}{\omega}$$

Where

$$d = \frac{(Y - \mu)^2}{V(\mu)}$$

## Dispersion Model Form

- Dispersion adjustments
  - Pearson residual has excess variability (deviance residual has bias)

| Distribution | Adjustment |
|--------------|------------|
| Normal       | 0          |
| Poisson      | $f/(2m)$   |
| Gamma        | 3f         |

- Parameter in the adjustment term is the scale parameter from the original response model

# Dispersion Model Results

- Dispersion model is integrated with original response model

|                        | Response                 | Weight                             |
|------------------------|--------------------------|------------------------------------|
| Initial Response Model | Loss / Exposure          | Exposure                           |
| Dispersion Model       | Squared Pearson Residual | Exposure/ (Exposure + Adjustment)  |
| Final Response Model   | Loss / Exposure          | Exposure/ Squared Pearson Residual |

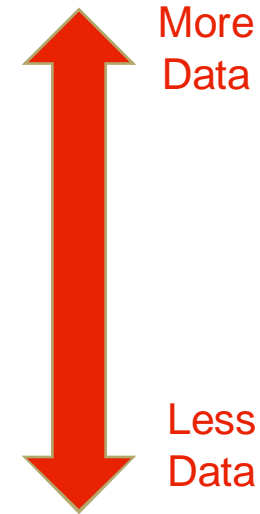


## Agenda

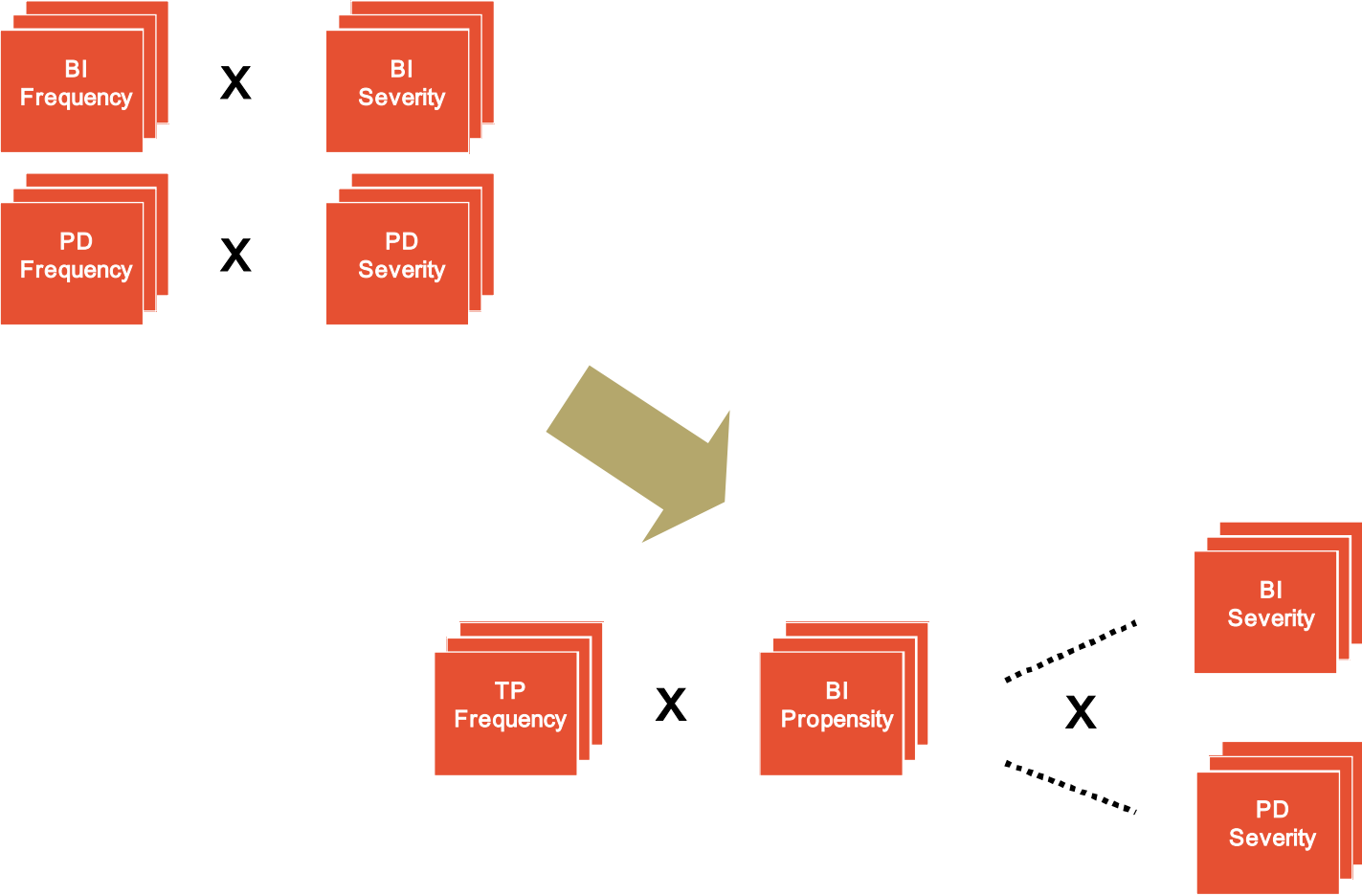
- "Quadrant Saddles"
- The Tweedie Distribution
- "Emergent Interactions"
- Dispersion Modeling
- **Modelling sparse claim types**
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- Model Validation
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## Amplification of the BI signal using PD experience

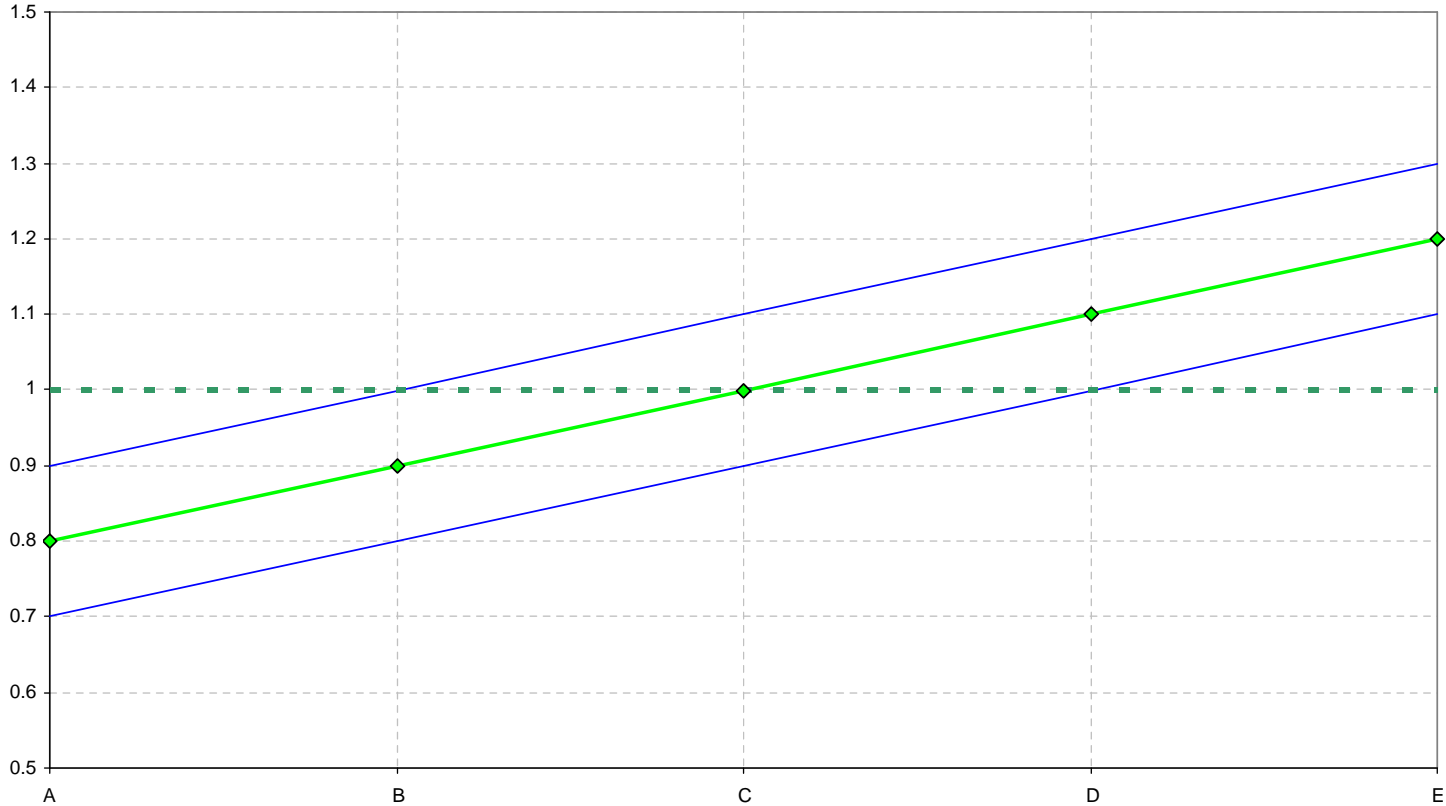
- Fit straight to BI
- Use PD model as a guide in free fitting BI
- Use PD model structure
- Offset PD relativities onto BI data as starting point
- BI/PD proportion model:
  - BI frequency = BI/PD proportion \* PD frequency



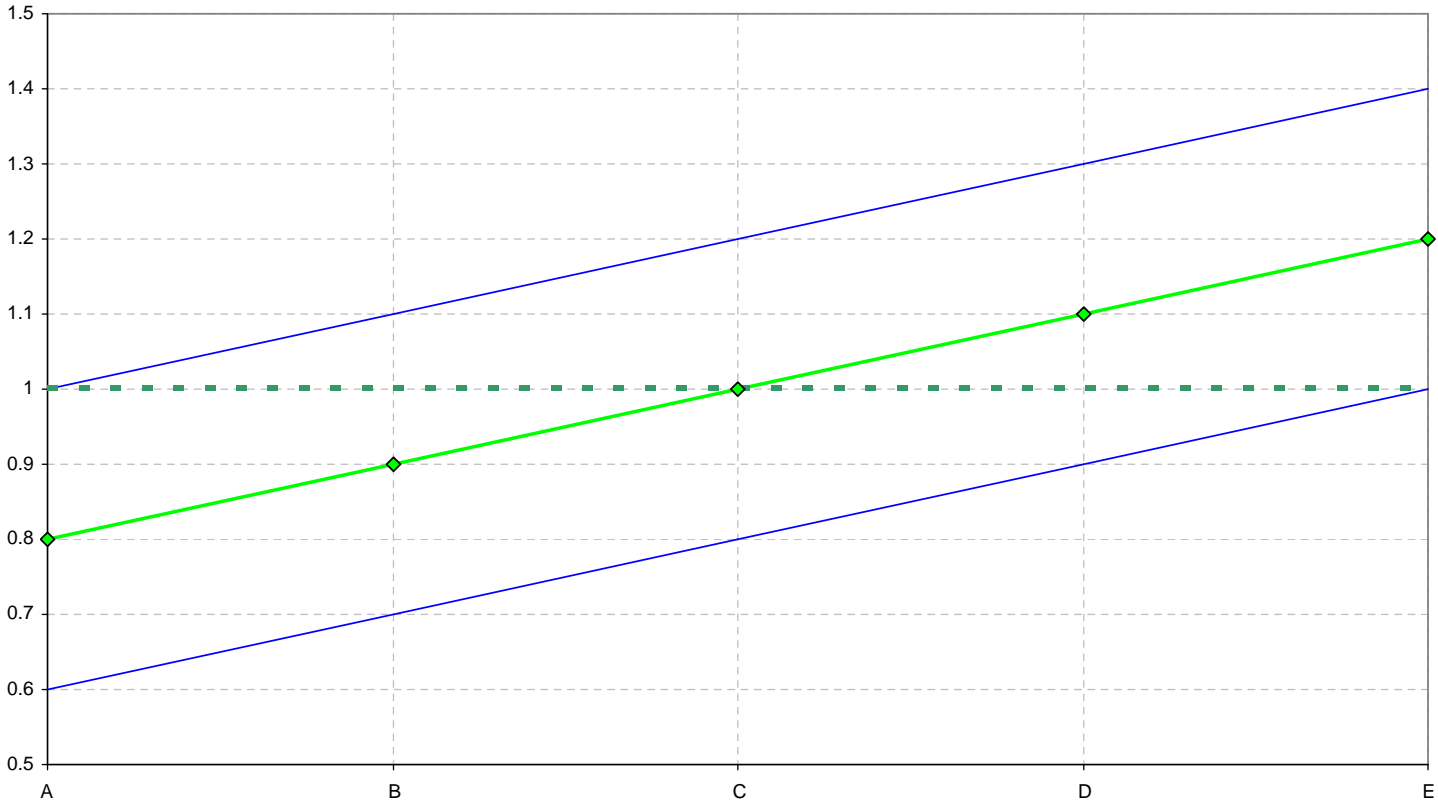
# Reference models



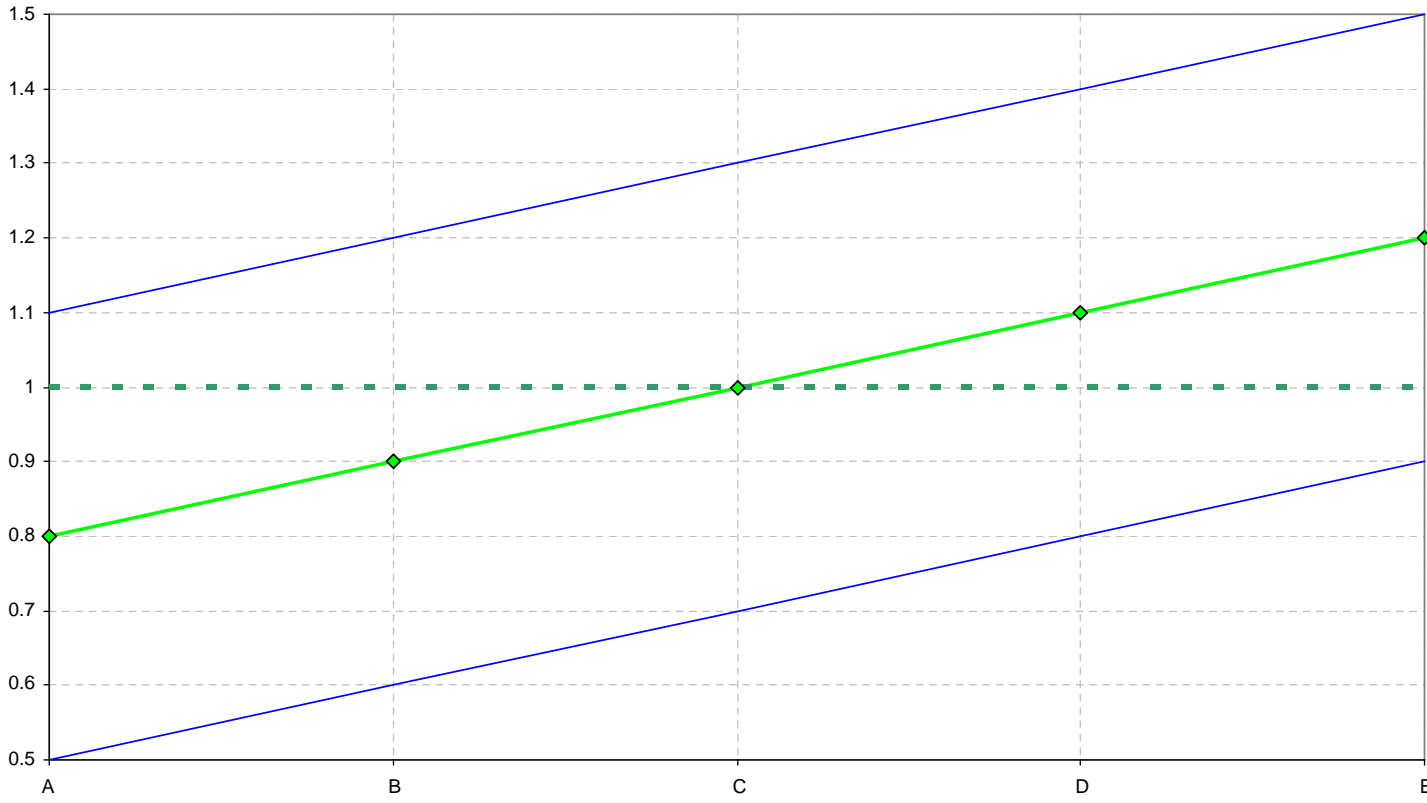
# Reference models



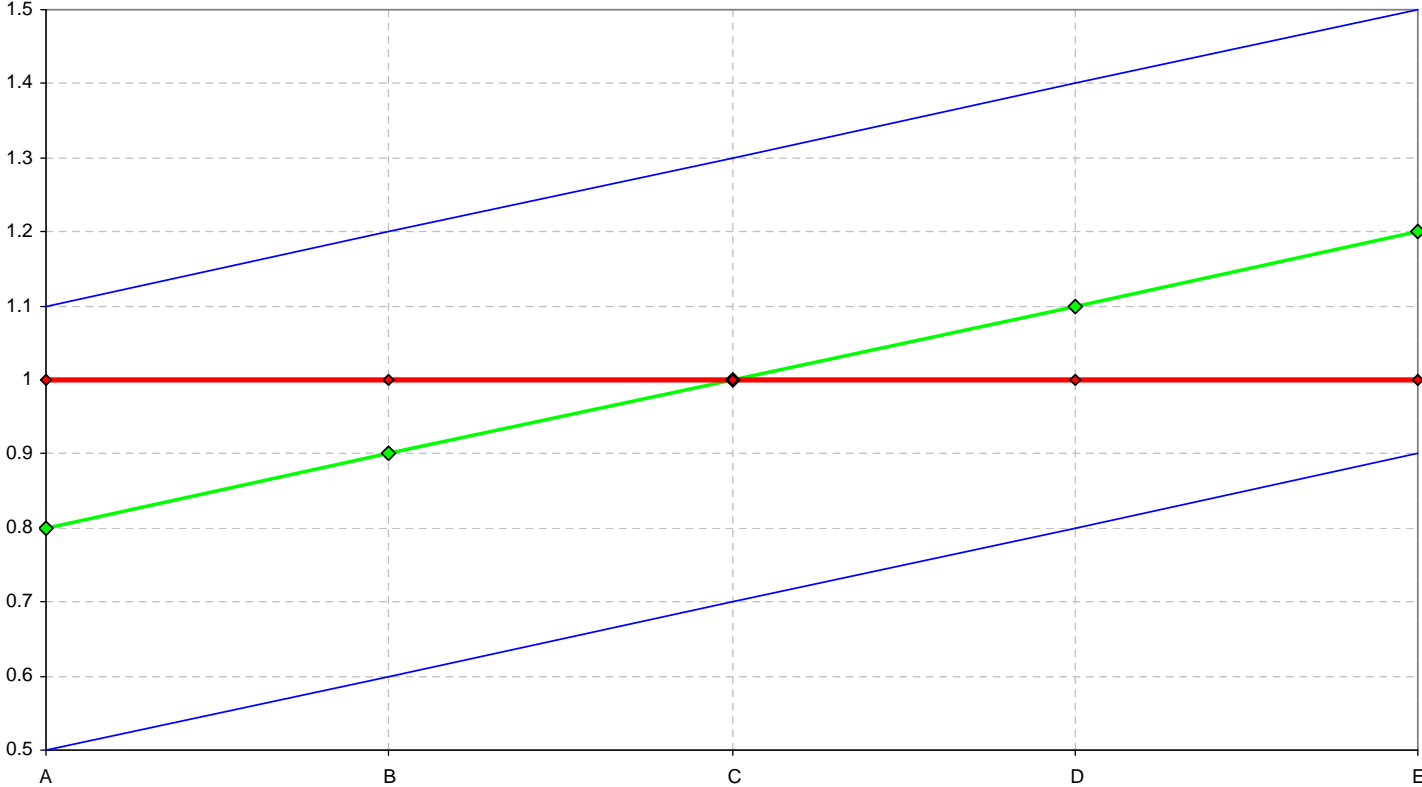
# Reference models



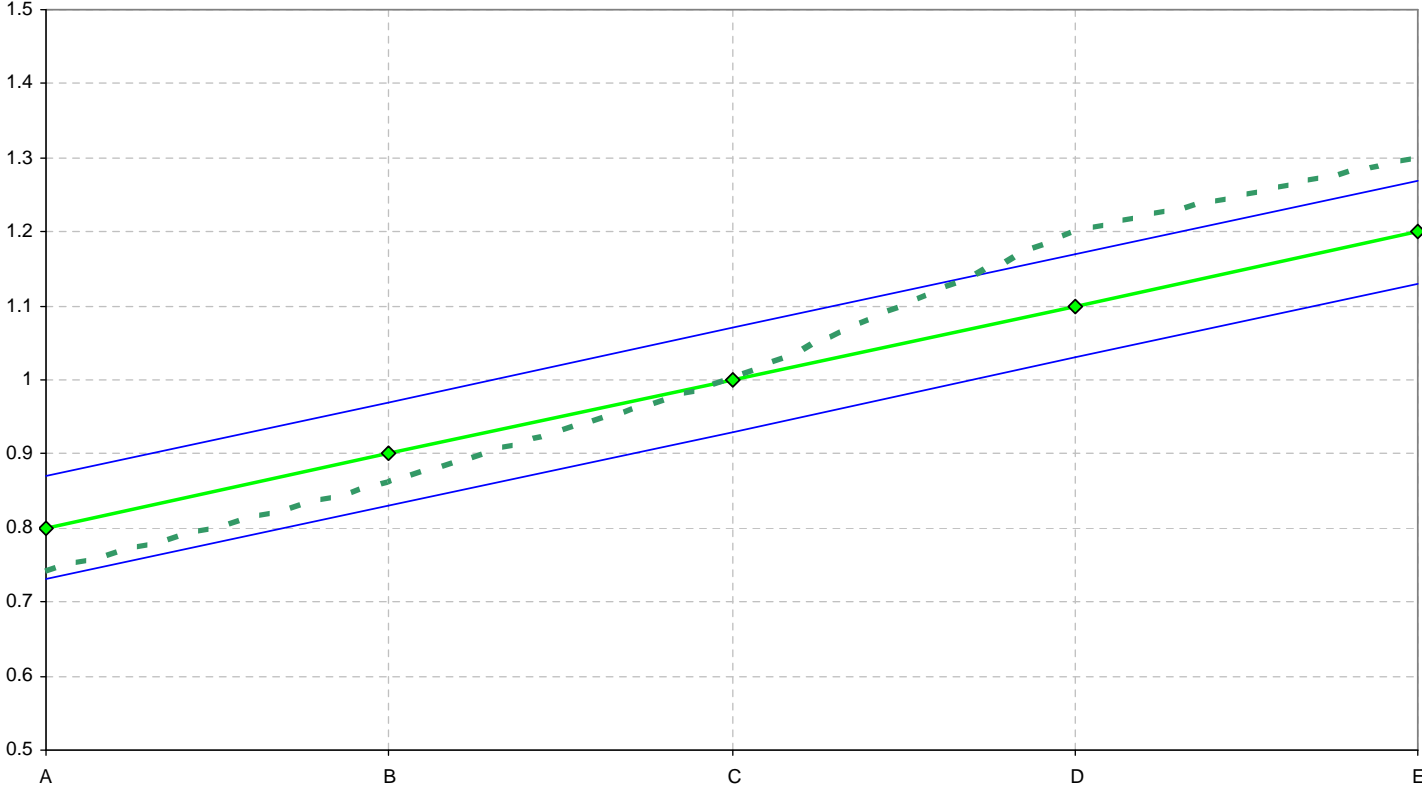
# Reference models



# Reference models

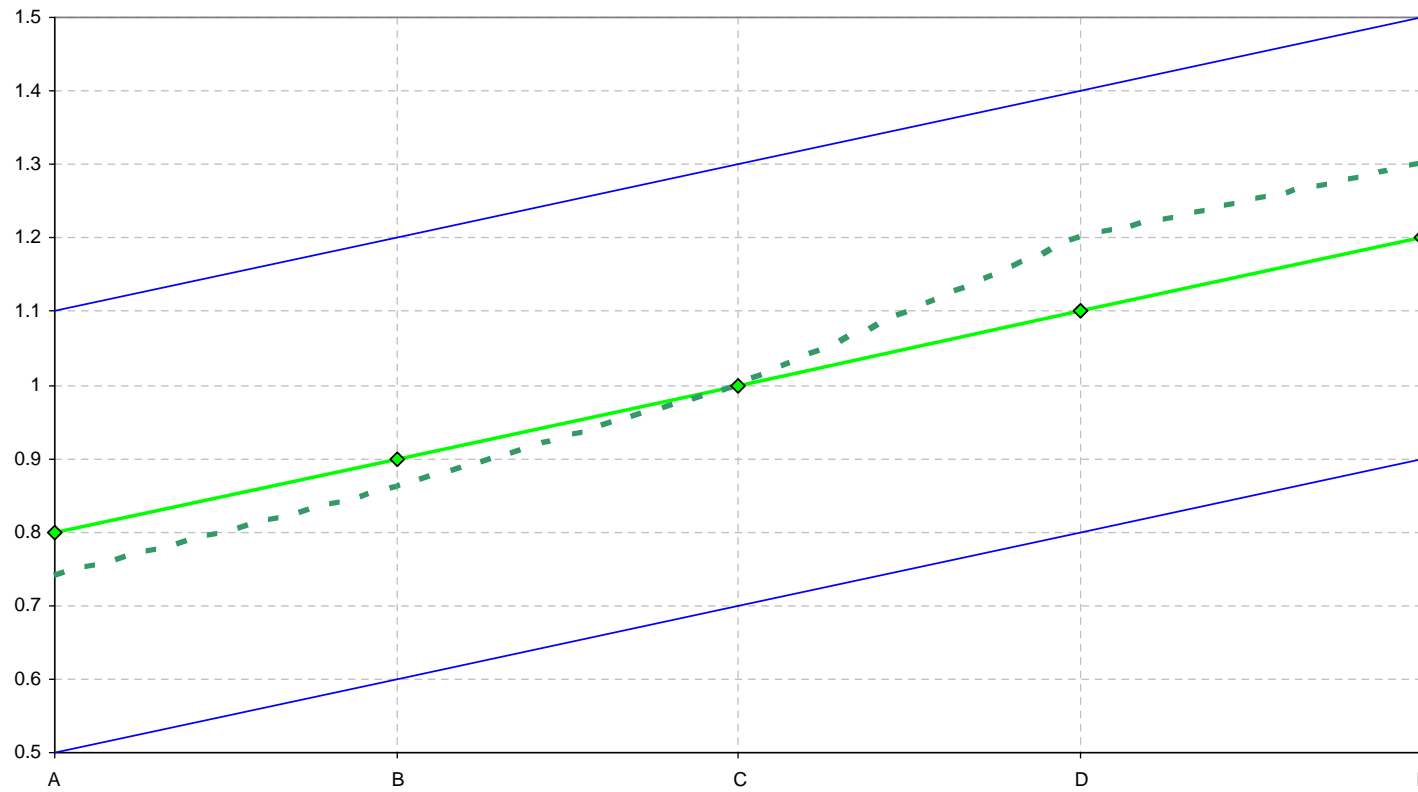


# Reference models

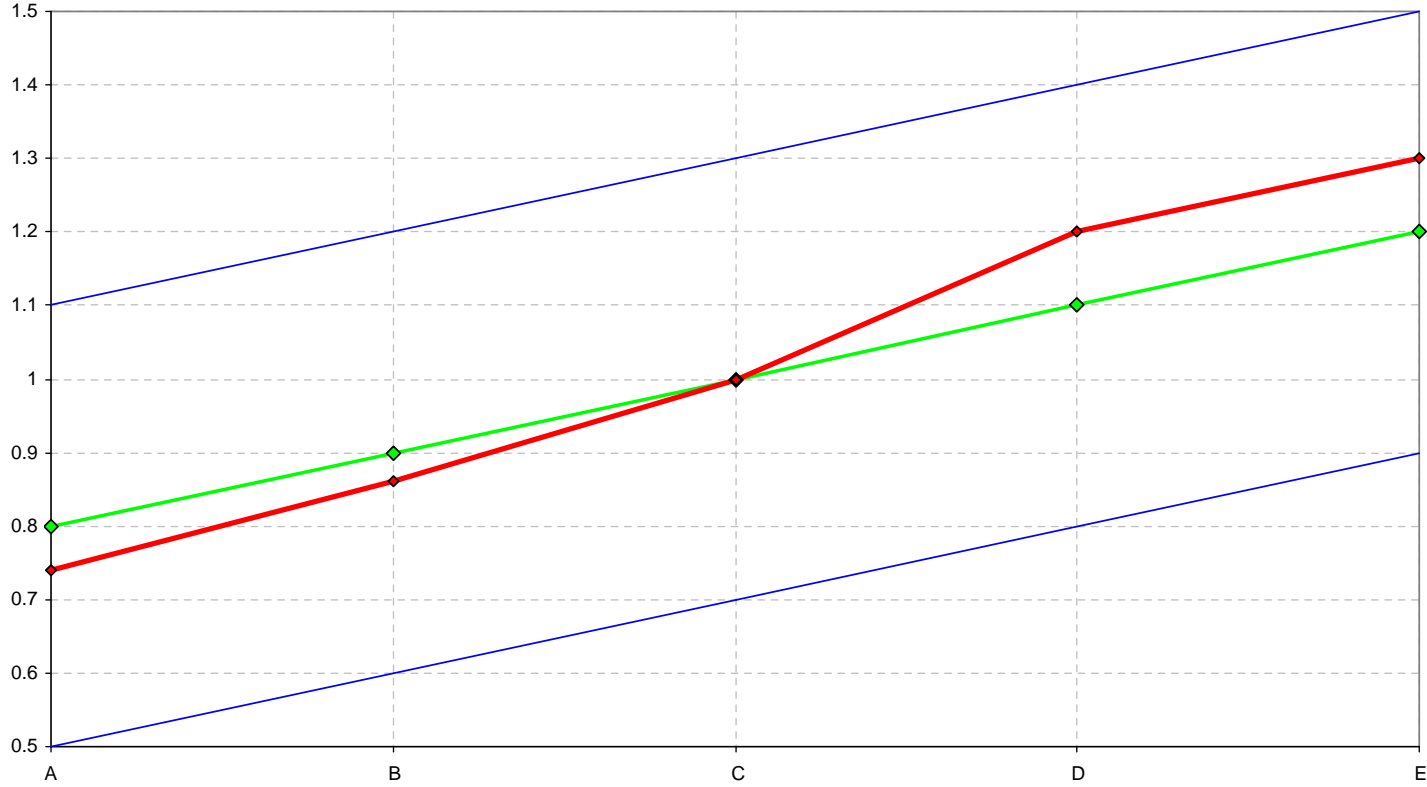




# Reference models



# Reference models



## Reference models

$$E[Y_i] = \mu_i = g^{-1}(\sum X_{ij} \cdot \beta_j + \xi_i)$$

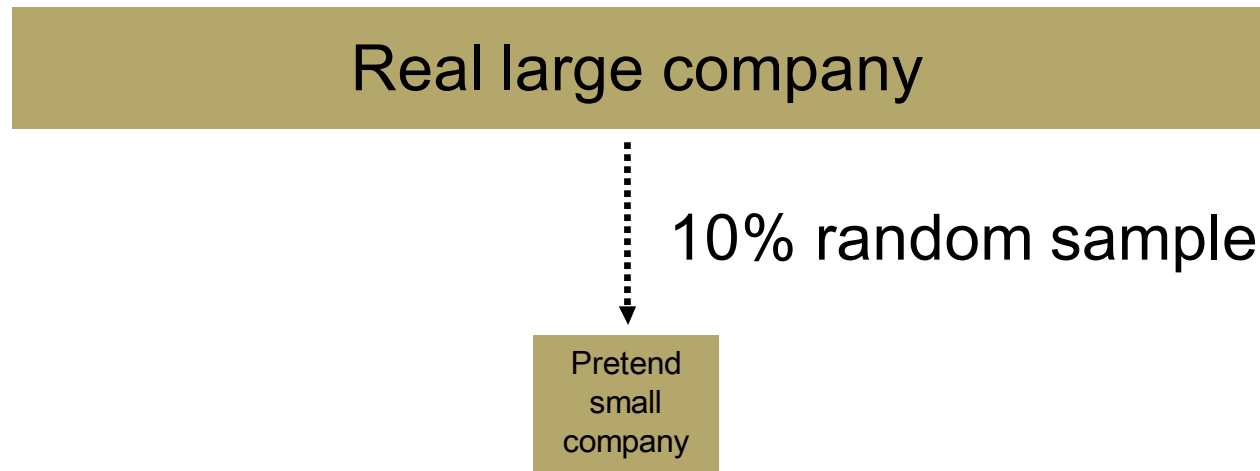


Offset term

- When modeling BI set PD fitted values to be offset term
- GLM will seek effects over and above assumed PD effect

# Experiment

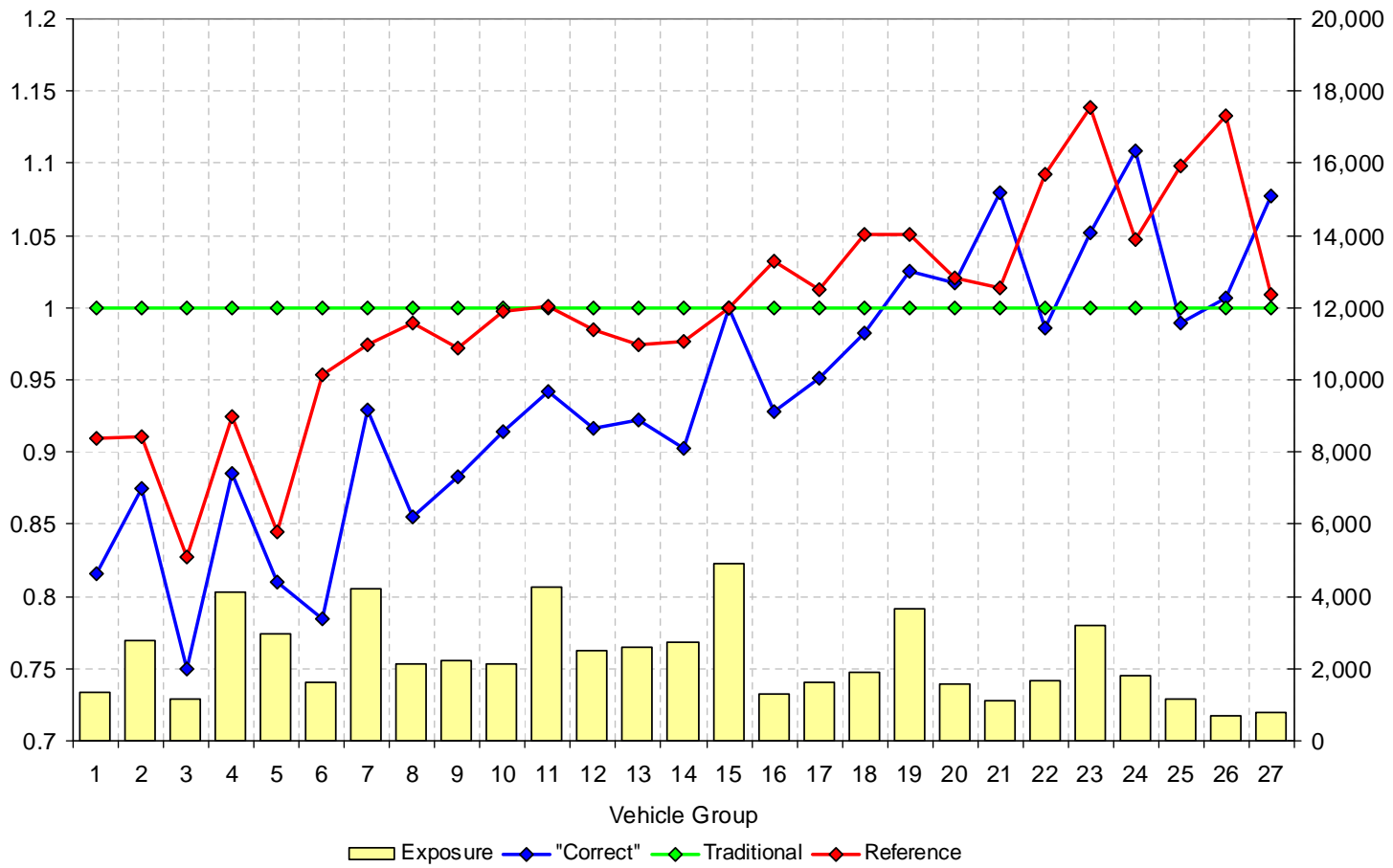
(1) GLM on BI claims on all the data - the "correct" answer



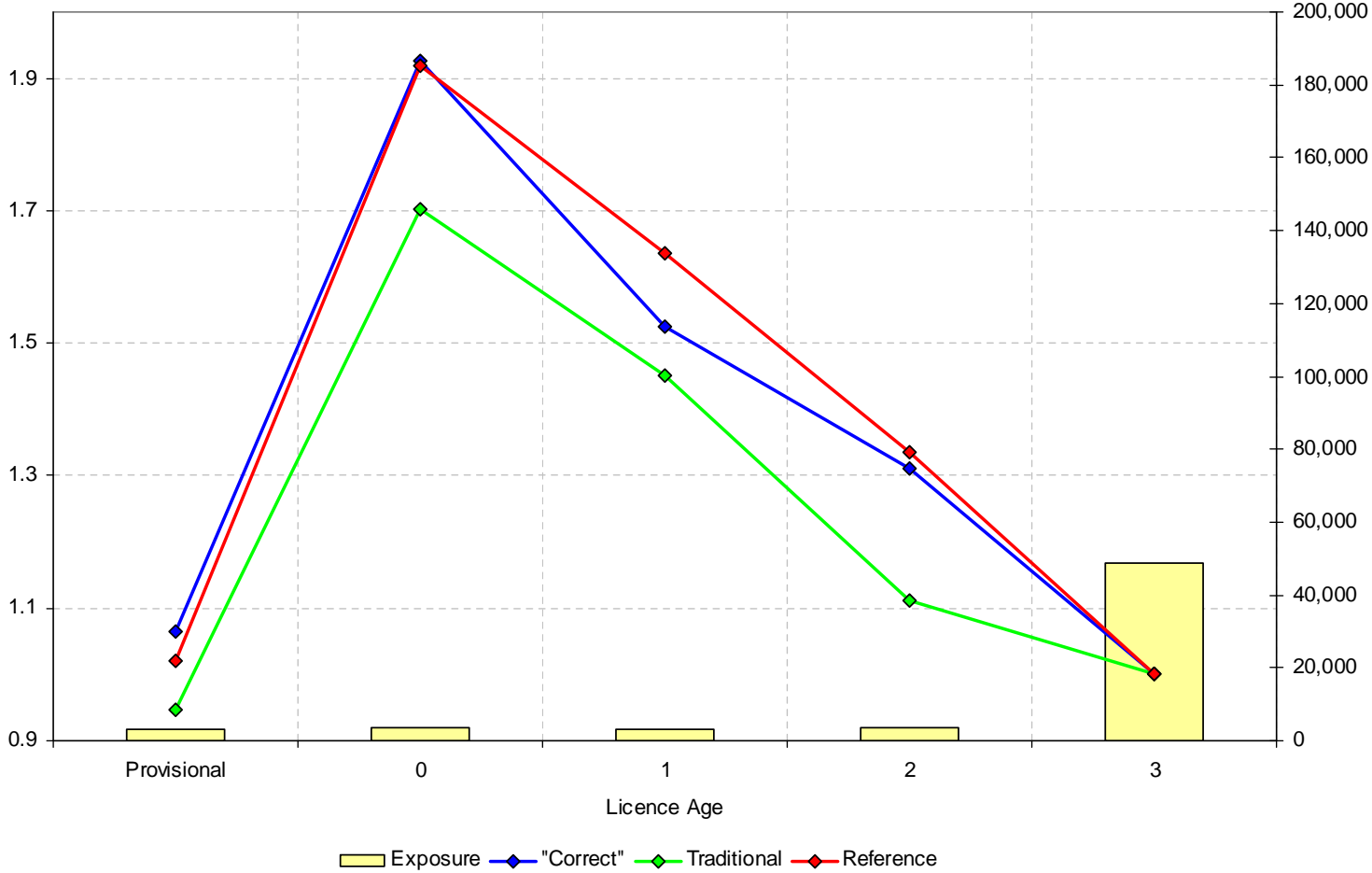
(2) Traditional GLM on BI claims on the "small company"

(3) Propensity reference model on BI claims cf PD claims

# Example result



# Example result



## Agenda

- "Quadrant Saddles"
- The Tweedie Distribution
- "Emergent Interactions"
- Dispersion Modeling
- Modelling sparse claim types
- **Driver Averaging**
- Model Validation
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## Household Averaging

- Historically companies assigned operators to vehicles for the purpose of rating
- More recently driver averaging strategies are deployed to capture household
  - Straight vs. geometric average
  - Weighted average
  - Modified
  - Average/assignment hybrid
- Modeling data needs to mimic the transaction



## Model Design

- In all modeling projects, it is imperative that the data set up mimic the rating
- Consider the following example...

| Vehicle | Operator | Vehicle Rate |
|---------|----------|--------------|
| V1      | Dad      | \$500        |
| V2      | Mom      | \$450        |

| Operator | Class Factor |
|----------|--------------|
| Dad      | 0.80         |
| Mom      | 0.85         |
| Junior   | 2.80         |

- Assume Mom had a \$1000 claim in Dad's car

# Assignment

- Actual assignment methodology each record represents a single vehicle with one assigned operator

| Veh | Op     | Sym | MYR  | Age | Sex | Type | Yths | Drvrs | Vehs | Exp | Clm | Losses | Prem     |
|-----|--------|-----|------|-----|-----|------|------|-------|------|-----|-----|--------|----------|
| V1  | Junior | 17  | 2006 | 16  | M   | OO   | 1    | 3     | 3    | 1   | 1   | 1,000  | 1,400.00 |
| V2  | Mom    | 17  | 2005 | 43  | F   | PO   | 1    | 3     | 3    | 1   | 0   | 0      | 382.50   |

- Operator characteristics based on assigned operator
- Vehicle characteristics based on vehicle
- Policy characteristics “catch” other drivers
- Losses assigned to vehicle

## Straight Average

- Straight average methodology:

$$VehicleFactor \times \frac{(Op1Factor + Op2Factor + Op3Factor)}{3}$$

- Which can be deconstructed:

$$VehicleFactor \times \frac{(Op1Factor)}{3}$$

$$VehicleFactor \times \frac{(Op2Factor)}{3}$$

$$VehicleFactor \times \frac{(Op3Factor)}{3}$$

## Straight Average

- Straight average methodology each record represents a single vehicle and operator combination

| Veh | Op     | Sym | MYR  | Age | Sex | Yths | Drvrs | Vehs | Exp | Clm | Losses | Prem   |
|-----|--------|-----|------|-----|-----|------|-------|------|-----|-----|--------|--------|
| V1  | Dad    | 17  | 2006 | 45  | M   | 1    | 3     | 3    | 1/3 | 0   | 0      | 133.33 |
| V1  | Mom    | 17  | 2006 | 43  | F   | 1    | 3     | 3    | 1/3 | 1   | 1,000  | 141.67 |
| V1  | Junior | 17  | 2006 | 16  | M   | 1    | 3     | 3    | 1/3 | 0   | 0      | 466.67 |
| V2  | Dad    | 17  | 2005 | 45  | M   | 1    | 3     | 3    | 1/3 | 0   | 0      | 120.00 |
| V2  | Mom    | 17  | 2005 | 43  | F   | 1    | 3     | 3    | 1/3 | 0   | 0      | 127.50 |
| V2  | Junior | 17  | 2005 | 16  | M   | 1    | 3     | 3    | 1/3 | 0   | 0      | 420.00 |

- Policy characteristics are same, but less predictive
- Exposure split amongst the vehicle
- Losses assigned to vehicle/operator combination
- iid is a major concern
- What about Comprehensive?

## Geometric Average

- Geometric average methodology:

$$VehicleFactor \times (Op1Factor + Op2Factor + Op3Factor)^{1/3}$$

- No direct decomposition

## Geometric Average

- Geometric methodology each record represents a single vehicle

| Veh | Sym | MYR  | # of Dads | # of Moms | # of Juniors | Exp | Clm | Losses | Prem   |
|-----|-----|------|-----------|-----------|--------------|-----|-----|--------|--------|
| V1  | 17  | 2006 | 1/3       | 1/3       | 1/3          | 1   | 1   | 1,000  | 619.72 |
| V2  | 17  | 2005 | 1/3       | 1/3       | 1/3          | 1   | 0   | 0      | 557.74 |

- Policy characteristics are same, but less predictive
- Predictors are translated to counts
- Losses assigned to vehicle
- More challenging to add operator interactions or variates

# Weighted Average

- Weighted average methodology for a straight average approach

| Veh | Op     | Sym | MYR  | Age | Sex | Type | Yths | Drvrs | Vehs | Exp | Clm | Losses | Prem   |
|-----|--------|-----|------|-----|-----|------|------|-------|------|-----|-----|--------|--------|
| V1  | Dad    | 17  | 2006 | 45  | M   | PO   | 1    | 3     | 3    | 1/3 | 0   | 0      | 133.33 |
| V1  | Mom    | 17  | 2006 | 43  | F   | OC   | 1    | 3     | 3    | 1/3 | 1   | 1,000  | 141.67 |
| V1  | Junior | 17  | 2006 | 16  | M   | OC   | 1    | 3     | 3    | 1/3 | 0   | 0      | 466.67 |
| V2  | Dad    | 17  | 2005 | 45  | M   | OC   | 1    | 3     | 3    | 1/3 | 0   | 0      | 120.00 |
| V2  | Mom    | 17  | 2005 | 43  | F   | PO   | 1    | 3     | 3    | 1/3 | 0   | 0      | 127.50 |
| V2  | Junior | 17  | 2005 | 16  | M   | OC   | 1    | 3     | 3    | 1/3 | 0   | 0      | 420.00 |

- Creates a relationship between the vehicle and the operator
- Uses the model to determine the weights
- More *accurate* as it requires more information

$$VehicleFactor1 \times \frac{(Op1Factor * PO + Op2Factor * OC + Op3Factor * OC)}{3}$$

## Agenda

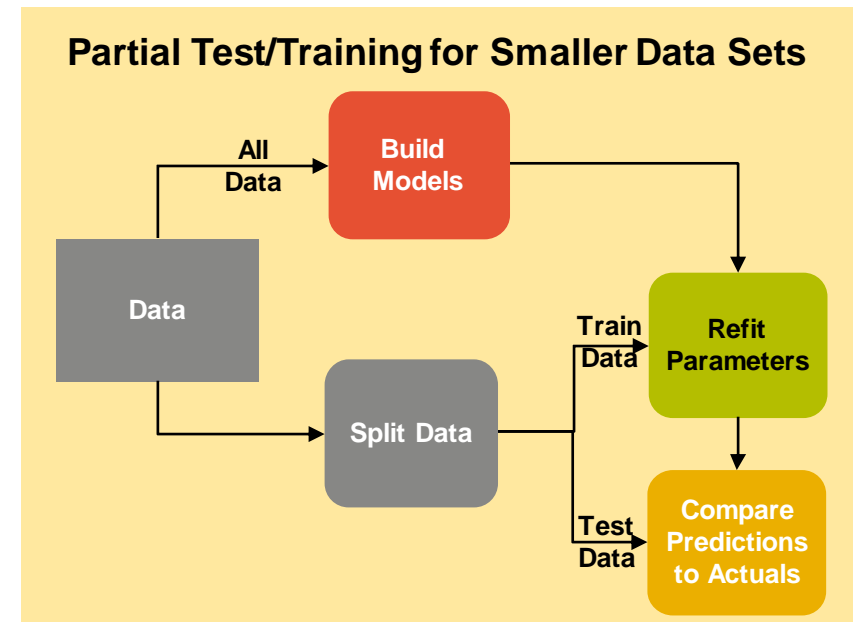
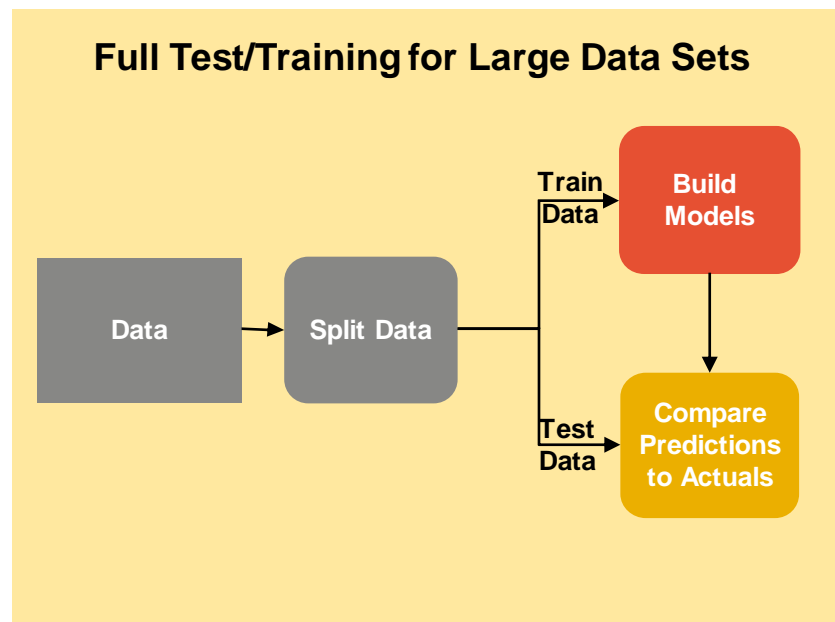
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# Validate Models

Holdout samples

- Holdout samples are effective at validating model
  - Determine estimates based on part of data set
  - Uses estimates to predict other part of data set

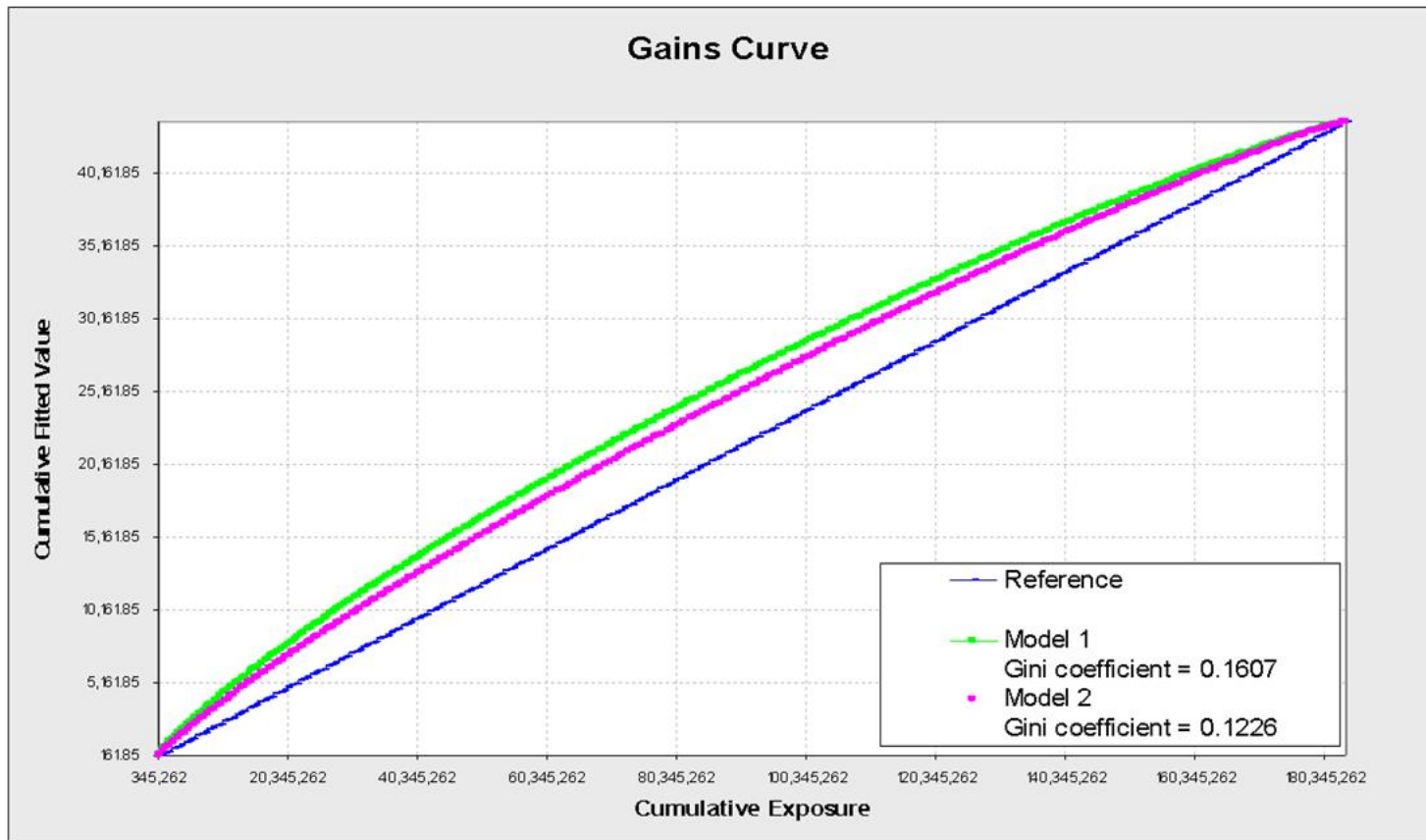


*Predictions should be close to actuals for heavily populated cells*

# Validate Models

## Gains curves

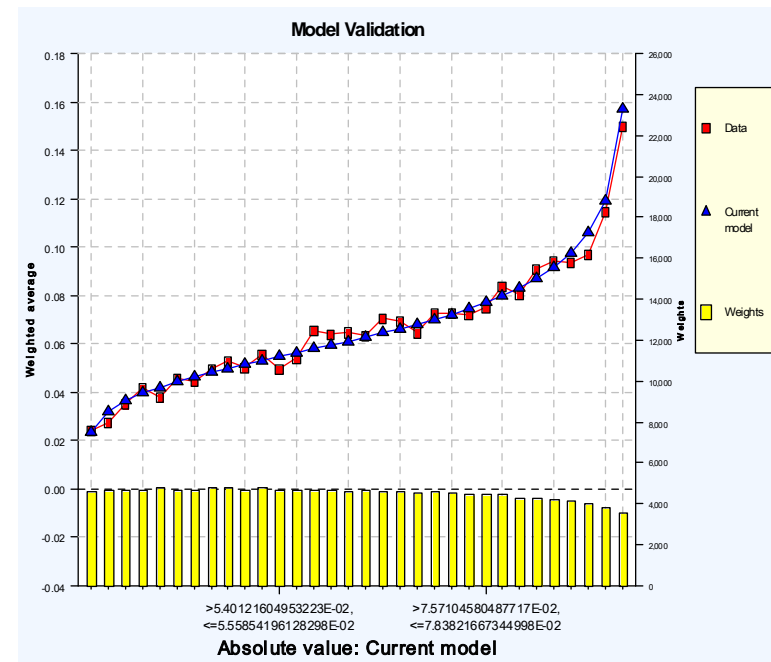
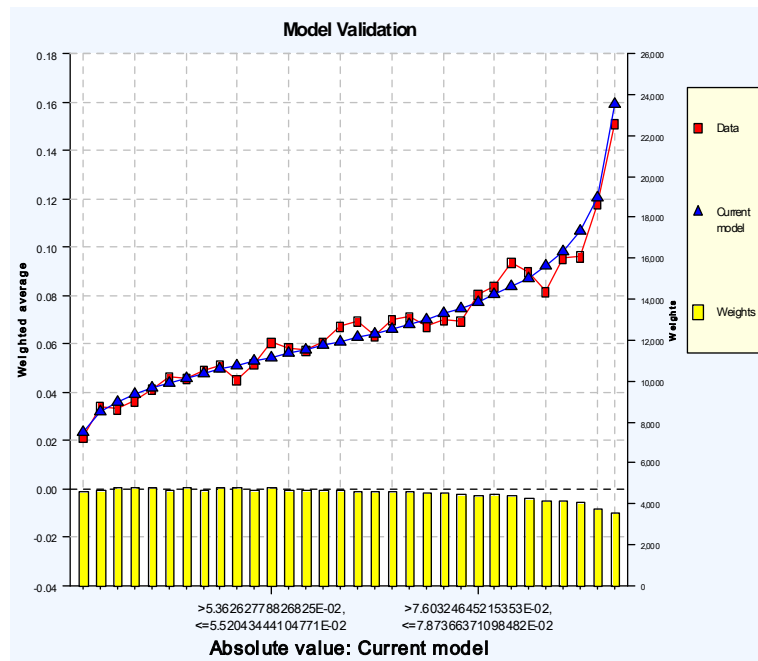
- Compare predictiveness of models



# Validate Models

## Lift curves

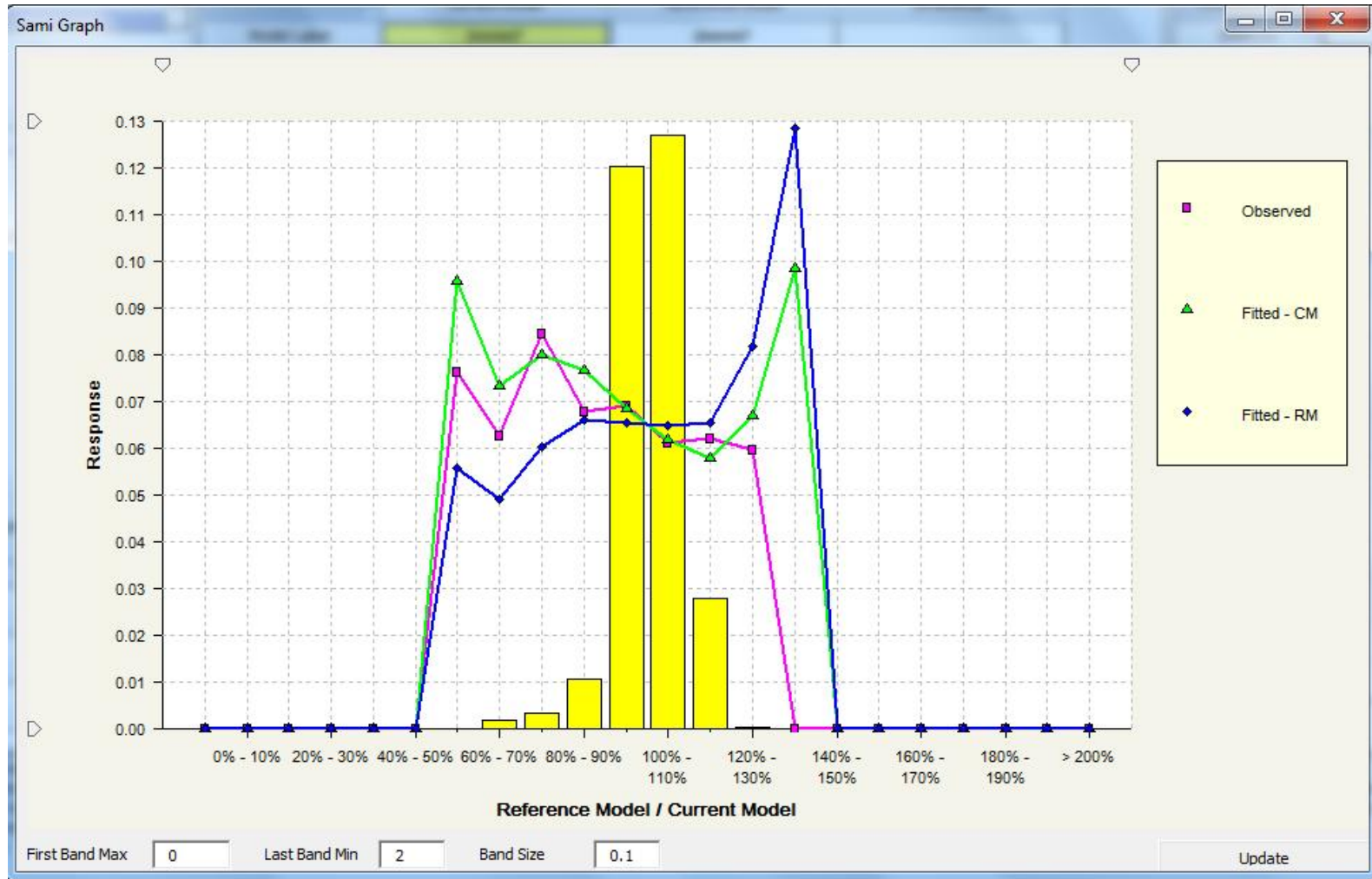
- Compare predictiveness of models



- More intuitive but difficult to assess performance

# Validate Models

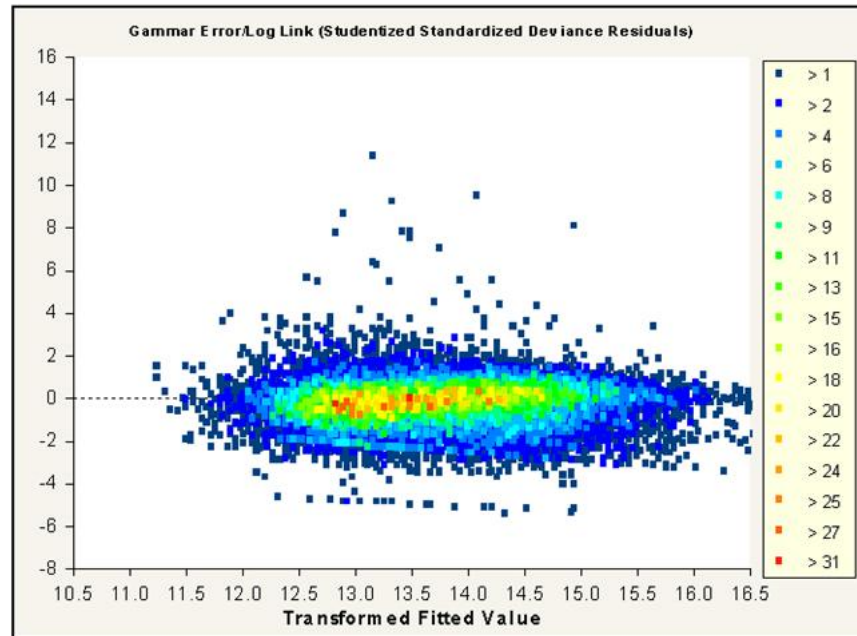
## X-Graphs



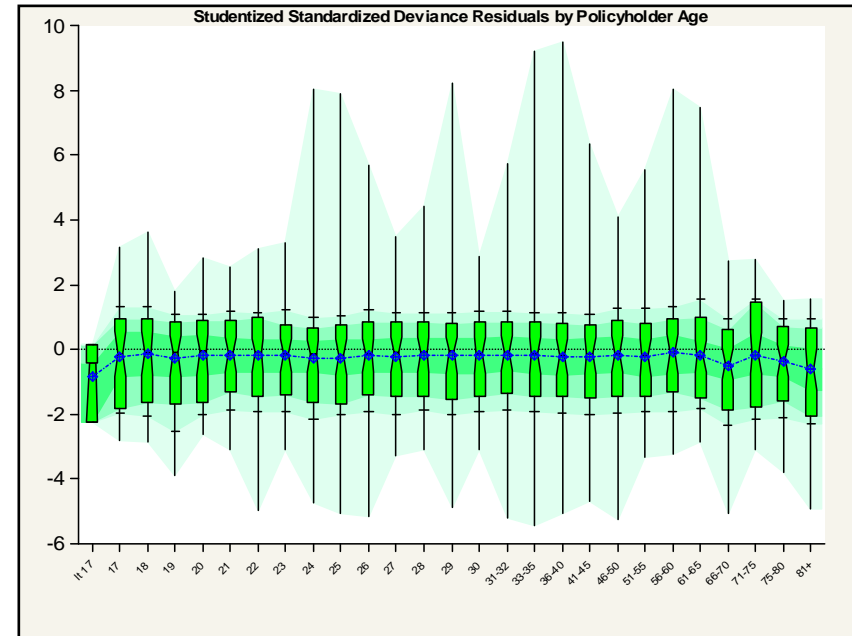
# Validate Models

## Residual analysis

- Recheck residuals to ensure appropriate shape



Is the contour plot symmetric?



Does the Box-Whisker show symmetry across levels?

## Agenda

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# Machine vs man



VS



# Machine vs man



VS





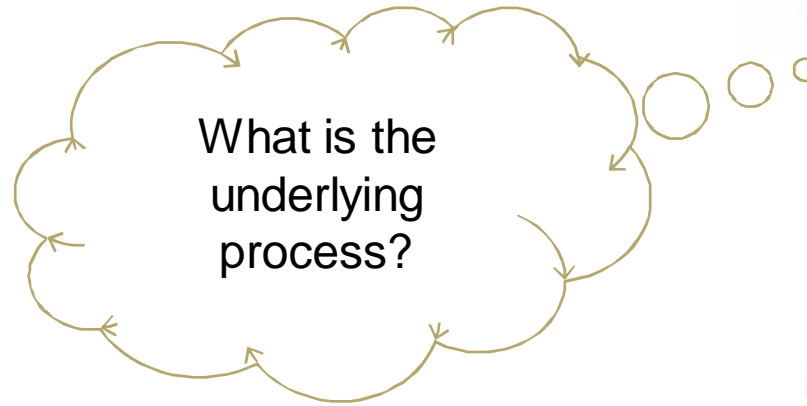
# Machine vs man



VS

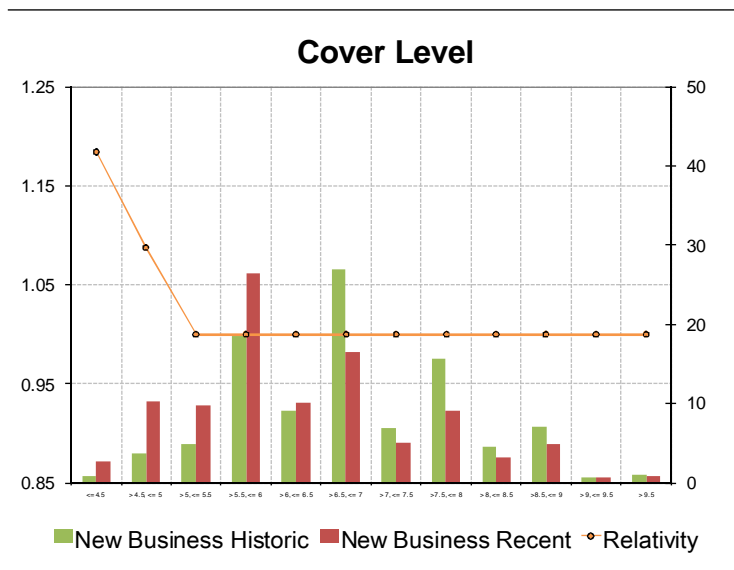


# Machine vs man

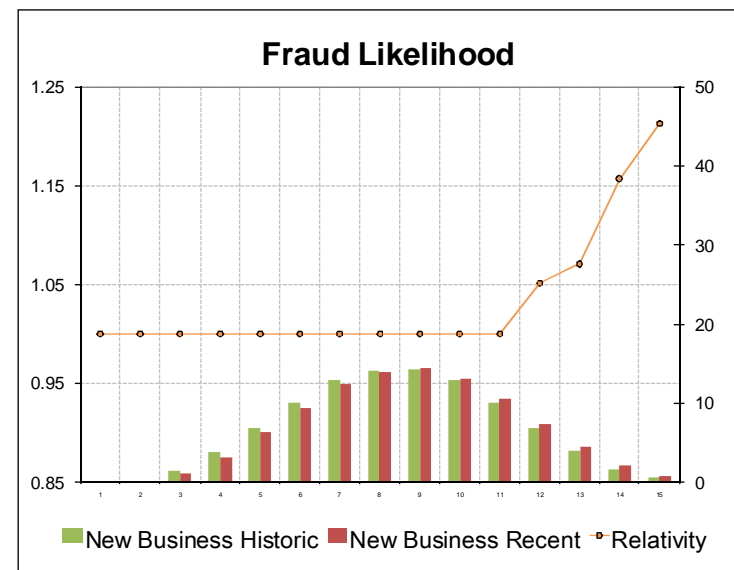


# Machine vs man

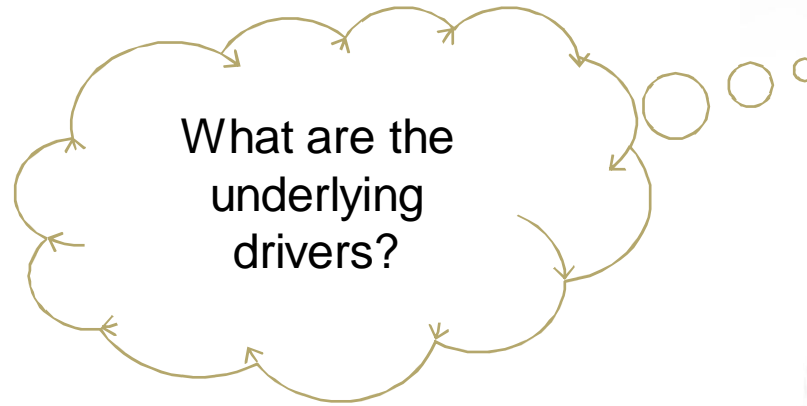
## Underwriting



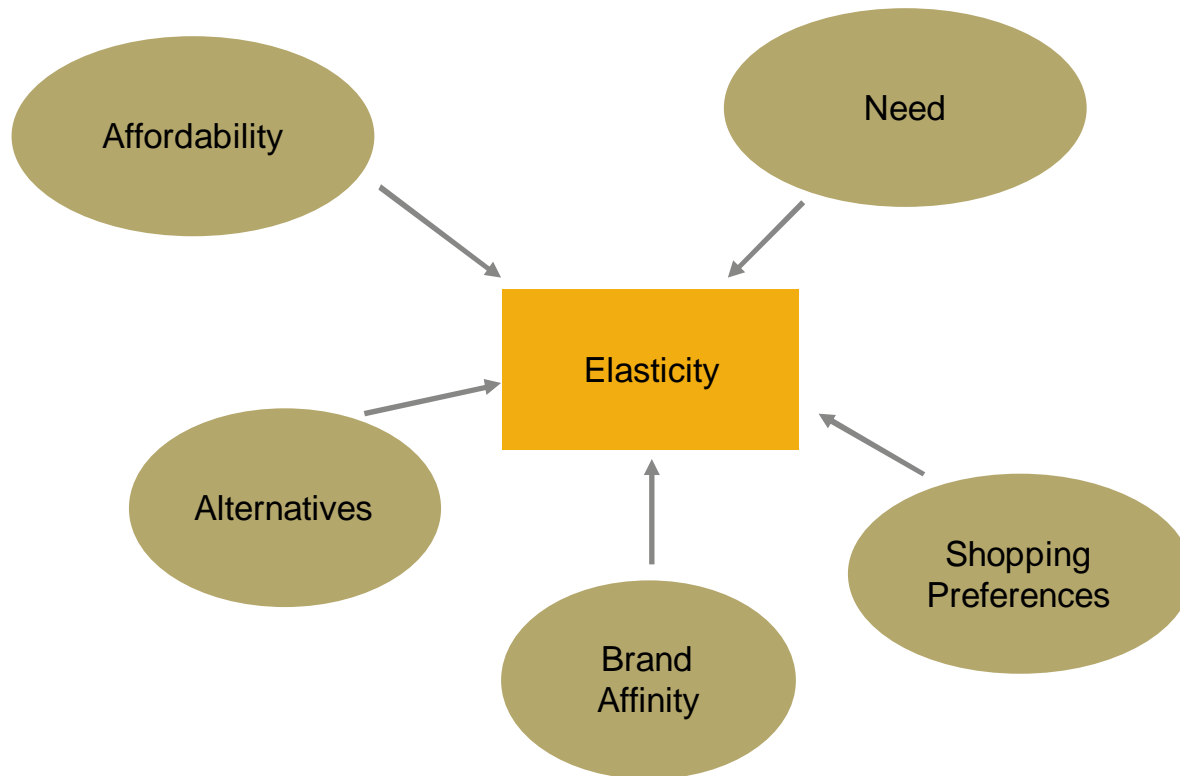
## Claims



# Machine vs man



# Drivers of elasticity

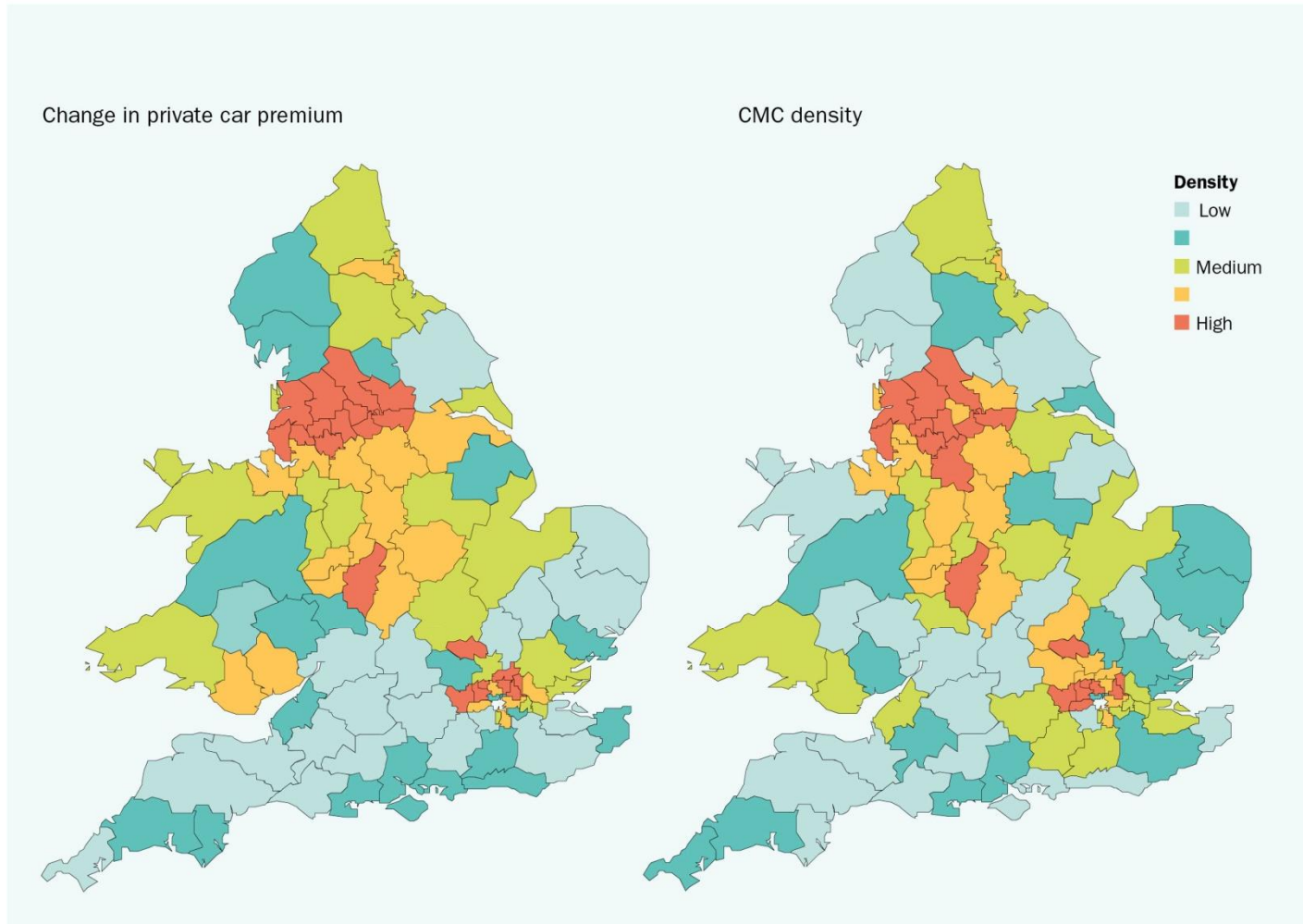


# Machine vs man

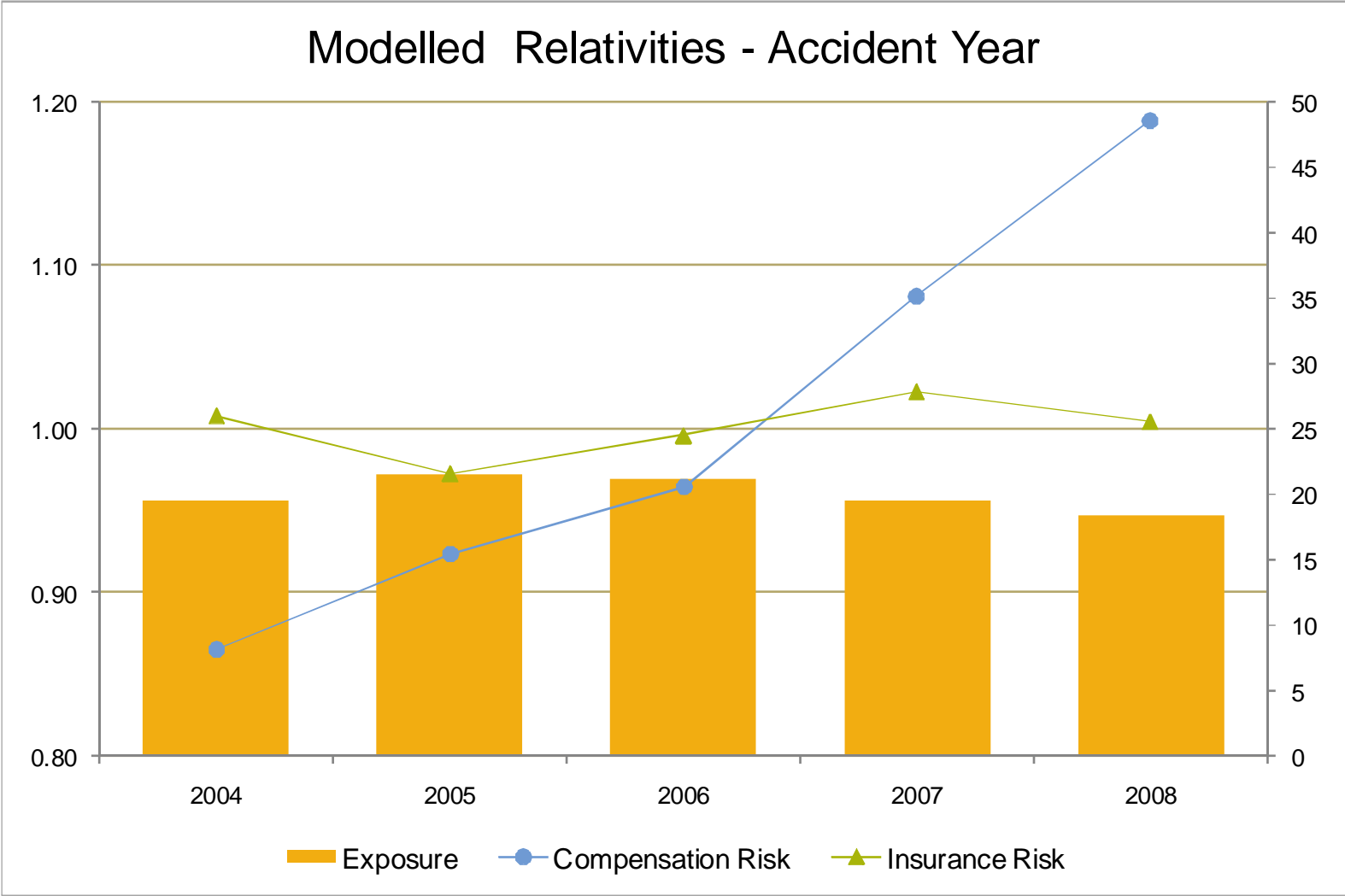
I reckon that lots of recent bodily injury changes are down to new types of claims



# Claims management companies

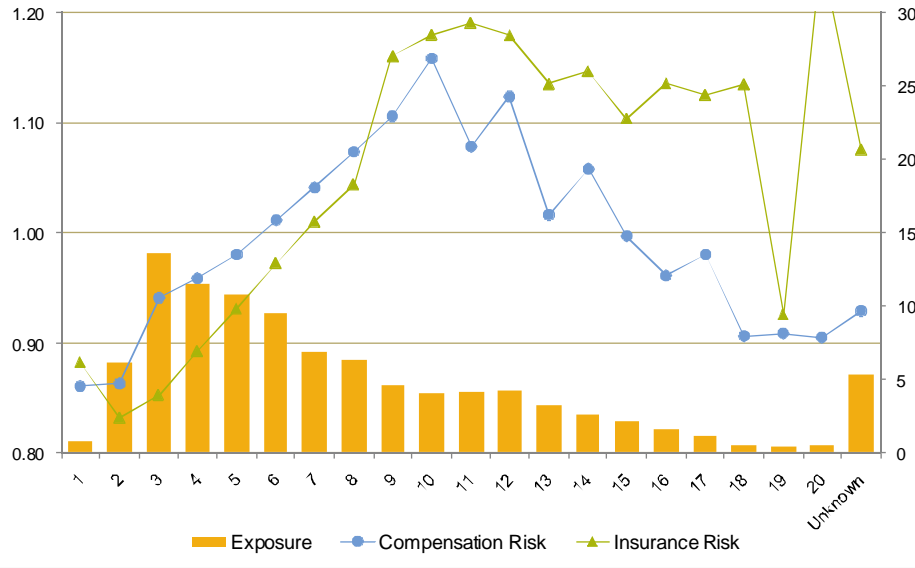


# BI models - "insurance" and "compensation" risk

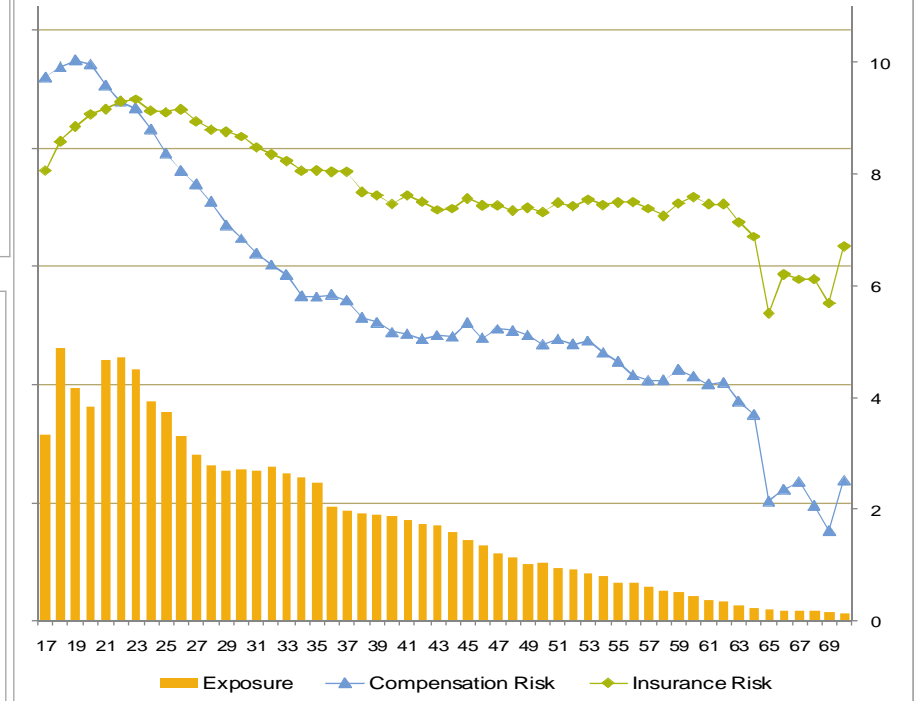




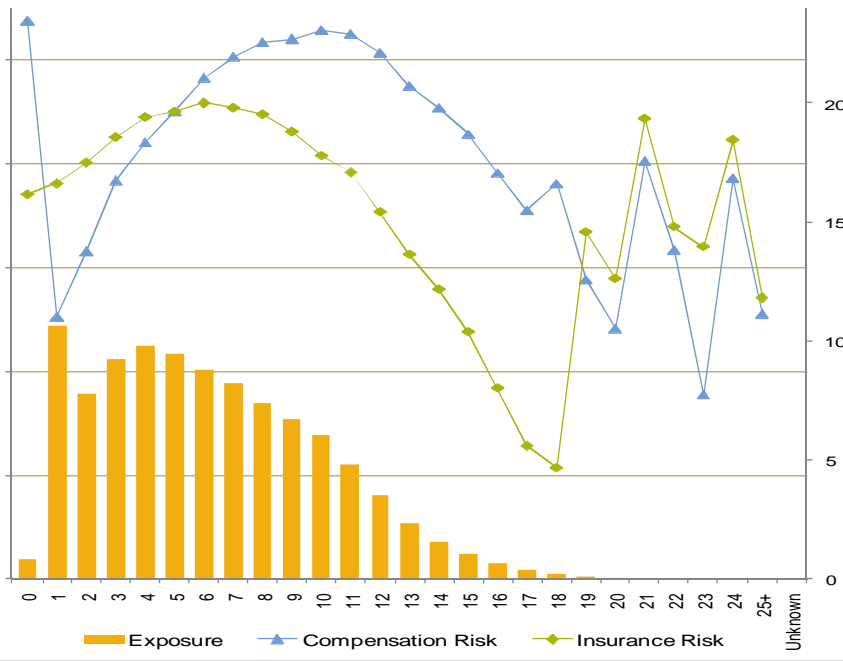
Modelled Relativities - ABI20 Vehicle Group



Proportion TPD with BI - Rated Driver Age



Proportion TPD with BI - Car Age





# GLM III - The Matrix Reloaded

Duncan Anderson, Serhat Guven